Summary Structures for Massive Data

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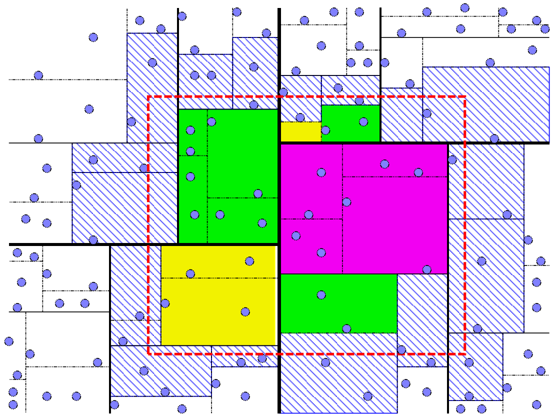
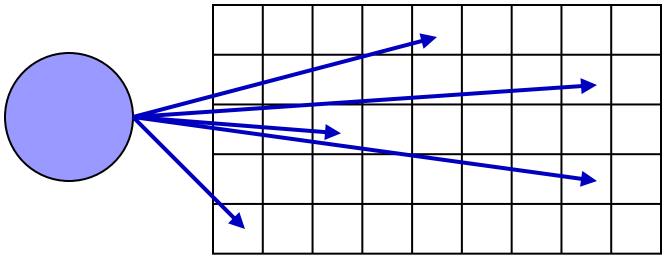
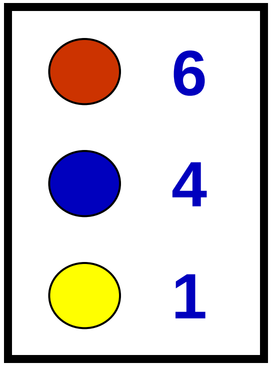






Massive Data

- "Big" data arises in many forms:
  - Physical Measurements: from science (physics, astronomy)
  - Medical data: genetic measurements, detailed time series
  - Activity data: GPS location, social network activity
  - Business data: customer behavior tracking at fine detail
- Common themes:
  - Data is large, and growing
  - There are important patterns and trends in the data
  - We don't fully know how to find them

Making sense of Big Data

- Want to be able to interrogate data in different use-cases:
  - Routine Reporting: standard set of queries to run
  - Analysis: ad hoc querying to answer ‘data science’ questions
  - Monitoring: identify when current behavior differs from old
  - Mining: extract new knowledge and patterns from data
- In all cases, need to answer certain basic questions quickly:
  - Describe the distribution of particular attributes in the data
  - How many (distinct) X were seen?
  - How many X < Y were seen?
  - Give some representative examples of items in the data

Summary Structures

- Much work on building a summary to (approximately) answer such questions
- To earn the name, should be (very) small!
  - Can keep in fast storage
- Should be able to build, update and query efficiently
- Key methods for summaries:
  - Create an empty summary
  - Update with one new tuple: streaming processing
  - Merge summaries together: distributed processing
  - Query: may tolerate some approximation

Techniques in Summaries

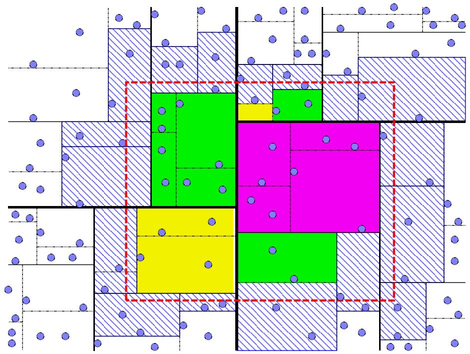
- Several broad classes of techniques generate summaries:
  - Sketch techniques: linear projections
  - Sampling techniques: (complex) random selection
  - Other special-purpose techniques
- In each class, will outline ‘classic’ and ‘recent’ results
- Conclude with “state of the union” of summaries

Random Sampling

- Basic idea: draw random sample, answer query on sample (and scale up if needed)
- Update: include new item in sample with probability $1/n$
  (and kick out an old item if sample is full)
- Merge: draw items from each input sample with the probability proportional to relative input size
- Query: run query on the sample (and possibly rescale result)

- Accuracy: answers any “predicate query” with additive error
  - E.g. What fraction of input items satisfy property X?
  - Error $+/- \varepsilon$ with 95% probability for sample size $O(1/\varepsilon^2)$

Structure-aware Sampling



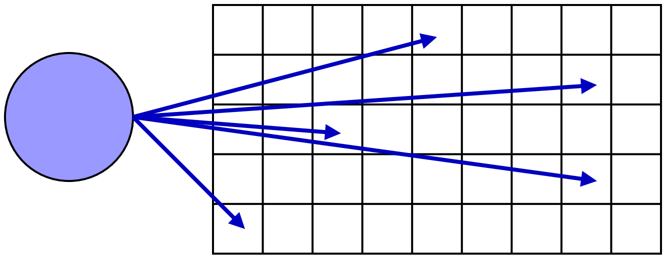
- Most queries are actually range queries:
  - "How much traffic from region X to region Y at 2am to 4am?"
- Much structure in data [Cohen, C, Duffield 11]
  - Order (e.g. ordered timestamps, durations etc.)
  - Hierarchy (e.g. geographic and network hierarchies)
  - (Multidimensional) products of structures
- Make sampling structure-aware when ejecting keys
  - Carefully pick subset of keys to subsample from
  - Empirically: constant factor improvement from same size sample

Sampling Pros and Cons

- Samples are very general, but have some limitations
- Uniform samples are no good for many problems
  - Anything to do with number of distinct items
- For some queries, other summaries have better performance
  - Technically: $O(1/\varepsilon^2)$ vs $O(1/\varepsilon)$ size
  - Practically: may be factors of 10s or 100s

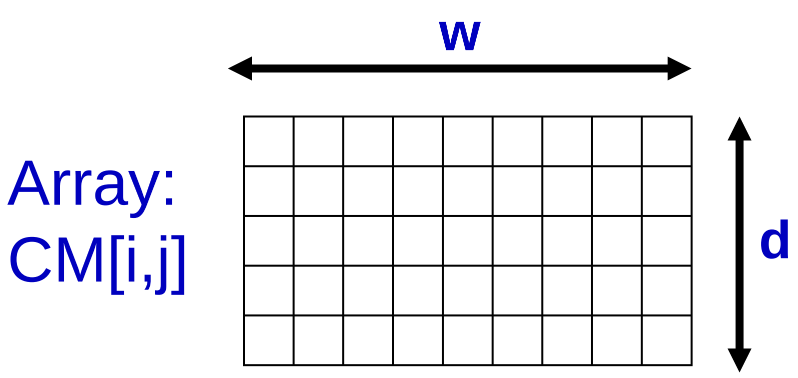
Sketch Summaries

- Subclass of summaries that are linear transforms of input
  - Merge = sum
  - Easy to extend to inputs that have negative weights
- Efficient sketches approximate quantities of interest:
  - $O(\varepsilon^{-1})$ space for point queries with ε L_1 error [CM]
  - $O(\varepsilon^{-2})$ space for point queries with ε L_2 error [CCFC]
  - $O(\varepsilon^{-2})$ space to estimate L_2 with ε relative error [AMS]

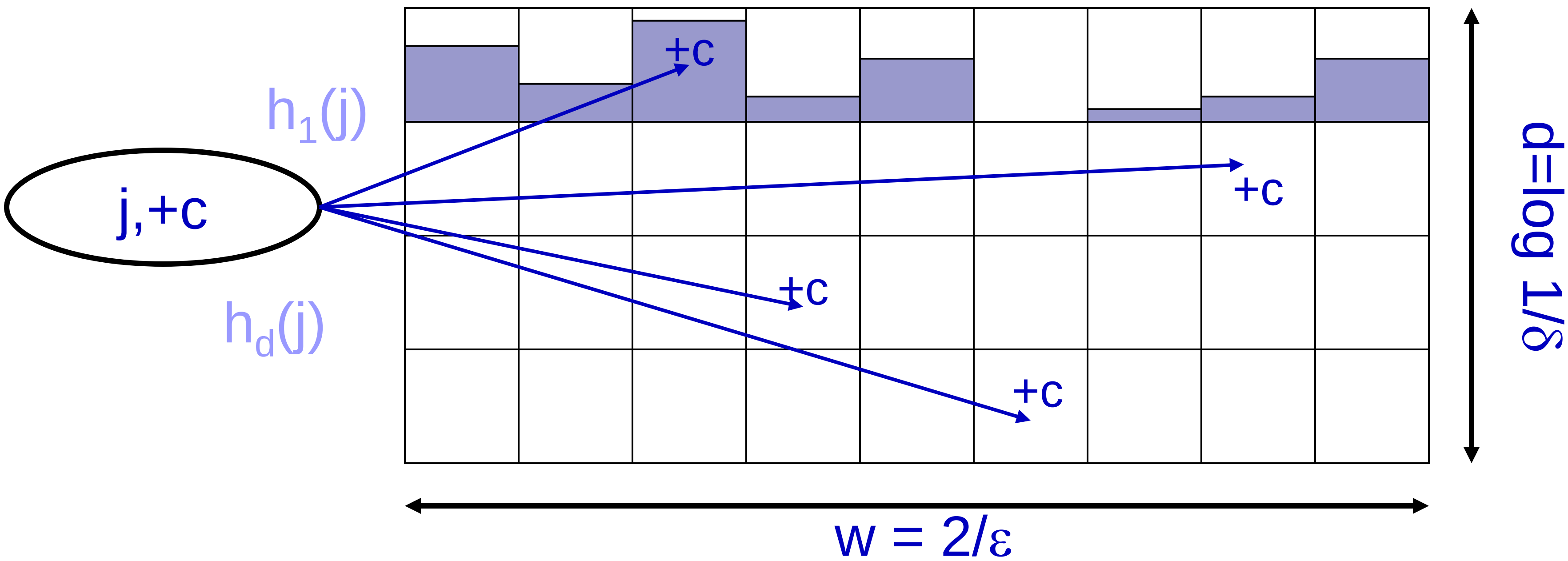


Count-Min Sketch [C, Muthukrishnan '03]

- Simple(st?) sketch idea, used in many different tasks
- Applicable when input data modeled as vector x of dimension m
- Creates a small summary as an array of $w \times d$ in size
- Use d (simple) hash function to map vector entries to [1..w]
- (Implicit) linear transform of input vector, so flexible



Count-Min Sketch Operations



- **Update**: each entry in vector x is mapped to one bucket per row
- **Merge**: combine two sketches by entry-wise summation
- **Query**: Estimate x[j] by taking $\min_k CM[k, h_k(j)]$
  - Guarantees error less than εN in size $O(1/\varepsilon \log 1/\delta)$ (Markov ineq)
  - Probability of more error is less than $1-\delta$

Lp Sampling

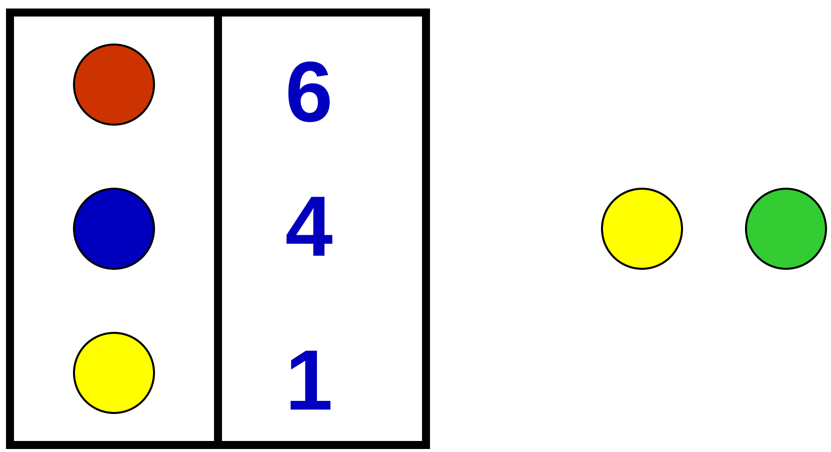
- L_p sampling: use sketches to sample i w/prob $(1\pm\varepsilon)\ f_i^p/|f|_p^p$
- "Efficient" solutions developed of size $O(\varepsilon^{-2} \log^2 n)$
  - [Monemizadeh, Woodruff 10] [Jowhari, Saglam, Tardos 11]
- Enable novel "graph sketching" techniques
  - Sketches for connectivity, sparsifiers [Ahn, Guha, McGregor 12]

- Challenge: improve space efficiency of L_p sampling
  - Empirically or analytically

Sketching Pros and Cons

- "Linear" summaries: can add, subtract, scale easily
  - Useful for forecasting models, large feature vectors in ML
- Other sketches have been designed for:
  - Count-distinct, Set sizes (Flajolet-Martin and beyond)
  - Set membership (Bloom Filter)
  - Vector operations: Euclidean norm, cosine similarity
- Some sketch types are large, slow to update (but parallel)
- Tricky to adapt to large domains (e.g. strings)
- Don't support complex operations (e.g. arbitrary queries)

Special-purpose Summaries



- Misra-Gries (MG) algorithm finds up to k items that occur more than 1/k fraction of the time in the input
- Update: Keep k different candidates in hand. For each item:
  - If item is monitored, increase its counter
  - Else, if < k items monitored, add new item with count 1
  - Else, decrease all counts by 1

Streaming MG analysis

- N = total weight of input
- M = sum of counters in data structure
- Error in any estimated count at most (N-M)/(k+1)
  - Estimated count a lower bound on true count
  - Each decrement spread over (k+1) items: 1 new one and k in MG
  - Equivalent to deleting (k+1) distinct items from stream
  - At most (N-M)/(k+1) decrement operations
  - Hence, can have “deleted” (N-M)/(k+1) copies of any item
  - So estimated counts have at most this much error

Merging two MG Summaries [ACHPWY '12]

- Merge algorithm:
  - Merge the counter sets in the obvious way
  - Take the $(k+1)$th largest counter $= C_{k+1}$, and subtract from all
  - Delete non-positive counters
  - Sum of remaining counters is M_{12}
- This keeps the same guarantee as Update:
  - Merge subtracts at least $(k+1)C_{k+1}$ from counter sums
  - So $(k+1)C_{k+1} \leq (M_1 + M_2 - M_{12})$
  - By induction, error is

    $((N_1 - M_1) + (N_2 - M_2) + (M_1 + M_2 - M_{12}))/(k+1) = ((N_1 + N_2) - M_{12})/(k+1)$

    (prior error) (from merge) (as claimed)

Special Purpose Summaries: Pros and Cons

- Tend to work very well for their target domain
- But only work for certain problems, not general
- Other special purpose summaries for:
  - Summarize distributions (medians): q-digest, GK summary
  - Graph distances, connectivity: limited results so far
  - (Multidimensional) geometric data: for clustering, range queries
    - Coresets, ε-approximations, ε-kernels, ε-nets

Applications shown for Summaries

- Machine learning over huge numbers of features
- Data mining: scalable anomaly/outlier detection
- Database query planning
- Password quality checking [HSM 10]
- Large linear algebra computations
- Cluster computations (MapReduce)
- Distributed Continuous Monitoring
- Privacy preserving computations
- … [Your application here?]

More speculative

Summary of Summary Issues

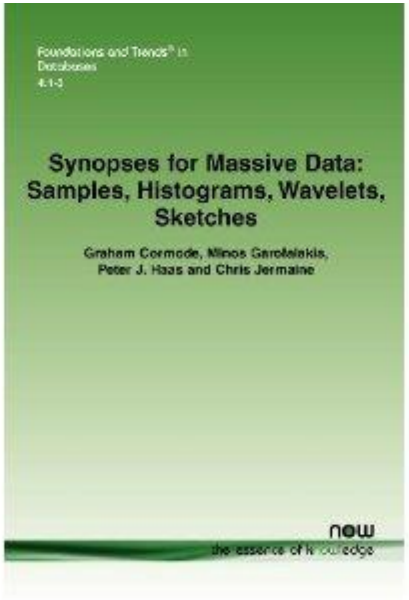
Strengths

- (Often) easy to code and use
  - Can be easier than exact algs
- Small — cache-friendly
  - So can be very fast
- Open source implementations
  - (maybe barebones, rigid)
- Easily teachable
  - As intro to probabilistic analysis
- (Mostly) highly parallel

Weaknesses

- (Still) resistance to random, approx algs
  - Less so for Bloom filter, hashes
- Memory/disk is cheap
  - So can do it the slow way
- Not yet in standard libraries
  - Developing: MadLib, Stream-lib
- Not yet in courses / textbooks
  - "this CM sketch sounds like the bomb! (although I have not heard of it before)"
- Few *public* success stories

Resources



- Sample implementations on web
  - Ad hoc, of varying quality
- Technical descriptions
  - Original papers
  - Surveys, comparisons
- (Partial) wikis and book chapters
  - Wiki: **`sites.google.com/site/countminsketch/`**
  - "Sketch Techniques for Approximate Query Processing"
    `dimacs.rutgers.edu/~graham/pubs/papers/sk.pdf`

Example: Bloom Filters (1970)

- A well-known and widely used summary
- Bloom filters compactly encode set membership
  - Create: Pick k hash functions to map items to (empty) bit vector
  - Update: Hash and set k entries to **1** to indicate item is present
  - Merge: Take bit-wise OR of two Bloom Filter vectors
  - Query: Hash item to vector, assume in set if all k entries are 1
- Analysis: store set size n in ~10n bits with few false positive