Finding Frequent Items in Data Streams

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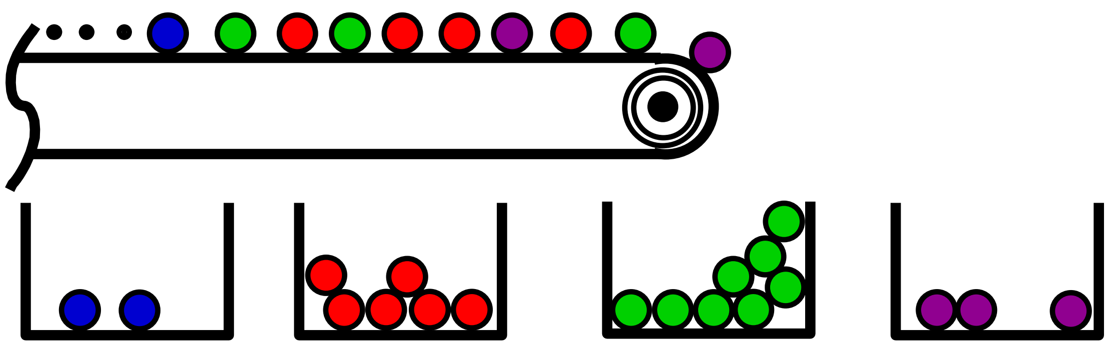
Data Streams

- Many large sources of data are best modeled as data streams
  - E.g. streams of network packets, defining traffic distributions
- Impractical and undesirable to store and process all data exactly
- Instead, seek algorithms to find approximate answers
  - With one pass over data, quickly build a small summary
- Active research area for last decade, history goes back 30 years

The Frequent Items Problem

- The Frequent Items Problem (aka Heavy Hitters): given stream of N items, find those that occur most frequently
- E.g. Find all items occurring more than 1% of the time
- Formally "hard" in small space, so allow approximation
- Find all items with count $\geq \phi N$, none with count $< (\phi - \varepsilon)N$
  - Error $0 < \varepsilon < 1$, e.g. $\varepsilon = 1/1000$
  - Related problem: estimate each frequency with error $\pm\varepsilon N$



Why Frequent Items?

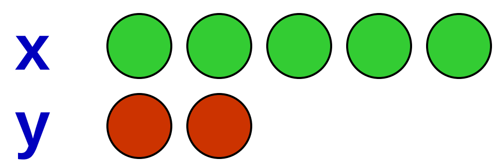
- A natural question on streaming data
  - Track bandwidth hogs, popular destinations etc.
- The subject of much streaming research
  - Scores of papers on the subject
- A core streaming problem
  - Many streaming problems connected to frequent items (itemset mining, entropy estimation, compressed sensing)
- Many practical applications
  - Search log mining, network data analysis, DBMS optimization

This Talk

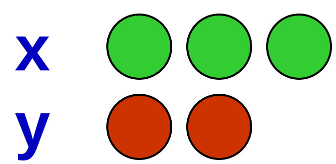
- A brief history of the frequent items problem
- A tour of some of the most popular algorithms
  - Counter-based algorithms: Frequent, LossyCounting, SpaceSaving
  - Sketch algorithms: Count-Min Sketch, Count Sketch
- Experimental comparison of algorithms
- Extensions, new results and future directions

Data Stream Models

- We model data streams as sequences of simple tuples
- Complexity arises from massive length of streams
- Arrivals only streams:
  - Example: (x, 3), (y, 2), (x, 2) encodes the arrival of 3 copies of item x, 2 copies of y, then 2 copies of x.



  - Could represent eg. packets on a network; power usage
- Arrivals and departures:
  - Example: (x, 3), (y,2), (x, -2) encodes final state of (x, 1), (y, 2).



  - Can represent fluctuating quantities, measure differences between two distributions, or represent general signals

The Start of The Problem?

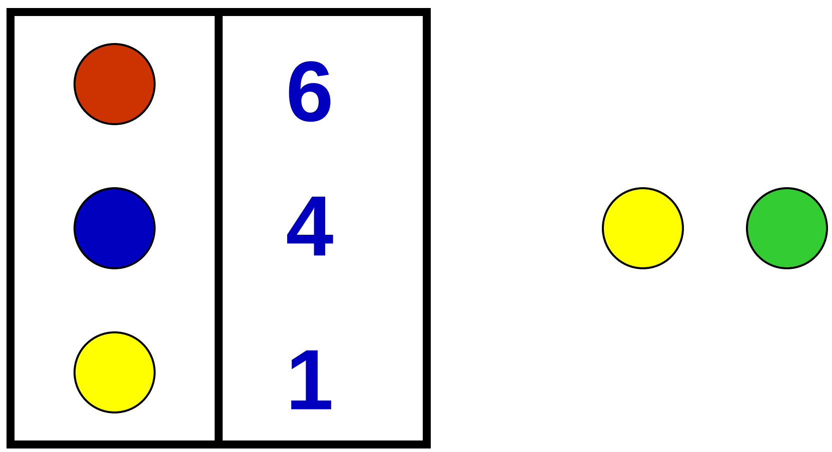
[J.Alg 2, P208-209] Suppose we have a list of *n* numbers, representing the "votes" of *n* processors on the result of some computation. We wish to decide if there is a majority vote and what the vote is.

- Problem posed by J. S. Moore in Journal of Algorithms, in 1981
- Does not require a streaming solution, but first solutions were

MAJORITY algorithm

- MAJORITY algorithm solves the problem in arrivals only model
- Start with a counter set to zero. For each item:
  - If counter is zero, pick up the item, set counter to 1
  - Else, if item is same as item in hand, increment counter
  - Else, decrement counter
- If there is a majority item, it is in hand
  - Proof outline: each decrement pairs up two different items and cancels them out
  - Since majority occurs > N/2 times, not all of its occurrences can be canceled out.

“Frequent” algorithm



- FREQUENT generalizes MAJORITY to find up to k items that occur more than 1/k fraction of the time
- Keep k different candidates in hand. For each item in stream:
  - If item is monitored, increase its counter
  - Else, if < k items monitored, add new item with count 1
  - Else, decrease all counts by 1

Frequent Analysis

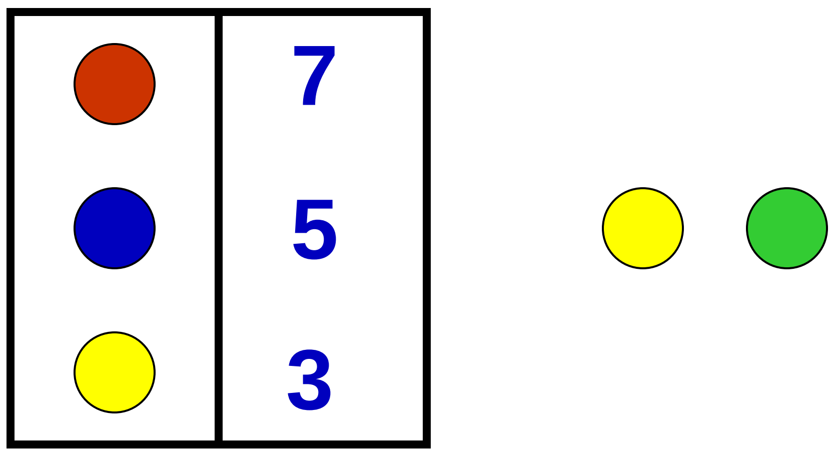
- Analysis: each decrease can be charged against k arrivals of different items, so no item with frequency N/k is missed
- Moreover, $k=1/\varepsilon$ counters estimate frequency with error εN
  - Not explicitly stated until later [Bose et al., 2003]

- Some history: First proposed in 1982 by Misra and Gries, rediscovered twice in 2002
  - Later papers showed how to make fast implementations

Lossy Counting

- LossyCounting algorithm proposed in [Manku, Motwani '02]
- Simplified version:
  - Track items and counts
  - For each block of $1/\varepsilon$ items, merge with stored items and counts
  - Decrement all counts by one, delete items with zero count
- Easy to see that counts are accurate to εN
- Analysis shows $O(1/\varepsilon \log \varepsilon N)$ items are stored
- Full version keeps extra information to reduce error

SpaceSaving Algorithm



- "SpaceSaving" algorithm [Metwally, Agrawal, El Abaddi 05] merges Lossy Counting and FREQUENT algorithms
- Keep k = 1/ε item names and counts, initially zero
  Count first k distinct items exactly
- On seeing new item:
  - If it has a counter, increment counter
  - If not, replace item with least count, increment count

SpaceSaving Analysis

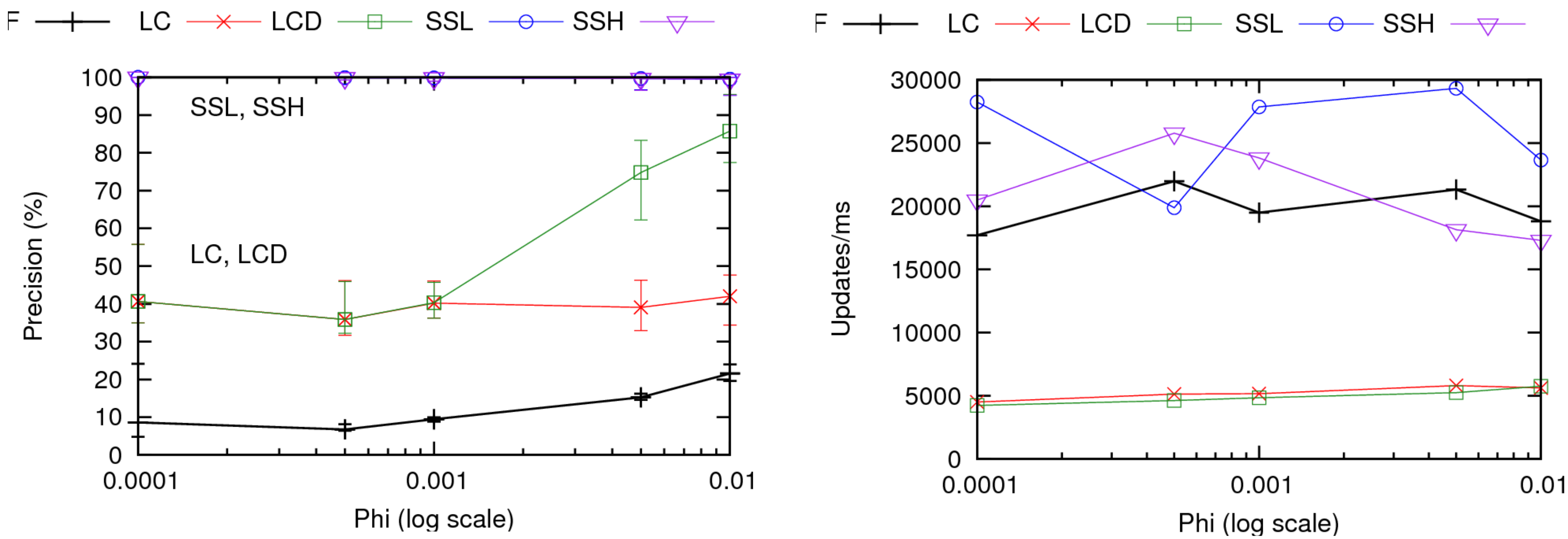
- Smallest counter value, min, is at most εn
  - Counters sum to n by induction
  - $1/\varepsilon$ counters, so average is εn: smallest cannot be bigger
- True count of an uncounted item is between 0 and min
  - Proof by induction, true initially, min increases monotonically
  - Hence, the count of any item stored is off by at most εn
- Any item x whose true count $> \varepsilon n$ is stored
  - By contradiction: x was evicted in past, with count $\leq min_t$
  - Every count is an overestimate, using above observation
  - So est. count of $x > \varepsilon n \geq min \geq min_t$, and would not be evicted

So: Find all items with count $> \varepsilon n$, error in counts $\leq \varepsilon n$

Experimental Comparison

- Implementations of all these algorithms (and more!) at **http://www.research.att.com/~marioh/frequent-items**
- Experimental comparison highlights some differences not apparent from analytic study
  - All counter algorithms seem to have similar worst-case performance ($O(1/\varepsilon)$ space to give εN guarantee)
  - Algorithms are often more accurate than analysis would imply
- Compared on a variety of web, network and synthetic data

Counter Algorithms Experiments



- Two implementations of SpaceSaving (SSL, SSH) achieve perfect accuracy in small space (10KB – 1MB)
- Very fast: 20M – 30M updates *per second*

Counter Algorithms Summary

- Counter algorithms very efficient for arrivals-only case
  - Use $O(1/\varepsilon)$ space, guarantee εN accuracy
  - Very fast in practice (many millions of updates per second)
- Similar algorithms, but a surprisingly clear "winner"
  - Over many data sets, parameter settings, SpaceSaving algorithm gives appreciably better results
- Many implementation details even for simple algorithms
  - "Find if next item is monitored": search tree, hash table…?
  - "Find item with smallest count": heap, linked lists…?
- Not much room left for improvement in core problem?

Outline

- Problem definition and background
- “Counter-based” algorithms and analysis
- “Sketch-based” algorithms and analysis
- Further Results
- Conclusions

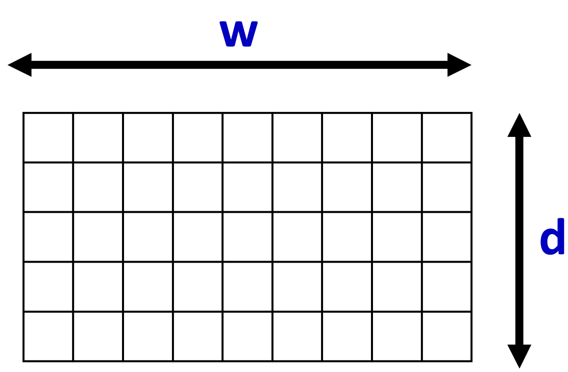
Sketch Algorithms

- Counter algorithms are for the "arrivals only" model, do not handle "arrivals and departures"
  - Deterministic solutions not known for the most general case
- Sketch algorithms compute a summary that is a linear transform of the frequency vector
  - Departures are naturally handled by such algorithms
- Sketches solve core problem of estimating item frequencies
  - Can then use to find frequent items via search algorithm

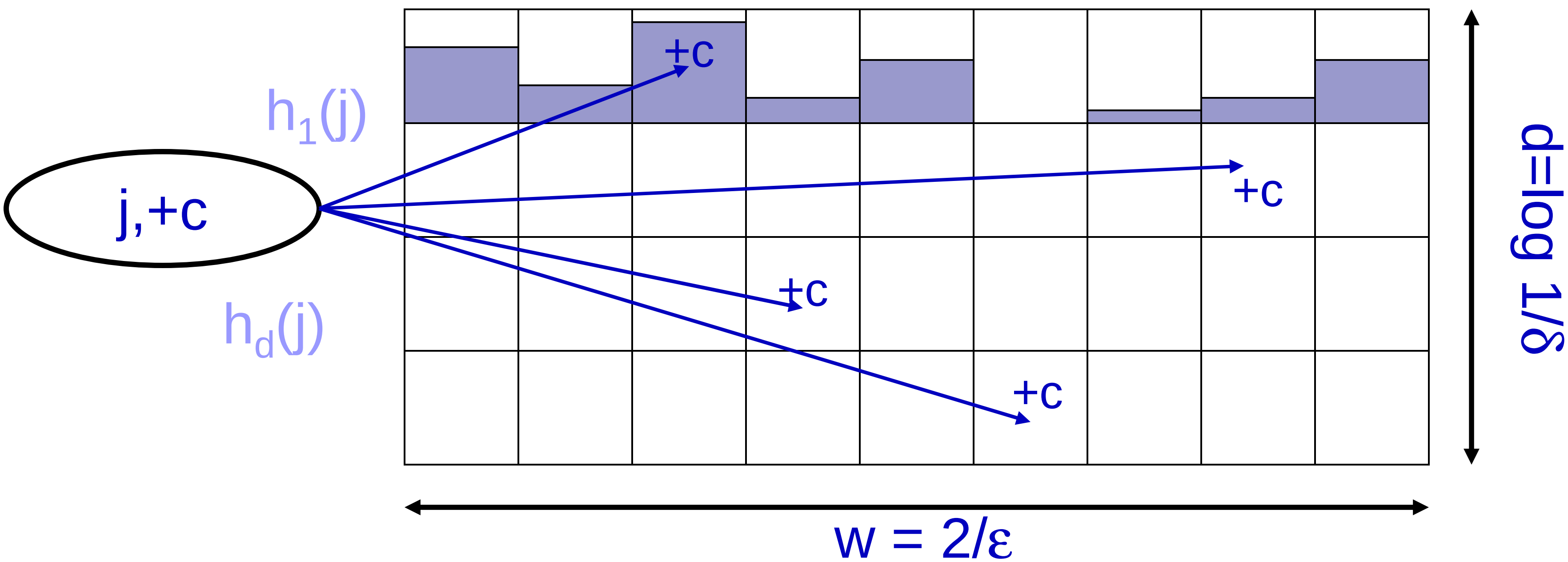
Count-Min Sketch

- Count-Min Sketch proposed in [C, Muthukrishnan '04]
- Model input stream as a vector x of dimension U
  - x[i] is frequency of item i
- Creates a small summary as an array of $w \times d$ in size
- Use d hash function to map vector entries to [1..w]

Array:
CM[i,j]



Count-Min Sketch Structure



- Each entry in vector x is mapped to one bucket per row.
- Estimate $x[j]$ by taking $\min_k CM[k,h_k(j)]$
  - Guarantees error less than $\varepsilon||x||_1$ in size $O(1/\varepsilon \log 1/\delta)$
  - Probability of more error is less than $1-\delta$

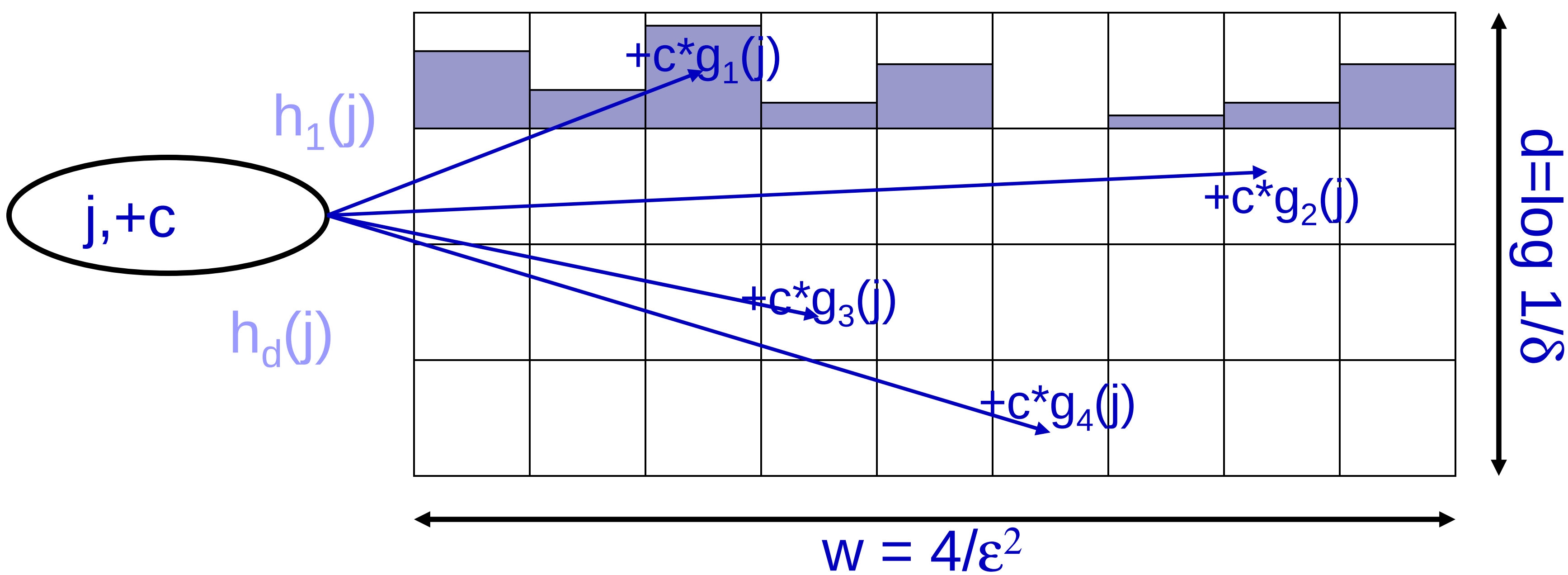
Count-Min Sketch Analysis

Approximate $x'[j] = \min_k CM[k,h_k(j)]$

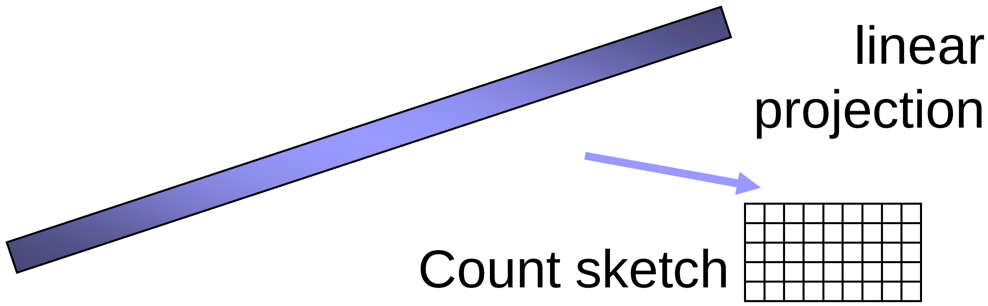
- Analysis: In k'th row, $CM[k,h_k(j)] = x[j] + X_{k,j}$
  - $X_{k,j} = \Sigma\, x[i] \mid h_k(i) = h_k(j)$
  - $E(X_{k,j}) \quad = \Sigma\, x[k]{*}Pr[h_k(i)=h_k(j)]$
    $\leq Pr[h_k(i)=h_k(k)] * \Sigma\, a[i]$
    $= \varepsilon\, ||x||_1/2$ by pairwise independence of h
  - $Pr[X_{k,j} \geq \varepsilon||x||_1] = Pr[X_{k,j} \geq 2E(X_{k,j})] \leq 1/2$ by Markov inequality
- So, $Pr[x'[j] \geq x[j] + \varepsilon||x||_1] = Pr[\forall\, k.\ X_{k,j} > \varepsilon||x||_1] \leq 1/2^{\log 1/\delta} = \delta$
- Final result: with certainty $x[j] \leq x'[j]$ and
  with probability at least $1-\delta$, $x'[j] < x[j] + \varepsilon||x||_1$
  - Estimate is biased, can correct easily

Count Sketch

- Count Sketch proposed in [Charikar, Chen, Farach-Colton '02]
- Uses extra hash functions $g_1 \ldots g_{\log 1/\delta}$ $\{1\ldots U\} \rightarrow \{+1,-1\}$
- Now, given update (j,+c), set $CM[k,h_k(j)]$ += $c^*g_k(j)$



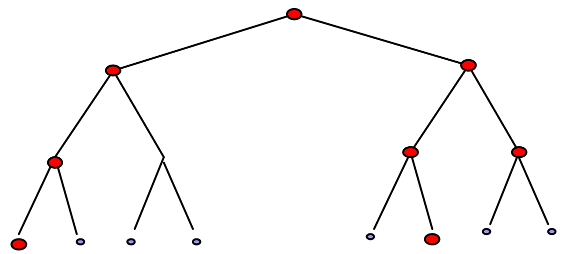
Count Sketch Analysis



- Estimate $x'_k[j] = CM[k,h_k(j)]*g_k(j)$
- Analysis shows estimate is correct in expectation
- Bound error by analyzing the variance of the estimator
  - Apply Chebyshev inequality on the variance
- With probability $1-\delta$, error is at most $\varepsilon||x||_2 < \varepsilon N$
  - $||x||_2$ could be much smaller than N, at cost of $1/\varepsilon^2$

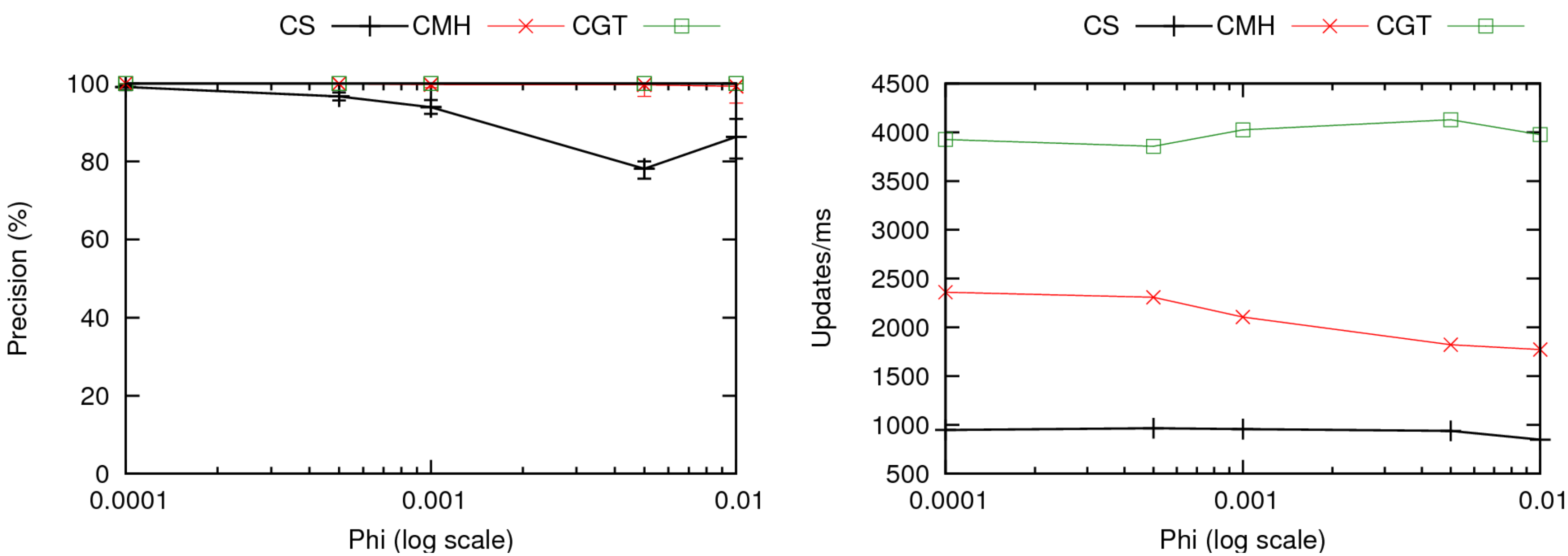
Hierarchical Search

- Sketches estimate the frequency of a single item
  - How to find frequent items without trying all items?
- Divide-and-conquer approach limits search cost
  - Impose a binary tree over the domain
  - Keep a sketch of each level of the tree
  - Descend if a node is heavy, else stop



- Correctness: all ancestors of a frequent item are also frequent
- Alternate approach based on "group testing"
  - Use sketches to determine identities of frequent items by running multiple tests.

Sketch Algorithms Experiments



- Less clear which sketch is best: depends on data, parameters
- Speed less by factor of 10, size more by factor 10:
  - A necessary trade off for flexibility to handle departures?

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Tighter Bounds

- Observation: algorithms outperform worst case guarantees
- Analysis: can prove stronger guarantees than εN
  - Define n_1 = highest frequency, n_2 = second highest, etc.
  - Then define $F_1^{res(k)} = N - (n_1 + n_2 + \ldots n_k)$, $\ll N$ for skewed dbns
  - Result [Berinde, C, Indyk, Strauss, '09] :
    Frequent, SpaceSaving (and others) guarantee $\varepsilon F_1^{res(k)}$ error
- Similar bounds for sketch algorithms
  - CountMin sketch also has $F_1^{res(k)}$ bound
  - Count sketch has $(F_2^{res(k)})^{1/2} = (\sum_{i=k+1}^{m} n_i^2)^{1/2}$ bound
  - Related to results in Compressed Sensing for signal recovery

Weighted Updates

- Weighted case: find items whose total weight is high
  - Sketch algorithms adapt easily, counter algs with effort
- Simple solution: all weights are integer multiples of small δ
- Full solution: define appropriate generalizations of counter algs to handle real valued weights [Berinde et al '09]
  - Straightforward to extend SpaceSaving analysis to weighted case
  - Frequent more complex, action depends on smallest counter value
  - No positive results known for LossyCounting

Mergability of Summaries

- Want to merge summaries, to summarize the union of streams
- Sketches with shared hash fns are easy to merge together
  - Via linearity, sum of sketches = sketch of sums
- Counter-based algorithms need new analysis [Berinde et al'09]
  - Merging two summaries preserves accuracy, but space may grow
  - With pruning of the summary, can merge indefinitely
  - Space remains bounded, accuracy degrades by at most a constant

Other Extensions

- Heavy Changers
  - Which items have largest (absolute, relative) change over two streams?
- Assumptions on frequency distribution, order
  - Give tighter space/accuracy tradeoff for skewed distributions
  - Worst case arrival order vs. random arrival order
- Distinct Heavy Hitters
  - E.g. which sources contact the most distinct addresses?
- Time Decay
  - “Weight” of items decay (exponentially, polynomially) with age

Conclusions

- Finding the frequent items is one of the most studied problems in data streams
  - Continues to intrigue researchers
  - Many variations proposed
  - Algorithms have been deployed in Google, AT&T, elsewhere…
- Still some room for innovation, improvements

- Survey and experiments in VLDB [C, Hadjieleftheriou '08]
  - Code, synthetic data and test scripts at **`http://www.research.att.com/~marioh/frequent-items`**
  - Shorter, broader write up in CACM 2009