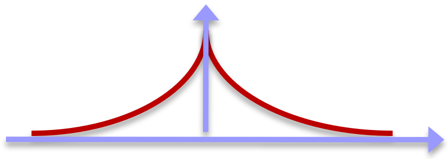
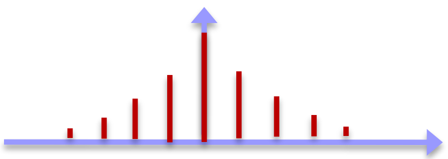
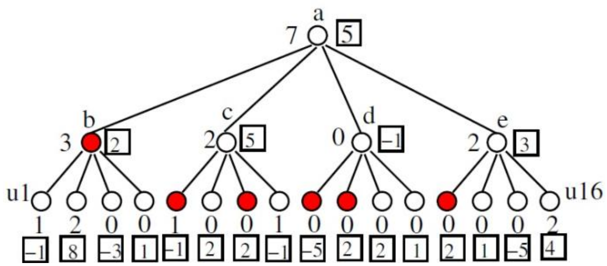




Building Blocks of Privacy: Differentially Private Mechanisms



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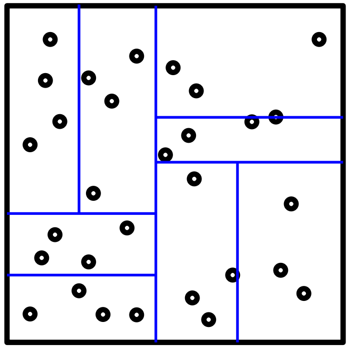
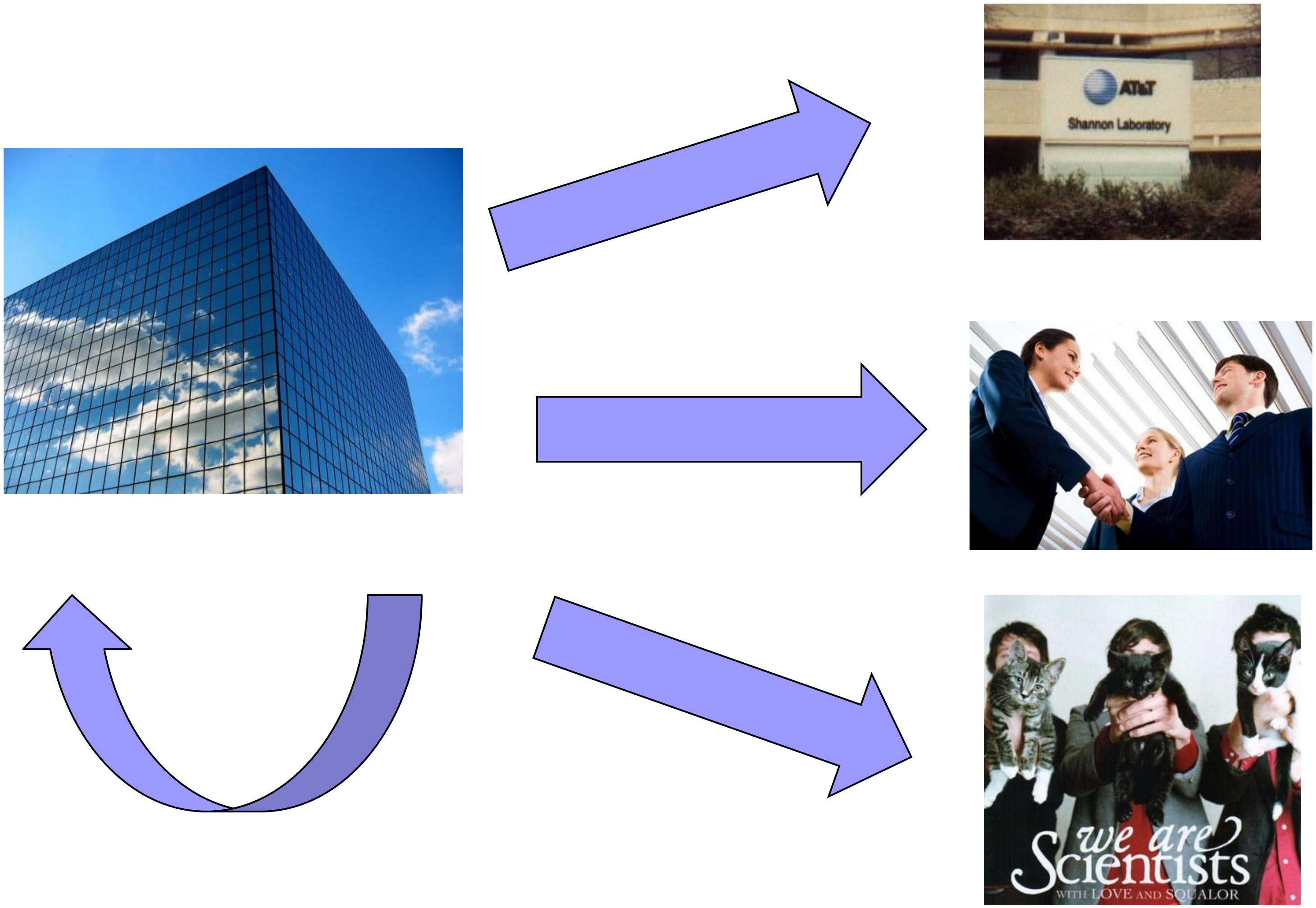


The data release scenario



Data Release

- Much interest in private data release
  - Practical: release of AOL, Netflix data etc.
  - Research: hundreds of papers
- In practice, many data-driven concerns arise:
  - How to design algorithms with a meaningful privacy guarantee?
  - Trading off noise for privacy against the utility of the output?
  - Efficiency / practicality of algorithms as data scales?
  - How to interpret privacy guarantees?
  - Handling of common data features, e.g. sparsity?
- This talk: describe some tools to address these issues

Differential Privacy

- Principle: released info reveals little about any individual
  - Even if adversary knows (almost) everything about everyone else!
- Thus, individuals should be secure about contributing their data
  - What is learnt about them is about the same either way
- Much work on providing differential privacy (DP)
  - Simple recipe for some data types e.g. numeric answers
  - Simple rules allow us to reason about composition of results
  - More complex algorithms for arbitrary data (many DP mechanisms)
- Adopted and used by several organizations:
  - US Census, Common Data Project, Facebook (?)

Differential Privacy Definition

The output distribution of a differentially private algorithm changes very little whether or not any individual's data is included in the input – so you should contribute your data

A randomized algorithm K satisfies ε-differential privacy if:
Given any pair of neighboring data sets,
D and D', and S in Range(K):

$$\Pr[K(D) = S] \leq e^{\varepsilon} \Pr[K(D') = S]$$

Neighboring datasets differ in one individual: we say $|D{-}D'|=1$

Achieving Differential Privacy

- Suppose we want to output the number of left-handed people in our data set
  - Can reduce the description of the data to just the answer, n
  - Want a randomized algorithm $K(n)$ that will output an integer
  - Consider the distribution $\Pr[K(n) = m]$ for different m
- Write $\exp(\varepsilon) = \alpha$, and $\Pr[K(n) = n] = p_n$. Then:

  $\Pr[K(n) = n-1] \leq \alpha \Pr[K(n-1)=n-1] = \alpha\, p_{n-1}$

  $\Pr[K(n) = n-2] \leq \alpha \Pr[K(n-1) = n-2] \leq \alpha^2 \Pr[K(n-2)=n-2] = \alpha^2\, p_{n-2}$

  $\Pr[K(n) = n-i] \leq \alpha^i\, p_{n-i}$

  Similarly, $\Pr[K(n) = n+i] \leq \alpha^i\, p_{n+i}$

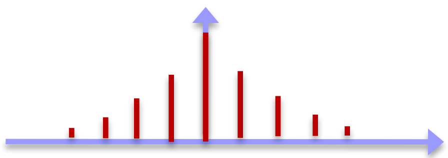
Achieving Differential Privacy

- We have $Pr[K(n) = n\text{-}i] \leq \alpha^i p_{n-i}$ and $Pr[K(n) = n+i] \leq \alpha^i p_{n+i}$
- Within these constraints, we want to maximize p_n
  - This maximizes the probability of returning "correct" answer
  - Means we turn the inequalities into equalities
- For simplicity, set $p_n = p$ for all n
  - Means the distribution of "shifts" is the same whatever n is
- Yields: $Pr[K(n) = n\text{-}i] = \alpha^i p$ and $Pr[K(n) = n+i] \leq \alpha^i p$
  - Sum over all shifts i:

$$p + \sum_{i=1}^{\infty} 2\alpha^i p = 1$$
$$p + 2p\,\alpha/(1-\alpha) = 1$$
$$p(1 - \alpha + 2\alpha)/(1-\alpha) = 1$$
$$p = (1-\alpha)/(1+\alpha)$$

Geometric Mechanism

- What does this mean?
  - For input n, output distribution is $\Pr[K(n) = m] = \alpha^{|m-n|} \cdot (1-\alpha)/(1+\alpha)$
- What does this look like?



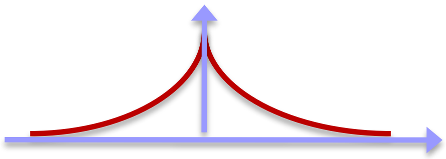
  - Symmetric geometric distribution, centered around n
  - We draw from this distribution centered around zero, and add to the true answer
  - We get the "true answer plus (symmetric geometric) noise"
- A first differentially private mechanism for outputting a count
  - We call this "the geometric mechanism"

Truncated Geometric Mechanism

- Some practical concerns:
  - This mechanism could output any value, from $-\infty$ to $+\infty$
- Solution: we can "truncate" the output of the mechanism
  - E.g. decide we will never output any value below zero, or above N
  - Any value drawn below zero is "rounded up" to zero
  - Any value drawn above N is "rounded down" to N
  - This does not affect the differential privacy properties
  - Can directly compute the closed-form probability of these outcomes

Laplace Mechanism

- Sometimes we want to output real values instead of integers
- The Laplace Mechanism naturally generalizes Geometric



  - Add noise from a symmetric continuous distribution to true answer
  - Laplace distribution is a symmetric exponential distribution
  - Is DP for same reason as geometric: shifting the distribution changes the probability by at most a constant factor
  - PDF: $Pr[X = x] = 1/2\lambda \exp(-|x|/\lambda)$
    Variance $= 2\lambda^2$

Sensitivity of Numeric Functions

- For more complex functions, we need to calibrate the noise to the influence an individual can have on the output
  - The (global) sensitivity of a function F is the maximum (absolute) change over all possible adjacent inputs
  - $S(F) = \max_{D, D' : |D-D'|=1} |F(D) - F(D')| = 1$
  - Intuition: S(F) characterizes the scale of the influence of one individual, and hence how much noise we must add
- S(F) is small for many common functions
  - S(F) = 1 for COUNT
  - S(F) = 2 for HISTOGRAM
  - Bounded for other functions (MEAN, covariance matrix…)

Laplace Mechanism with Sensitivity

- Release $F(x) + Lap(S(F)/\varepsilon)$ to obtain ε-DP guarantee
  - $F(x)$ = true answer on input x
  - $Lap(\lambda)$ = noise sampled from Laplace dbn with parameter λ
  - Exercise: show this meets ε-differential privacy requirement
- Intuition on impact of parameters of differential privacy (DP):
  - Larger $S(F)$, more noise (need more noise to mask an individual)
  - Smaller ε, more noise (more noise increases privacy)
  - Expected magnitude of $|Lap(\lambda)|$ is (approx) $1/\lambda$

Sequential Composition

- What happens if we ask multiple questions about same data?
  - We reveal more, so the bound on ε differential privacy weakens
- Suppose we output via K_1 and K_2 with ε_1, ε_2 differential privacy:

$$\Pr[\ K_1(D) = S_1\ ] \leq \exp(\varepsilon_1) \Pr[K_1(D') = S_1] \text{, and}$$

$$\Pr[\ K_2(D) = S_2\ ] \leq \exp(\varepsilon_2) \Pr[K_2(D') = S_2]$$

$$\begin{aligned}\Pr[\ (K_1(D) = S_1), (K_2(D) = S_2)] &= \Pr[K_1(D)=S_1] \Pr[K_2(D) = S_2] \\ &\leq \exp(\varepsilon_1) \Pr[K_1(D') = S_1] \exp(\varepsilon_2) \Pr[K_2(D') = S_2] \\ &= \exp(\varepsilon_1 + \varepsilon_2) \Pr[(K_1(D') = S_1), (K_2(D') = S_2)]\end{aligned}$$

  - Use the fact that the noise distributions are independent
- Bottom line: result is $\varepsilon_1 + \varepsilon_2$ differentially private
  - Can reason about sequential composition by just “adding the ε’s”

Parallel Composition

- Sequential composition is pessimistic
  - Assumes outputs are correlated, so privacy budget is diminished
- If the inputs are disjoint, then result is $\max(\varepsilon_1, \varepsilon_2)$ private
- Example:
  - Ask for count of people broken down by handedness, hair color

	Redhead	Blond	Brunette
Left-handed	23	35	56
Right-handed	215	360	493

  - Each cell is a disjoint set of individuals
  - So can release each cell with ε-differential privacy (parallel composition) instead of 6ε DP (sequential composition)

Exponential Mechanism

- What happens when we want to output non-numeric values?
- Exponential mechanism is most general approach
  - Captures all possible DP mechanisms
  - But ranges over all possible outputs, may not be efficient
- Requirements:
  - Input value x
  - Set of possible outputs O
  - Quality function, q, assigns "score" to possible outputs $o \in O$
    - $q(x, o)$ is bigger the "better" o is for x
  - Sensitivity of $q = S(q) = \max_{x,x',o} |q(x,o) - q(x',o)|$

Exponential Mechanism

- Sample output $o \in O$ with probability

  $$\Pr[K(x) = o] = \exp(\varepsilon\, q(x,o)) / (\sum_{o' \in O} \exp(\varepsilon q(x,o')))$$

- Result is $(2\varepsilon\, S(q))$-DP
  - Shown by considering change in numerator and denominator under change of x is at most a factor of $\exp(\varepsilon\, S(q))$
- Scalability: need to be able to draw from this distribution
- Generalizations:
  - O can be continuous, $\sum$ becomes an integral
  - Can apply a prior distribution over outputs as $P(o)$
    - We assume a uniform prior for simplicity

Exponential Mechanism Example 1: Count

- Suppose input is a count n, we want to output (noisy) n
  - Outputs O = all integers
  - $q(o,n) = -|o-n|$
  - $S(q) = 1$
  - Then $\Pr[K(n) = o] = \exp(-\varepsilon\ |o-n|)/(\sum_o -\varepsilon|o-n|) = \alpha^{-|o-n|} \cdot (1-\alpha)/(1-\alpha)$
  - Simplifies to the Geometric mechanism!
- Similarly, if O = all reals, applying exponential mechanism results in the Laplace Mechanism
- Illustrates the claim that Exponential Mechanism captures all possible DP mechanisms

Exponential Mechanism, Example 2: Median

- Let $M(X)$ = median of set of values in range $[0,T]$ (e.g. median age)
- Try Laplace Mechanism: $S(M) = T$
  - There can be datasets X, X' where $M(X) = 0$, $M(X') = T$, $|X-X'|=1$
  - Consider $X = [0^n, 0, T^n]$, $X' = [0^n, T, T^n]$
  - Noise from Laplace mechanism outweighs the true answer!
- Exponential Mechanism: set $q(o,X) = -|\ \mathrm{rank}_X(o) - |X|/2|$
  - Define $\mathrm{rank}_X(o)$ as the number of elements in X dominated by o
  - Note, $\mathrm{rank}_X(M(X)) = |X|/2$: median has rank half
  - $S(q) = 1$: adding or removing an individual changes q by at most 1
  - Then $\Pr[\ K(X) = o] = \exp(\varepsilon\ q(o,X))/(\sum_{o' \in O} \exp(\varepsilon\ q(o',X)))$
  - Problem: O could be very large, how to make efficient?

Exponential Mechanism, Example 2: Median

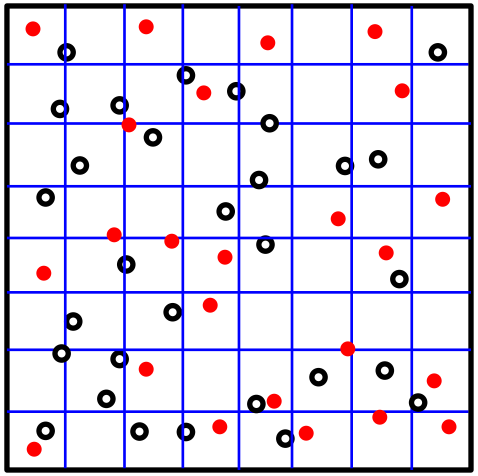
- Observation: for many values of o, $q(o, X)$ is the same:
  - Index X in sorted order so $x_1 \leq x_2 \leq x_3 \leq \ldots \leq x_n$
  - Then for any $x_i \leq o < o' \leq x_{i+1}$, $rank_X(o) = rank_X(o')$
  - Hence $q(o,X) = q(o',X)$
- Break possible outputs into ranges:
  - $O_0 = [0,x_1]$ $O_1 = [x_1, x_2]$ … $O_n = [x_n, T]$
  - Pick range O_j with probability proportional to $|O_j|\exp(\varepsilon q(O,X))$
  - Pick output $o \in O_j$ uniformly from the range
  - Time cost is proportional to number of ranges n (after sorting X)
- Similar tricks make exponential mechanism practical elsewhere

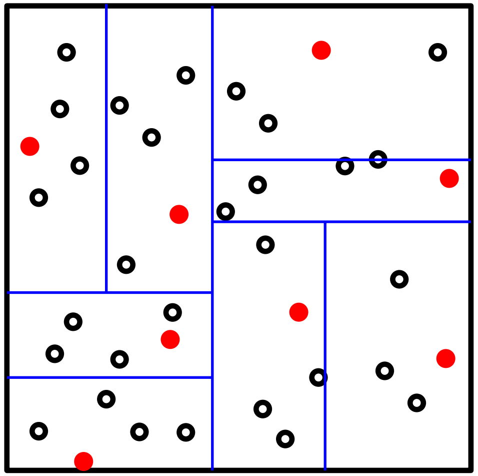
Recap

- Have developed a number of building blocks for DP:
  - Geometric and Laplace mechanism for numeric functions
  - Exponential mechanism for sampling from arbitrary sets
- And "cement" to glue things together:
  - Parallel and sequential composition theorems
- With these blocks and cement, can build a lot
  - Many papers arrive from careful combination of these tools!
- Useful fact: any post-processing of DP output remains DP
  - (so long as you don't access the original data again)
  - Helps reason about privacy of data release processes

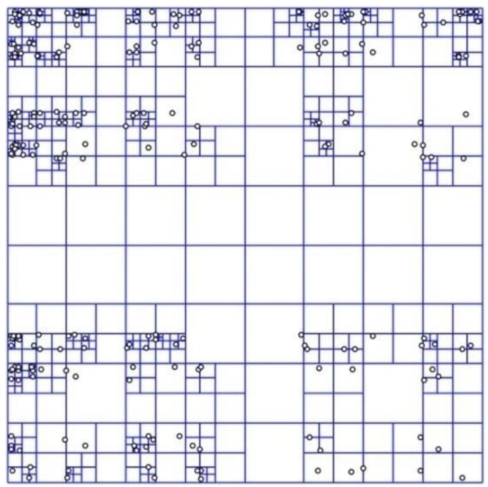
Case Study: Sparse Spatial Data

- Consider location data of many individuals
  - Some dense areas (towns and cities), some sparse (rural)
- Applying DP naively simply generates noise
  - lay down a fine grid, signal overwhelmed by noise
- Instead: compact regions with sufficient number of points

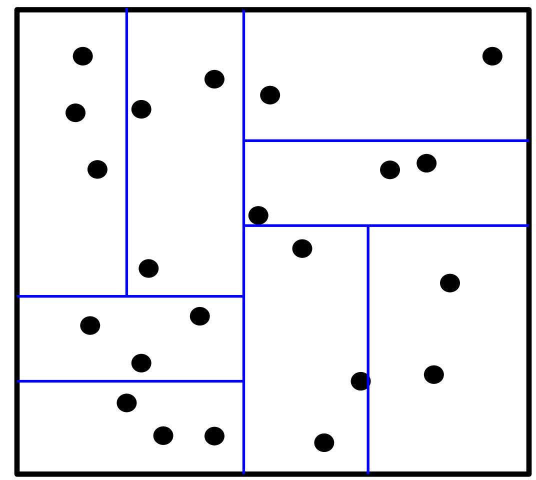




Private Spatial decompositions



quadtree



kd-tree

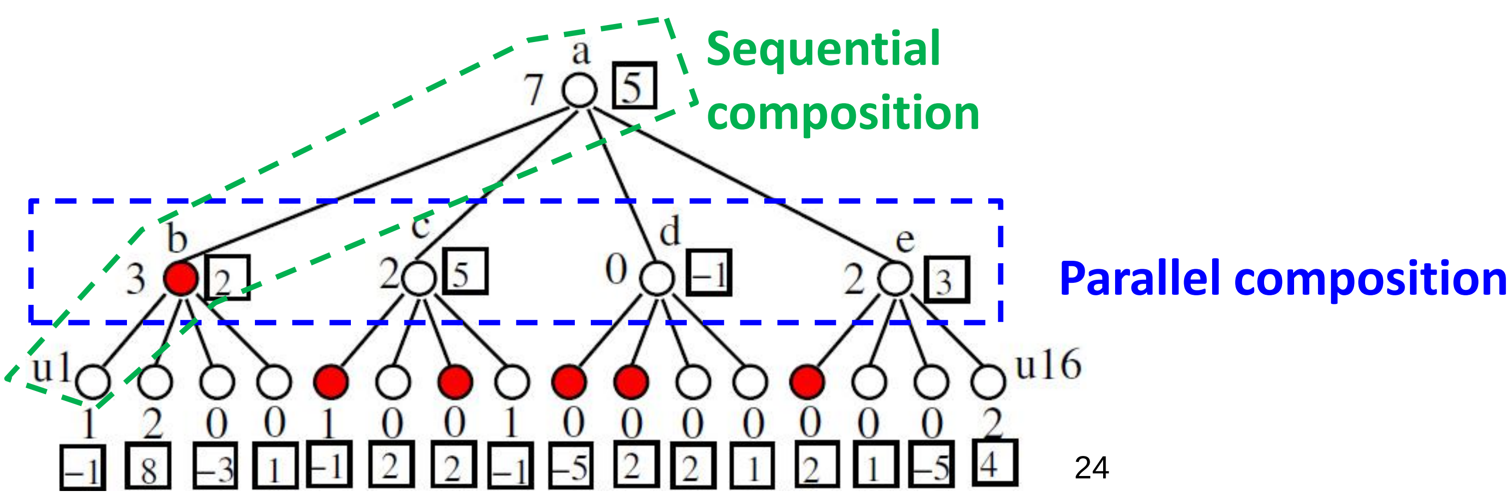
- Build: adapt existing methods to have differential privacy
- Release: a private description of data distribution (in the form of bounding boxes and noisy counts)

Building a Private kd-tree

- Process to build a private kd-tree
  - Input: maximum height *h*, minimum leaf size *L*, data set
  - Choose dimension to split
  - Get (private) median in this dimension
  - Create child nodes and add noise to the counts
  - Recurse until:
    - Max height is reached
    - Noisy count of this node less than *L*
    - Budget along the root-leaf path has used up
- The entire PSD satisfies DP by the composition property

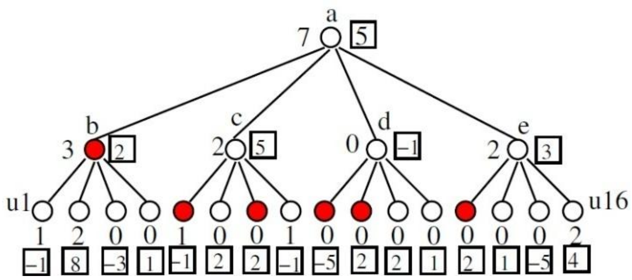
Building PSDs – privacy budget allocation

- Data owner specifies a total budget ε reflecting the level of anonymization desired
- Budget is split between medians and counts
  - Tradeoff accuracy of division with accuracy of counts
- Budget is split across levels of the tree
  - Privacy budget used along any root-leaf path should total ε



Privacy budget allocation

- How to set an ε_i for each level?
  - Compute the number of nodes touched by a 'typical' query
  - Minimize variance of such queries
  - Optimization: min $\sum_i 2^{h-i} / \varepsilon_i^2$ s.t. $\sum_i \varepsilon_i = \varepsilon$
  - Solved by $\varepsilon_i \propto (2^{(h-i)})^{1/3}\varepsilon$: more to leaves
  - Total error (variance) goes as $2^h/\varepsilon^2$



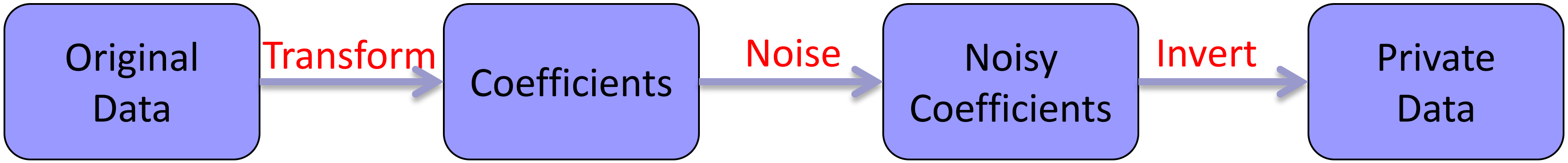
- Tradeoff between noise error and spatial uncertainty
  - Reducing h drops the noise error
  - But lower h increases the size of leaves, more uncertainty

Post-processing of noisy counts

- Can do additional post-processing of the noisy counts
  - To improve query accuracy and achieve consistency
- Intuition: we have count estimate for a node and for its children
  - Combine these independent estimates to get better accuracy
  - Make consistent with some true set of leaf counts
- Formulate as a linear system in n unknowns
  - Avoid explicitly solving the system
  - Expresses optimal estimate for node v in terms of estimates of ancestors and noisy counts in subtree of v
  - Use the tree-structure to solve in three passes over the tree
  - Linear time to find optimal, consistent estimates

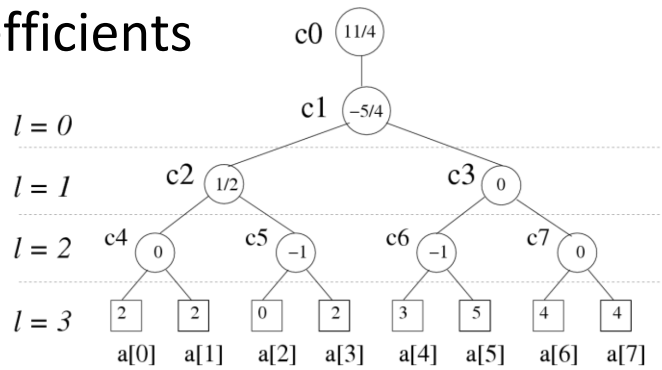
Data Transformations

- Can think of trees as a 'data-dependent' transform of input
- Can apply other data transformations
- General idea:
  - Apply transform of data
  - Add noise in the transformed space (based on sensitivity)
  - Publish noisy coefficients, or invert transform (post-processing)
- Goal: pick a transform that preserves good properties of data
  - And which has low sensitivity, so noise does not corrupt



Wavelet Transform

- Haar wavelet transform commonly used to approximate data
  - Any 1D range is expressed using log n coefficients
  - Each input point affects log n coefficients
  - Is a linear, orthonormal transform
- Can add noise to wavelet coefficients
  - Treat input as a 1D histogram of counts
  - Bounded sensitivity: each individual affects coefficients by O(1)
  - Can transform noisy coefficients back to get noisy histogram
- Range queries are answered well in this model
  - Each range query picks up noise (variance) $O(\log^3 n / \varepsilon)$
  - Directly adding noise to input would give noise $O(n / \varepsilon)$



(a)

Other Transforms

Many other transforms can be applied within DP

- (Discrete) Fourier Transform: also bounded sensitivity
  - Often need only a fixed set of coefficients: further reduces S(F)
  - Used for representing data cube counts, time series
- Hierarchical Transforms: binary trees and quadtrees
- Randomized Transforms: sketches and compressed sensing

$$A_8 = \sqrt{\frac{1}{8}} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Local Sensitivity

- A common fallacy: using local sensitivity instead of global
  - Global sensitivity $S(F) = \max_{x,x' : |x-x'|=1} |F(x)-F(x')|$
  - Local sensitivity $S(F,x) = \max_{x' : |x-x'|=1} |F(x)-F(x')|$
  - These can be very different: local can be much smaller than global
  - It is tempting (but incorrect) to calibrate noise to local sensitivity
- Bad case for local sensitivity: Median
  - Consider $X = [0^n, 0, 0, T^{n-1}]$, $X' = [0^n, 0, T^n]$, $X'' = [0^n, T, T^n]$
  - $S(F,X) = 0$ while $S(F, X') = T$
  - Scale of the noise will reveal exactly which case we are in
- Still, there has to be something better than always using global?
  - Such bad cases seem artificial, rare

Smooth Sensitivity

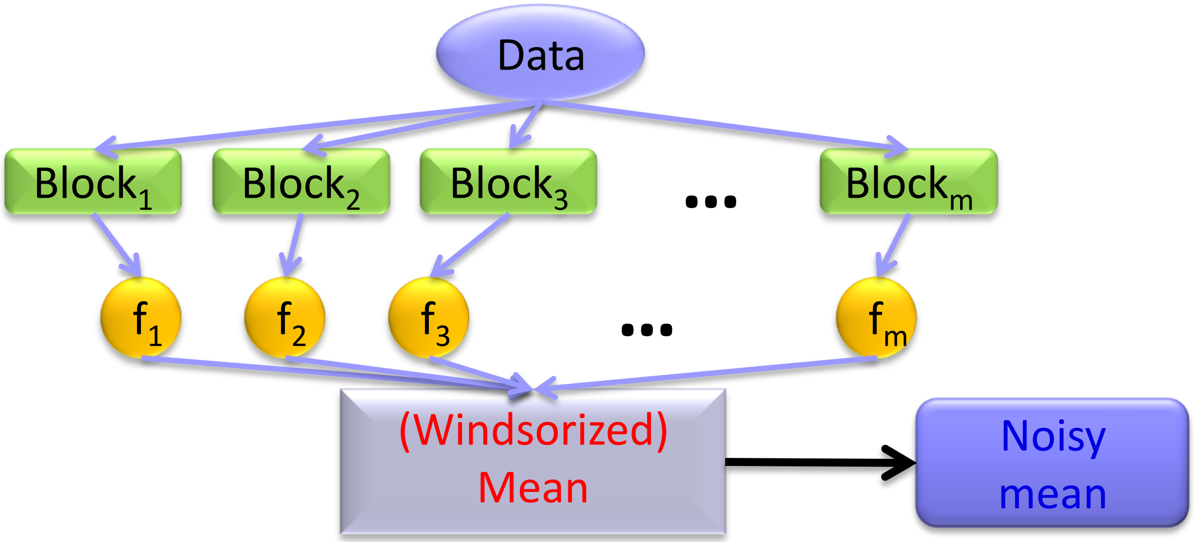
- Previous case was bad because local sensitivity was low, but "close" to a case where local sensitivity was high
- "Smooth sensitivity" combines sensitivity from all neighborhoods (based on parameter β)
  - $SS(F,x) = \max_{o \in O} LS(F,o) \exp(-\beta\ |o - x|)$
  - Contribution of output o is decayed exponentially based on distance of o from x, $|o - x|$
  - Can add Laplace noise scaled by $SS(F,x)$ to obtain (variant of) DP

Smooth Sensitivity: Example

- Consider the median function M over n items again
  - Compute the maximum change in the median for each distance d
  - LS measures when median changes from x_i to x_{i+1}
- So LS at distance d is at most $\max_{0\leq j\leq d} (x_{n/2+j} - x_{n/2+j-d-1})$
  - Largest gap that can be created by inserting/deleting at most d items
- Gives $SS(M,x) = \max_{0\leq d\leq n} \exp(-d\beta) \max_{0\leq j\leq d} (x_{n/2+j} - x_{n/2+j-d-1})$
  - Can compute in time $O(n^2)$
  - Empirically, exponential mechanism seems preferable
  - No generic process for computing smooth sensitivity

Sample-and-aggregate

- Sample-and-aggregate gives a useful template
  - Intuition: sampling is almost DP - can't be sure who is included
  - Break input into moderate number of blocks, m
  - Compute desired function on each block
  - Snap to some range [min, max] and aggregate (e.g. mean)
  - Add Laplace noise scaled by sensitivity (max-min)



Sparse Data

- Suppose we have many (overlapping) queries, most of which have a small answer, but we don't know which
  - We are only interesting in large answers (e.g. frequent itemsets)
  - Two problems: time efficiency, and "privacy efficiency"
- Time efficiency:
  - Don't want to add noise to every single zero-valued query
  - Assume we can materialize all non-zero query answers
  - Count how many are zero
  - Compute probability of noise pushing a zero-query past threshold
  - Sample from Binomial distribution how many to "upgrade"
  - Sample noisy value conditioned on passing threshold

Sparse Data – Privacy Efficiency

- Only want to pay for c queries with that exceed threshold T
  - Assume all queries have sensitivity S
- Compute noisy threshold $T' = T + \text{Lap}(2S/\varepsilon)$
- For each query, add noise $\text{Lap}(2Sc/\varepsilon)$, only output if above T'
- Result is ε-DP
  - For "suppressed" answers, probability of seeing same output is about the same as if T' was a little higher on neighboring input
  - For released answers, DP follows from Laplace mechanism
- Result is reasonably accurate: with high probability,
  - All suppressed answers are smaller than $T + \alpha$
  - All released answers have error at most α

  for parameter $\alpha(c, 1/\varepsilon, S)$, and at most c query answers $> T - \alpha$

Multiplicative weights

- The idea of "multiplicative weights" widely used in optimization
  - Up-weight 'good' answers, down-weight 'poor' answers
  - Applied to output of DP mechanism
- Set-up:
  - (Private) input, represented as vector D with n entries
  - Q, set of queries over x (matrix)
  - T, bound on number of iterations
  - Output: ε-DP vector A so that $Q(A) \approx Q(D)$

Multiplicative Weights Algorithm

- Initialize vector A_0 to assign uniform weight for each value
- For $i=1$ to T:
  - Exponential Mechanism ($\varepsilon/2T$) to sample j prop. to $|Q_j(A_i) - Q_j(D)|$
    - Try to find query with large error
  - Laplace Mechanism to estimate $\Delta = (Q_j(A) - Q_j(D)) + Lap(2T/\varepsilon)$
    - Error in the selected query
  - Set $A_i = A_{i-1} . \exp(\Delta\ Q_j(D)/2n)$, normalize so that A_i is a distribution
    - (Noisily) reward good answers, penalize poor answers
- Output $A = \text{average}_i\ nA_i$
  - Privacy follows via sequential composition of EM and LM steps
  - Accuracy (should) improve in each iteration, up to log iterations

Other topics

- Huge amount of work in DP across theory, security, DB…
- Many topics not touched on in this tutorial:
  - Connections to game theory and auction design
  - Mining primitives: regression, clustering, frequent itemsets
  - Efforts in programming languages and systems to support DP
  - Variant definitions: (ε, δ)-DP, other privacy/adversary models
  - Lower bounds for privacy (what is not possible)
  - Applications to graph data (social networks), mobility data etc.
  - Privacy over data streams: pan-privacy and continual observation

Concluding Remarks

- Differential privacy can be applied effectively for data release
- Care is still needed to ensure that release is allowable
  - Can't just apply DP and forget it: must analyze whether data release provides sufficient privacy for data subjects
- Many open problems remain:
  - Transition these techniques to tools for data release
  - Want data in same form as input: private synthetic data?
  - Allow joining anonymized data sets accurately
  - Obtain alternate (workable) privacy definitions

Thank you!

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