Trusting the Cloud with Practical Interactive Proofs

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There are no guarantees in life

"WE... MAKE NO REPRESENTATIONS OF ANY KIND... THAT THE SERVICE OR THIRD PARTY CONTENT WILL BE UNINTERRUPTED, ERROR FREE OR FREE OF HARMFUL COMPONENTS, OR THAT ANY CONTENT... WILL BE SECURE OR NOT OTHERWISE LOST OR DAMAGED."

- From the terms of service of a certain cloud computing service...
- Can we obtain guarantees of correctness of the computation?
  - Without repeating the computation?
  - Without storing all the input?

YES!

Interactive Proofs

What's the answer?

42

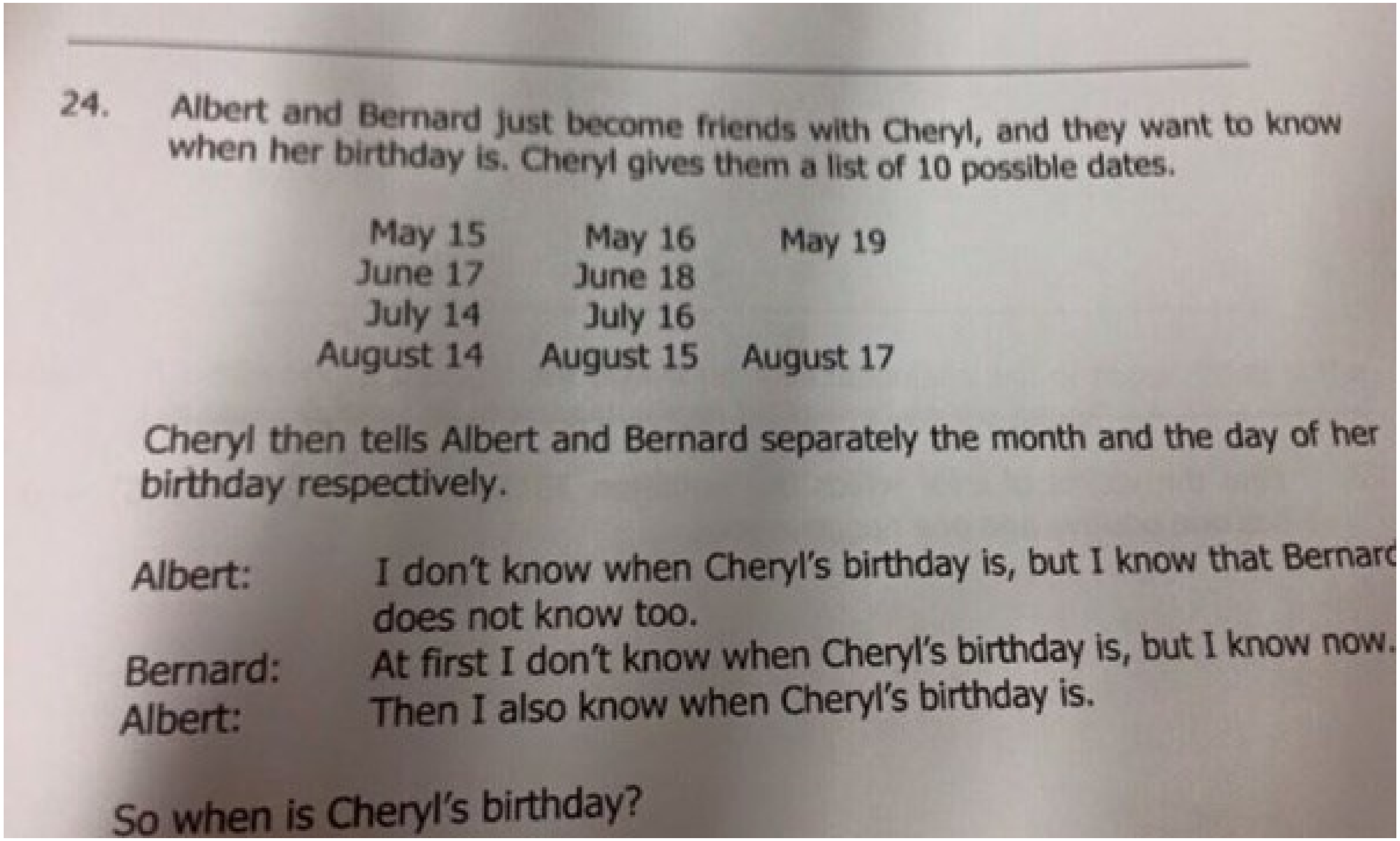
Prove it!

1010101001000110110101100010001

110101?

11010010001000110101010010001101

OK!


24. Albert and Bernard just become friends with Cheryl, and they want to know when her birthday is. Cheryl gives them a list of 10 possible dates.

May 15	May 16	May 19
June 17	June 18	
July 14	July 16	
August 14	August 15	August 17

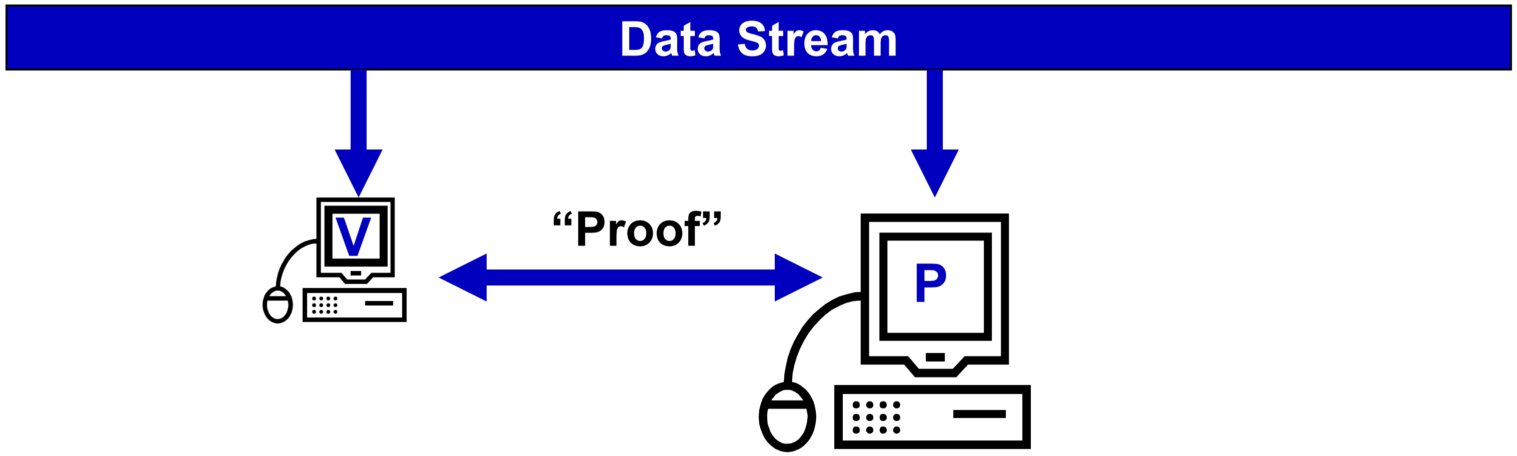
Cheryl then tells Albert and Bernard separately the month and the day of her birthday respectively.

Albert:	I don't know when Cheryl's birthday is, but I know that Bernar does not know too.
Bernard:	At first I don't know when Cheryl's birthday is, but I know now.
Albert:	Then I also know when Cheryl's birthday is.

So when is Cheryl's birthday?

(Streaming) Interactive Proofs

- Two party-model: outsource to a more powerful "prover"
  - Fundamental problem: how to be sure that the prover is honest?
- Prover provides "proof" of the correct answer
  - Ensure that "verifier" has very low probability of being fooled
  - Measure resources of the participants, rounds of interaction
  - Related to communication complexity Arthur-Merlin model, and Algebrization, with additional streaming constraints

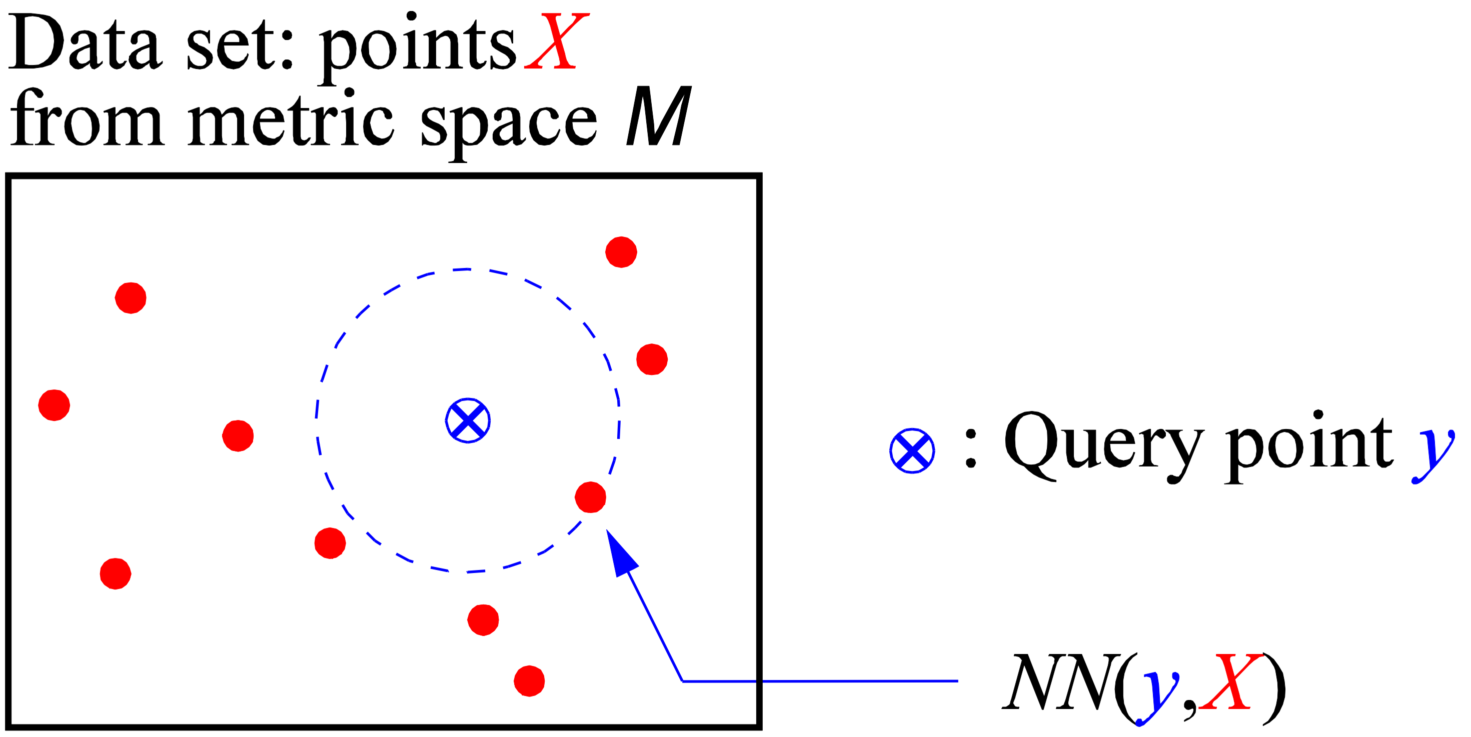


Starter Problem: Index

0 1 1 1 0 1 0 1 1 0 0 0 0 …	1258914

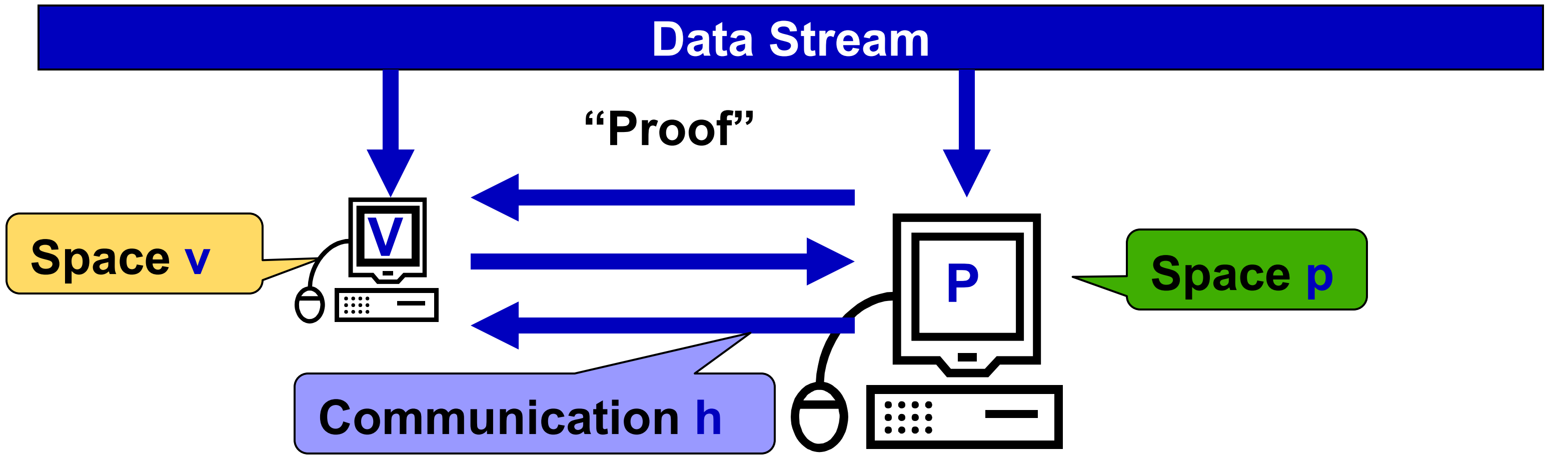
- Fundamental (hard) problem in data streams
  - Input is a length m binary string x followed by index y
  - Desired output is x[y]
  - Requires Ω(m) space even allowing error probability
- Can we find a protocol to allow recovery of arbitrary bits
  - Without having the verifier store the entire sequence?

Real problem: Nearest neighbor

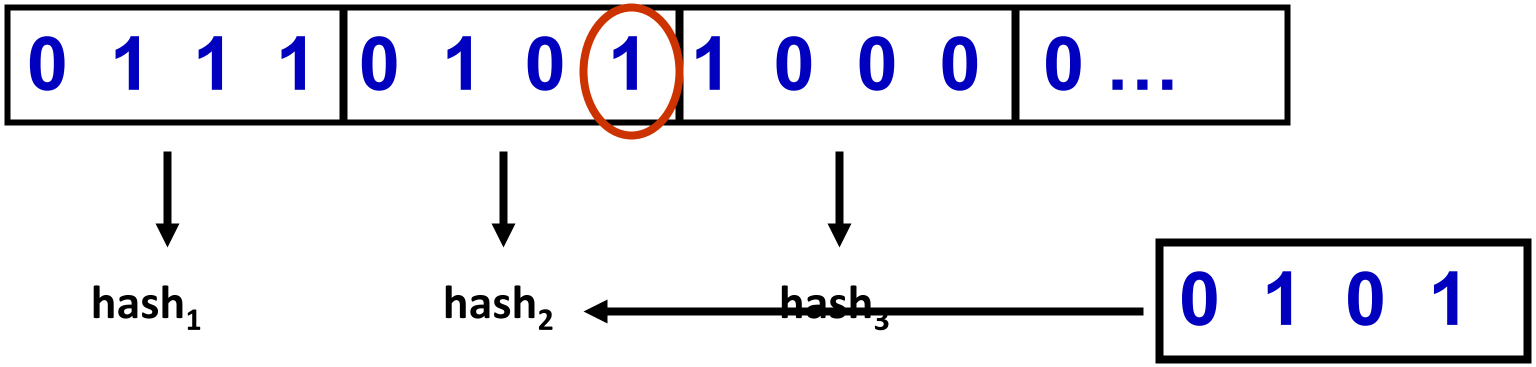


Parameters

- m data points (m very large)
  - Verifier V processes data using small space << m
  - Prover P processes data using space at least m
- V and P have a conversation to determine the answer
  - If P is honest, 0.99 probability that V accepts the answer
  - If P is dishonest, 0.99 probability that V rejects the answer
  - Measure the space used by V, P, communication used by both



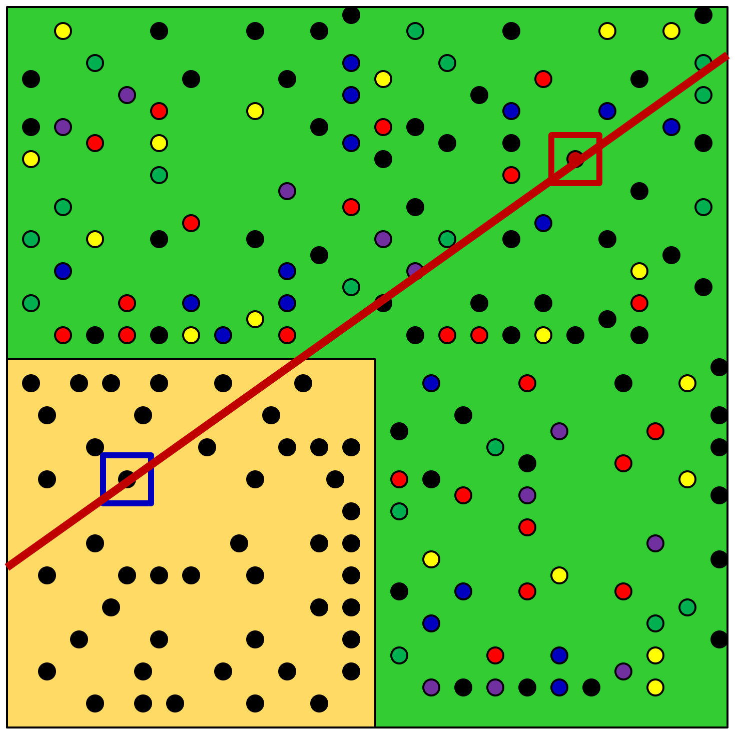
Index: 1 Round Upper Bound



- Divide the bit string into blocks of H bits
- Verifier remembers a hash on each block
- After seeing index, Prover replays its block
- Verifier checks hash agrees, and outputs x[y]

- Cost: H bits of proof from the prover, V = m/H hashes
  - So HV = O(m log m), any point on tradeoff is possible

2 Round Index Protocol



Challenge line λ

Random point $r \in \mathbf{F}^b$

Query point y

Data indexed in Boolean hypercube $\{0,1\}^b$

Extended to hypercube $\mathbf{F}^b$

1. V picks r and evaluates low-degree extension of input at r to get q
2. V sends λ to P
3. P sends polynomial p' which is input restricted to λ
4. V checks that p'(r) = q, and outputs p'(y)

Streaming LDE Computation

- Given query point $r \in F^b$, evaluate extension of input at r
- Initialize: $z = 0$
- Update with impact of each data point $y=(y_1, \dots y_b)$ in turn. Structure of polynomial means update causes

$$z \leftarrow z + \prod_{i=1}^{b} ((1-y_i)(1-r_i) + y_i r_i)$$

  – Lagrange polynomial, can be evaluated in small space

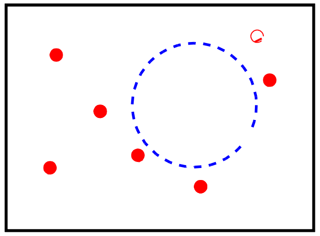
- Can be computed quickly, using appropriate precomputed look-up tables

Correctness and Cost

- Correctness of the protocol
  - If P is honest: V will always accept
  - If P is dishonest: V only accepts if $p'(r) = q$
    This happens with probability $b/|F|$: can make $|F|$ bigger
- Costs of the protocol
  - V's space: $O(b \log |F|) = O(\log n \log \log n)$ bits
  - P and V exchange λ and p' as $(b + 1)$ values in F,
    so communication cost is $O(\log n \log \log n)$ bits
  - Exponential improvement over one round
- Consequences: can do other computations via Index e.g. median
  - What about more complex functions?

Nearest Neighbour Search

- Basic idea: convert NNS into an (enormous) index problem
  - Work with input points in $[n]^d$
  - Assume all distances are multiples of $\varepsilon = 1/n^d$
- Let B = {all distinct balls}; note $|B| \leq n^{2d}$
  - Convert input points to **virtual set of balls** from B:
  - point $x \rightarrow$ all balls β such that $x \in \beta$
- V processes virtual stream σ through index protocol
- For query $y \in X$, P specifies point $z \in X$, claiming $z = NN(y,X)$
  - Show $ball(z,0) \in \sigma$ via Index Protocol
  - And $ball(z, dist(y, z)-\varepsilon) \notin \sigma$ via Index Protocol
- Protocol allows correct demonstration of nearest neighbour
- Drawback: blow-up of input size costs V a lot!



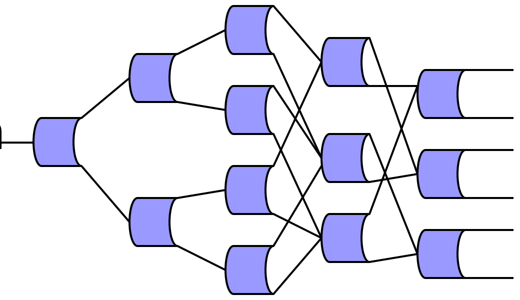
Practical Proof Protocol

- Exploit structure of the metric space containing the points
  - Let $\Phi(\beta,x)$ be the function that reports 1 iff x is in ball β
  - Goal: query the vector $v[\beta] = \sum_{x \text{ in input}} \Phi(\beta,x)$
  - $\Phi(\beta,x)$ has a simple circuit for common metrics (Hamming, L_1, L_2...)
  - "Arithmetize" the formula to compute distances
- Transform formula Φ to polynomial Φ' via

  $G_1 \wedge G_2 \mapsto G'_1\, G'_2$ and $\qquad G_1 \vee G_2 \mapsto 1-(1-G'_1)(1-G'_2)$
- Low-degree extension of v: $v'(B_1 \dots B_{2d \log n}) = \sum_x \Phi'(B_1 \dots B_{2d \log n}, x)$
  - Can then apply Index protocol to v' – v never materialized by P or V
- Final costs of the protocol:
  - Verifier can process each data point in time $\text{poly}(d,\log n)$
  - Communication cost and verifier space both $\text{poly}(d,\log m,\log n)$ bits

General Computations

- Want to be able to solve more general computations
- Framework: "Interactive Proofs for Muggles", STOC'08
  Goldwasser, Kalai, Rothblum [GKR08]
- Idea: computations modeled by arithmetic circuits
  - Arranged into layers of addition and multiplication gates
- (Super)Round i: Prover claims value of LDE of layer i at r_i
  Run multiround IP to reduce to a claim about layer $i-1$ at r_{i-1}
- Start with claimed output, end with LDE of input
  - Verifier can check against own calculated LDE
  - V only needs LDE + compact circuit description

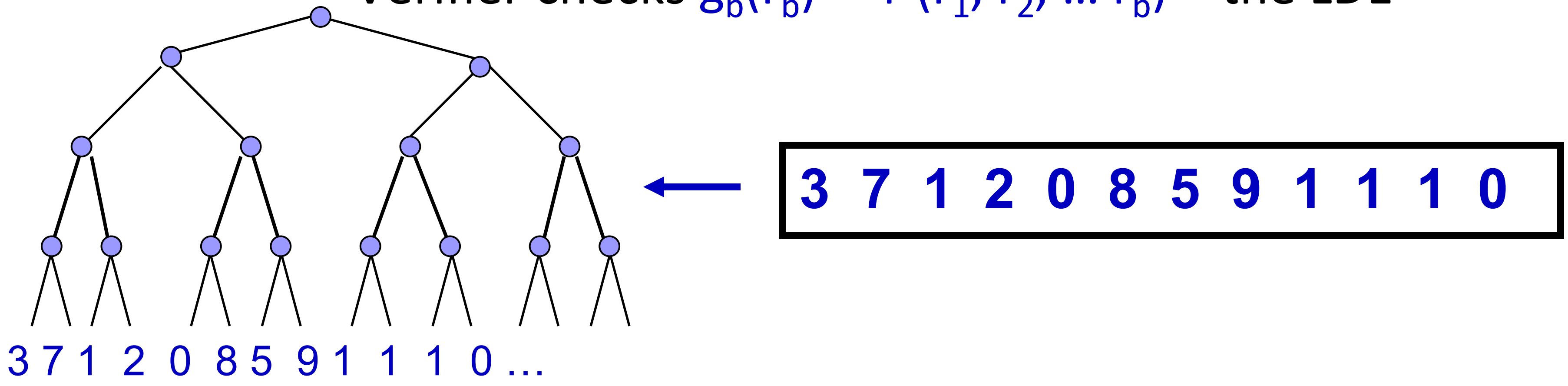


Primitive: Sum Check Protocol

Hold prover to a claim about a (low-degree) function of the input

Index data using $\{0,1\}^b$ in $b = \log N$ dimensional space

- Verifier picks one $(r_1 \ldots r_b) \in \mathbf{F}^b$, and evaluates $f^k(r_1, r_2, \ldots r_b)$
- Round 1: Prover sends $g_1(x_1)=\sum_{x_2 \ldots x_b} f^k(x_1, x_2 \ldots x_b)$, V sends r_1
- Round i: Prover sends $g_i(x_i) = \sum_{x_{i+1} \ldots x_b} f^k(r_1, r_2 \ldots r_{i-1}, x_i, x_{i+1} \ldots x_b)$
  Verifier checks $g_{i-1}(r_{i-1}) = g_i(0) + g_i(1)$, sends r_i
- Round d: Prover sends $g_b(x_b) = f^k(r_1, \ldots r_{d-1}, x_b)$
  Verifier checks $g_b(r_b) = f^k(r_1, r_2, \ldots r_b)$ = the LDE



Putting GKR08 into practice

- Verifier needs LDEs of "wiring polynomials" of the circuit
  - E.g. add(a, b, c) = 1 iff gate a at layer i has inputs b, c from layer i-1
  - Looks costly to evaluate directly, need to sum LDE over n^3 values?
  - Use the multilinear extension of the add() and mult() polynomials
  - Each gate contributes one term to the sum, so linear in circuit size
  - Going layer to layer keeps degree low: ensures probability holds
- Linear in circuit size is still slow – same as evaluating the circuit!
  - Take advantage of regularity in common wiring patterns
  - E.g. binary tree: compute contribution of all gates at once
  - Also holds for circuits for FFT, Matrix multiplication etc.

Engineering GKR08

- Include some "shortcut" gates in addition to add, mult
  - Wide-sum $\bigoplus$: add up a large number of inputs
    - Only needs a single sum-check protocol
  - Exponentiation: raise to a constant power (x^8, x^{16})
    - More efficient than repeated self-multiplication
- Choose the right field size for computations
  - Work modulo a large Mersenne prime allows efficient arithmetic

Experimental Results

Problem	Gates	Size (gates)	P time	V time	Rounds	Comm
F_2	+, ×	0.4M	8.5 s	.01 s	986	11.5 KB
F_2	+, ×, $\oplus$	0.2M	6.5 s	.01 s	118	2.5 KB
F_0	+, ×	16M	552.6 s	.01 s	3730	87.4 KB
F_0	+, ×, x^8, $\oplus$	8.2M	432.6 s	.01 s	1310	51.0 KB
F_0	+, ×, x^{16}, $\oplus$	6.2M	441.2 s	.01 s	1024	56.8 KB
PMwW	+, ×, x^8, $\oplus$	9.6M	482.2 s	.01 s	1513	56.1 KB

- (Relatively) efficient results for frequency moments (F_2, F_0), pattern matching with wildcards (PMwW)

Concluding Remarks

- These protocols are truly practical
  - No, really, they are
- Also provide insight into the theory of Arthur-Merlin communication games
- Many open problems around this area
  - Extend to other data mining/machine learning problems
  - Prove lower bounds: some problems are hard
  - Evaluations on real data, optimization of implementations
  - Variant models: power of two provers…

Connections

- Full papers and connection to computational complexity
  - "Verifiable Stream Computation and Arthur–Merlin Communication", Computational Complexity Conference 2015
  - "Practical Verified Computation with Streaming Interactive Proofs", Innovations in Theoretical Computer Science 2012
- Streaming Computation: what can be computed in one pass?
  - Typical focus on statistical security (no crypto assumptions)
- Fully Homomorphic Encryption: crypto security allows repetition
  - Make use of circuit garbling to provide secrecy as well as integrity
- Argument Systems (ginger/pepper/zaatar/pinnochio)
  - Need hefty preprocessing of computation
- Other directions: multiple provers, ZKP, public verifiability

Related Directions

- Prover's work is data parallel: can take use of GPU for acceleration [Thaler et al. HotCloud 2012]
- Further tricks shave log factors off prover's effort [Thaler, Crypto 2013]
- Reduce dependency on domain size when data is sparse [Chakrabarti et al., 2013]
- Three party model (data owner, server, clients) [Cormode et al., SIGMOD 2013]