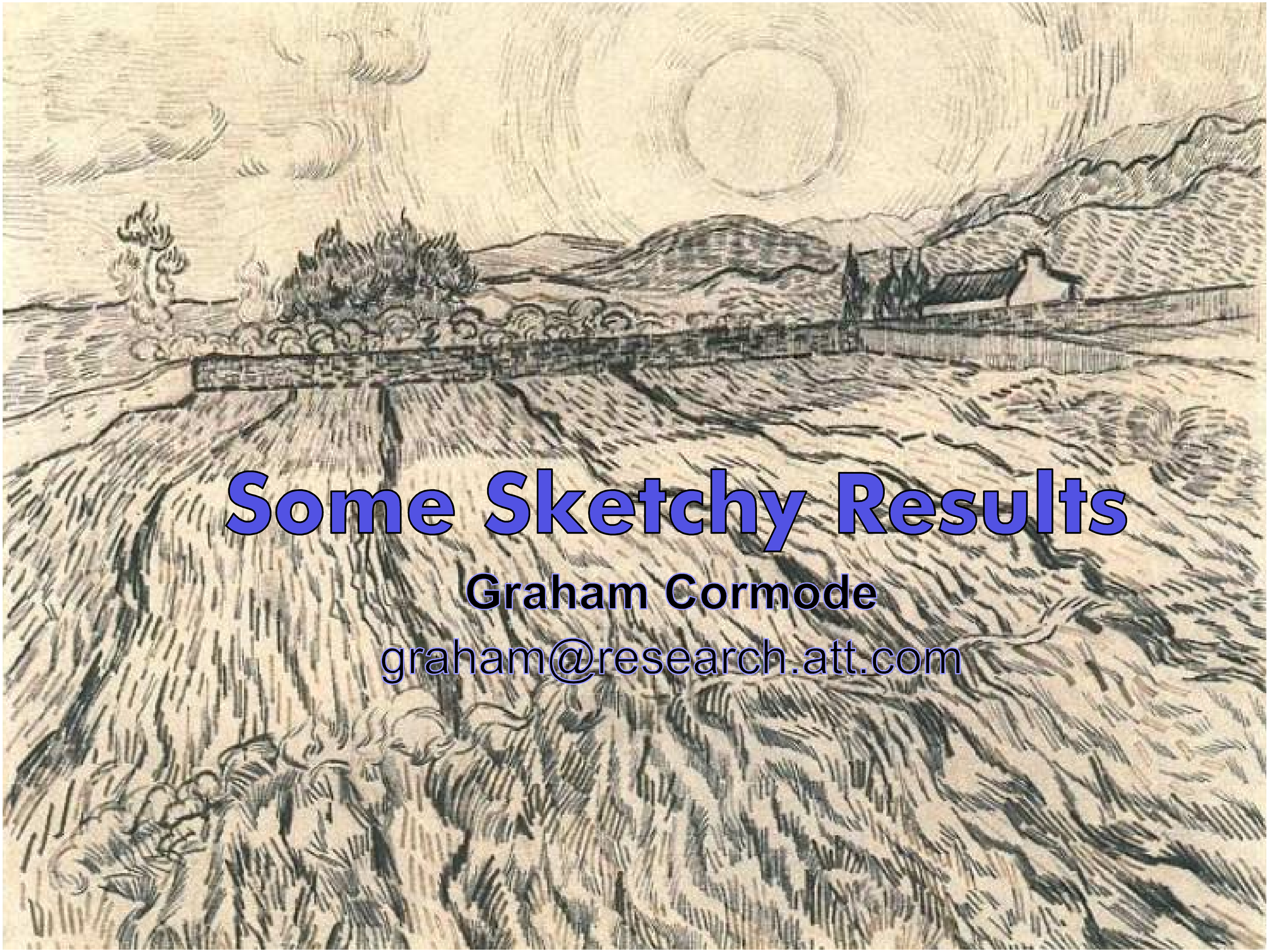

Some Sketchy Results

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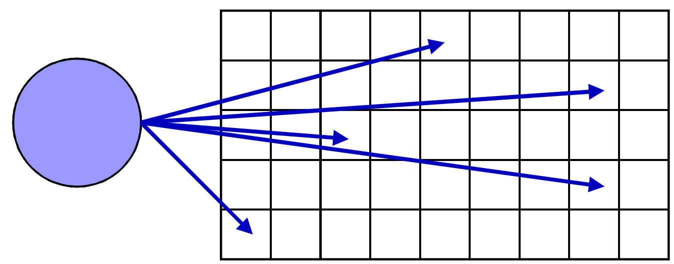
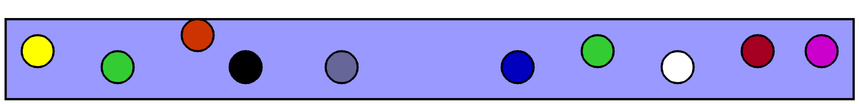
Intro to Sketches

- "Sketch" data structures are compact, randomized summaries
- Term coined by Broder in 1997
  - Exact interpretation varies
- Common sketch properties:
  - Approximate a holistic function
  - Sublinear in size of the input
  - Linear transform of input
  - Can easily merge sketches



Compact summary
Limited independence
Linear transform

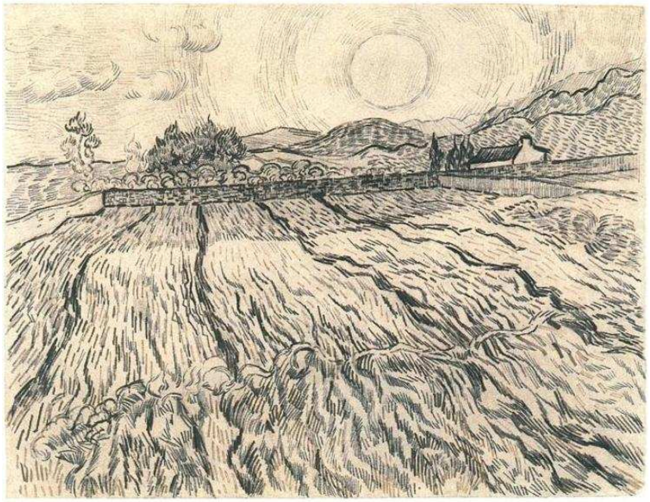


Sketch Types

- (Linear) Fingerprints for equality tests (~1981)
  - Gives updatable randomized equality tests in constant space
- Bloom filters for set membership queries (1970)
  - Can be made linear transforms of the input
- Min-wise hashes for (Jaccard) similarity and sampling (~1997)
  - Not linear, but mergeable / distributable
- Counting sketches summarize distributions (1996, 99, 02, 03)
  - Count sketch, AMS, Count-min etc.
- Count-Distinct sketches (1983, 2001, 2002)
  - Flajolet-Martin, Gibbons-Tirthapura, BJKST etc.

Sketches in the Field

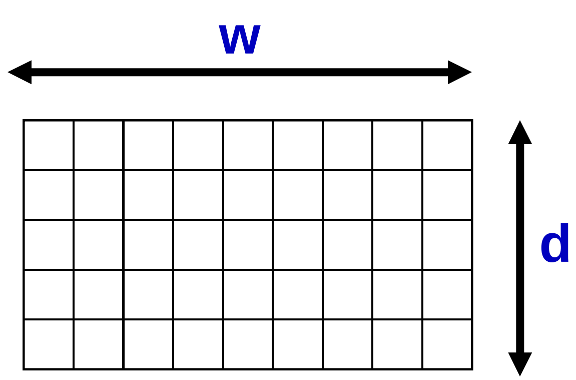
- Sketches have been widely used in many applications
- **Why** are they successful?
  - Often simple to implement
  - Solve foundational problems well
  - Can seem magical on first encounter
- Why aren't they **more successful**?
  - Primarily: not yet fully mainstream
- What can we do to **promote** their success?



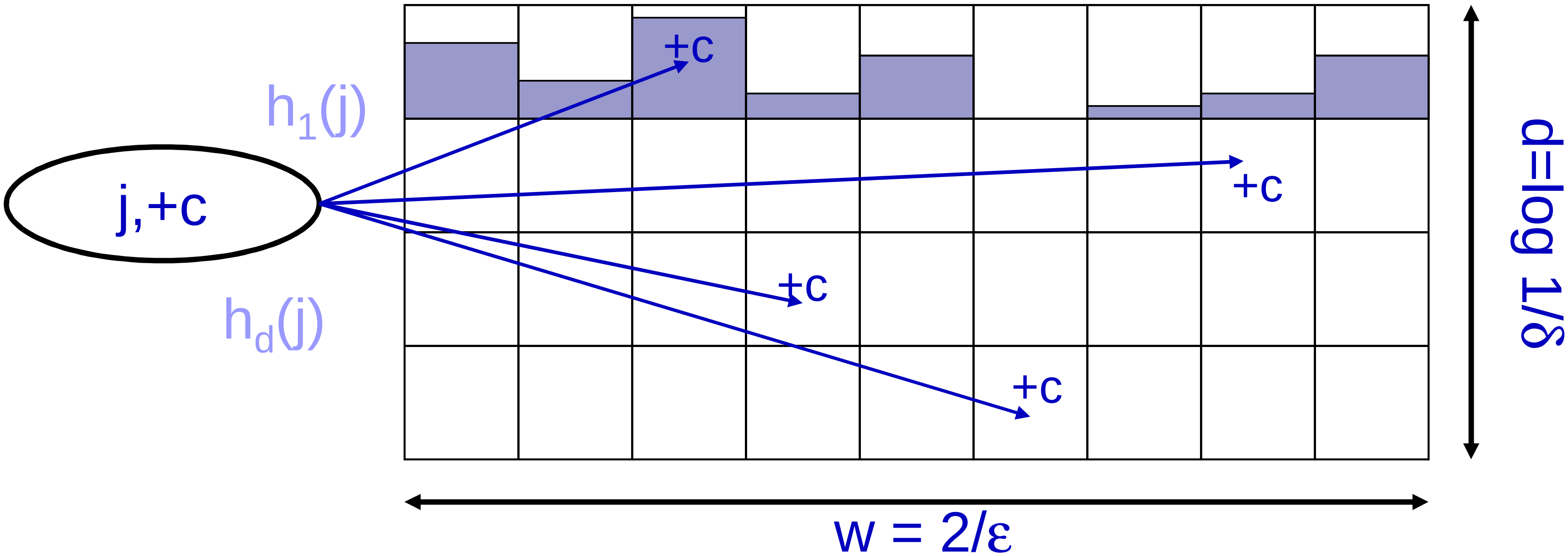
Count-Min Sketch

- Simple sketch idea, can be used within many different tasks
- Model input data as a vector x of dimension m
- Creates a small summary as an array of $w \times d$ in size
- Use d hash function to map vector entries to $[1..w]$
- (Implicit) linear transform of input vector, so flexible

Array: CM[i,j]



Count-Min Sketch Structure



- Each entry in vector x is mapped to one bucket per row.
- Merge two sketches by entry-wise summation
- Estimate $x[j]$ by taking $\min_k CM[k,h_k(j)]$
  - Guarantees error less than εF_1 in size $O(1/\varepsilon \log 1/\delta)$ (Markov ineq)
  - Probability of more error is less than $1-\delta$

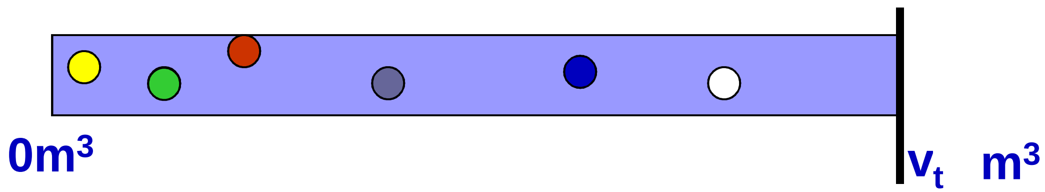
[C, Muthukrishnan '04]

Count-Min for "Heavy Hitters"

- After sequence of items, can estimate f_i for any i (up to εN)
- Heavy Hitters are all those i s.t. $f_i > \phi N$
- Slow way: test every i after creating sketch
- Faster way: test every i after it is seen, and keep largest f_i's
- Alternate way:
  - keep a binary tree over the domain of input items, where each node corresponds to a subset
  - keep sketches of all nodes at same level
  - descend tree to find large frequencies, discarding branches with low frequency

F_0 Sketch

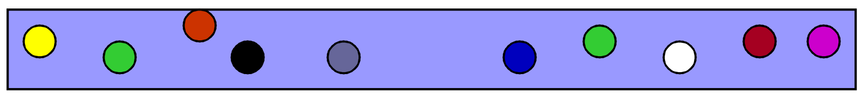
- F_0 is the number of distinct items in a multiset
  - a fundamental quantity with many applications
- [BJKST02] Pick random hash over items, $h: [m] \rightarrow [m^3]$



- For each item i, compute $h(i)$, and track the t distinct items achieving the smallest values of $h(i)$
  - Note: whenever i occurs, $h(i)$ is same
  - Let v_t = t'th smallest value of $h(i)$ seen.
- If $F_0 < t$, give exact answer, else estimate $F'_0 = tm^3/v_t$
  - $v_t/m^3 \approx$ fraction of hash domain occupied by t smallest
  - Analysis shows relative error $(1 \pm 1/\sqrt{t})$ via Chebyshev bound

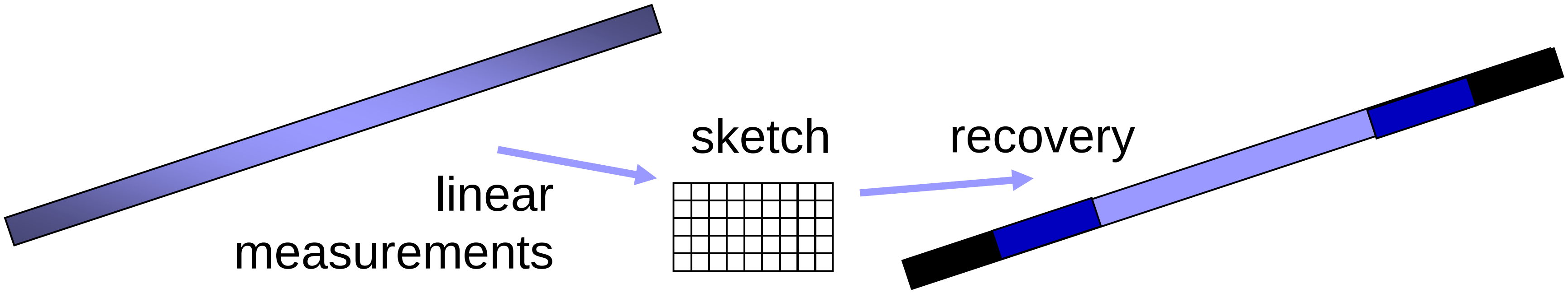
F_0 Sketch Properties

- Space cost for $1 \pm \varepsilon$ error:
  - Store $t=1/\varepsilon^2$ hash values, so $O(1/\varepsilon^2 \log m)$ bits
  - Can improve to $O(1/\varepsilon^2 + \log m)$ with additional tricks



- Time cost:
  - Hash i, update v_t and list of t smallest if necessary
  - Total time $O(\log 1/\varepsilon + \log m)$ worst case
- Generalization [Gibbons-Tirthapura 01, Beyer-HRSG09]:
  - Store t original items with their hash values ("distinct sample")
  - Estimate number of distinct items satisfying some predicate
  - Other extensions: can allow (multiset) deletions

Application: Compressed Sensing

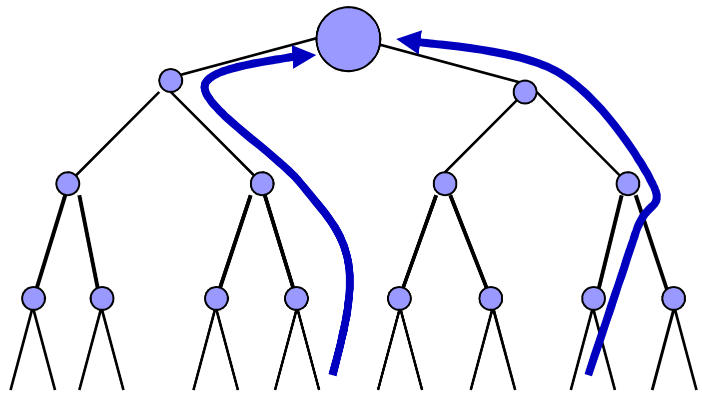


- "Compressed Sensing" has been rocking the EE world since 2004
  - Design a compact measurement matrix M
  - Given product (Mx), recover a good approximation of vector x
  - Optimize: rows of M, density of M, recovery time, error prob
- Sketch techniques yield compressed sensing techniques
  - Very sparse binary M, very fast decoding, but weaker error prob
- Has launched a line of research on sparse recovery
  - See Gilbert-Indyk survey, wiki

Application: Stream Data Analysis

- Many "big data" applications generate large data streams
  - Network traffic analysis, web log analysis
- Sketches allow complex reports on large streaming data
  - In GS-tool (AT&T), CMON (Sprint) for telecom/network data
  - In Sawzall (Google), the only permitted tool for any log analysis
- E.g. track popular queries, number of distinct destinations

Application: Sensor Networks



- Sensor networks distribute many small, weak sensors
  - (Mergeable) sketches fit in here exactly
- **Problem**: no one actually does anything like this [Welsh 10]
  - Most sensor deployments have few nodes, careful placement
  - Attempt to capture all data, no in-network processing
- Hundreds of papers, but algorithms not in this field (yet)

Other Emerging Applications

- Machine learning over huge numbers of features
- Data mining: scalable anomaly/outlier detection
- Database query planning
- Password quality checking [HSM 10]
- Large linear algebra computations
- Cluster computations (MapReduce)
- Distributed Continuous Monitoring
- Privacy preserving computations
- … [Your application here?]

More speculative

Sketch Issues

Strengths

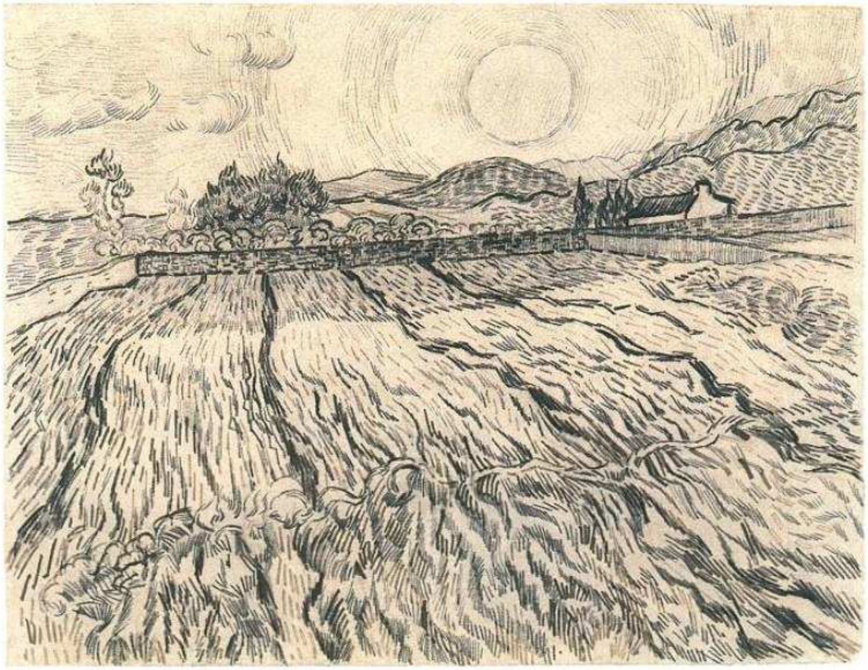
- Easy to code up and use
  - Easier than exact algs
- Small — cache-friendly
  - So can be very fast
- Open source implementations
  - (maybe barebones, rigid)
- Easily teachable
  - As intro to probabilistic analysis
- Highly parallel

Weaknesses

- (Still) resistance to random, approx algs
  - Less so for Bloom filter, hashes
- Memory/disk is cheap
  - Unless data is “too Big To File”
- Not yet in standard libraries
- Not yet in ugrad curricula/texts
  - “this CM sketch sounds like the bomb! (although I have not heard of it before)”
- Looking for killer parallel apps

Open Problems

- More sketches for applications
- More applications for sketches
- More outreach/PR for sketches



- More info:
  - Wiki: **`sites.google.com/site/countminsketch/`**
  - “Sketch Techniques for Approximate Query Processing” **`www.eecs.harvard.edu/~michaelm/CS222/sketches.pdf`**