Putting Brakes on the Rush to AP Calculus

by Joseph G. Rosenstein and Anoop Ahluwalia[1]

This article is an abbreviated version of two unpublished articles[2] that reported on two separate but related studies that involved different cohorts of Rutgers students. "The Rush to Calculus" by the first author involved an analysis of students' high school and college transcripts to determine how many students achieve and maintain the advanced placement in their first year of college (Hint: Not too many do). The second study, "Why Do Students Rush to Calculus?" by both authors surveyed students who had reported their AP exam scores to the university in order to investigate why students take AP Calculus, what are their expectations from the course and what are the consequences of their taking AP Calculus.

Over 250,000 high school students take AP Calculus each year, and the number increases by over 7% each year.[3] Should we be happy about this?

We should first ask what our objectives are in encouraging more students to take AP Calculus, and then evaluate whether our objectives are being met.

Two generations ago, essentially no one (including the first author) took calculus in high school; indeed, most students began college with "analytic geometry," a topic which is now incorporated into high school precalculus courses. AP calculus was introduced in order to provide the very best math students with "advanced placement," a way to start their college careers a semester or two ahead and get to more advanced mathematics more quickly.

However, for a variety of reasons, providing this option for a few students has, over the past 25 years, been transformed into a policy of accelerating the curriculum, including pushing Algebra 1 into 8th grade, so that large numbers of students can take calculus in high school and take advantage of this "advanced placement" option. After all, shouldn't everyone have the opportunity to be "advanced"?

One question that should therefore be asked is, What percentage of the students who take AP Calculus are actually advantaged by the acceleration? That is, those who take AP calculus begin college with second-semester (or third-semester) calculus and remain one semester (or two semesters) ahead of students who did not take AP calculus.

More recently, AP Calculus has been seen as a vehicle to significantly increase the number of students going into what is called the STEM pipeline, STEM being an acronym for science, mathematics, engineering, and technology. A 2007 report by the National Academy of Sciences, "Rising Above the Gathering Storm," sees offering more AP Calculus courses in high school as a major way of addressing the country's pipeline problem and recommends training large numbers of high school math teachers to teach

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[1] Joseph G. Rosenstein is professor of mathematics at Rutgers University and Anoop Ahluwalia is assistant professor of mathematics at Brookdale (NJ) Community College.

[2] "The Rush to Calculus" can be found at dimacs.rutgers.edu/~joer/The-Rush-to-Calculus.pdf and "Why Do Students Rush to Calculus?" can be found at dimacs.rutgers.edu/~joer/Why-Rush-to-Calculus.pdf

[3] This information is from the College Board which oversees the Advanced Placement Program – see http://apcentral.collegeboard.com/program/research

AP Calculus (ignoring the fact that there are already a large number of capable calculus teachers in the nation's colleges and universities).

A second question that should therefore be asked is, Does taking AP Calculus encourage students to pursue STEM careers?

Two studies conducted by the authors help shed light on these two questions and address the initial question of whether that the nation actually benefits when more and more students are taking AP Calculus in high school.

In the first study, conducted five years ago, the first author found that only a tiny percent (5.4%) of Rutgers College students who took a full-year calculus course in high school continued their acceleration through their first year of college. The other 94.6% of the students who took calculus in high school essentially used their years of acceleration in order to slow down (or even end) their mathematical studies.

Arriving at this conclusion involved substantiating four conjectures that he had formed some years earlier in a modest experiment:

1. About half of the students who take calculus in high school take AP Calculus,
2. About half of those who take the AP Calculus course take the AP Calculus exam,
3. About half of those who take the AP Calculus exam receive grades of 4 or 5 (sufficient to get college credit at Rutgers and comparable institutions), and
4. About half of those who receive grades of 4 or 5 actually continue on an accelerated path.

Assuming that these conjectures are correct, we are led to the following conclusion: Only about 1 out of 16 students (6.25%) who are accelerated into a calculus course in high school earn and make use of advanced placement in mathematics.

With the assistance of the Rutgers Office of Institutional Research (OIR)[4], a sample of 400 students was randomly selected from the 2130 students entering Rutgers College in the Fall of 2003, and Rosenstein reviewed all of their high school transcripts to see whether the first conjecture was correct. High school transcripts do not indicate whether students take the AP Calculus exam, but Rutgers OIR provided a list of all 332 Rutgers College students who took the AP calculus exam and their scores, so to test whether the second conjecture was correct, Rosenstein compared the list of students in the sample who took the AP calculus course in high school with the list of students whose scores on the AP Calculus exam were reported to Rutgers. He also used the second list to test the third conjecture about the scores of students on the AP Calculus exam. Finally, Rutgers OIR provided the math courses taken by all of the students in the second list, together with their grades, in their first two years of college, so that the fourth conjecture could also be tested.

[4] We would like to thank Tina Grycenkov of the Rutgers Office of Institutional Research for helping gain approval and cooperation for conducting this study, and for gathering and facilitating my use of the needed information.

The results were as follows:

1. 123 out of 217 students (56.7%) who took a full-year calculus course in high school took an AP Calculus course,
2. 59 out of 123 students (48.0%) who took an AP Calculus course took the AP Calculus exam,
3. 162 out of 332 students (48.8%) who took the AP Calculus exam received advanced placement (either for one semester, if they scored 4 or 5 on the AB exam, or for two semesters, if they scored 4 or 5 on the BC exam)[5], and
4. 66 out of the 162 students (40.1%) who received advanced placement continued with their acceleration (that is, took the next two math courses in their first year at Rutgers).

This enables us to conclude that the percentage of students who take a full-year calculus course in high school who then continue with their acceleration through their first year at Rutgers is the product of these four fractions, that is, (59/217) x (66/332), or 5.4%. Even when we look only at the students who took an AP Calculus course in high school, the percentage of those students who continue with their acceleration through their first year at Rutgers is only (59/123) x (66/332) or 9.5%.

Thus a very small percentage of the students who are mathematically accelerated throughout high school actually continue their acceleration in college and actually take advantage of that acceleration. This data suggests that only 1 of 10 students currently taking AP Calculus actually takes advantage of the "advanced placement" that taking AP Calculus makes possible. One might draw the conclusion that a district or school that encourages 10 times that many students into AP Calculus is misleading its students and distorting its curriculum.

On the other hand, one might argue that taking AP Calculus benefits students even if they do not earn advanced placement and continue with their acceleration. To examine this argument, we consider four groups of students whose acceleration was interrupted and see whether the data sheds light on what happened to these students in college. The remaining 94.6% of the students are made up out of those who:

a. Received a 4 or 5 on the AP Calculus exam but did not continue their acceleration.
b. Took the AP Calculus exam, but received a score of 3 or less.
c. Took the AP Calculus course, but didn't take the AP Calculus exam.
d. Took calculus, but didn't take the AP Calculus course.

As to (a), we noted above that 66 out of 162 students (40.1%) of the students who received advanced placement continued their acceleration. What happened to the other 96 students who didn't?

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[5] There are two Advanced Placement calculus exams, known as the AB exam and the BC exam. Students who take the high school equivalent of a one-semester college calculus course take the AB exam, whereas those who take the equivalent of a two-semester college calculus course take the BC exam; approximately 60% of the BC exam addresses topics on the AB exam.

- 52 took and successfully completed the next math course in their first semester at Rutgers and then took no additional math courses
- 17 took the next math course in their first semester at Rutgers, but received poor grades and then took no additional math courses
- 17 took a semester off – that is, they took no math in one of their first two semesters at Rutgers, but then resumed the calculus sequence
- 3 rejected advanced placement and started college with Calculus 1
- Of the remaining 7 students, 1 took no math courses, 1 took a math course for liberal arts students, and 5 had partial success in their math courses

The only benefit received by these students from taking AP Calculus is that it enabled 52 of them to stop taking mathematics a semester earlier – they were accelerated for four years so that they could get done with mathematics a semester earlier! On the other hand, it seems that 25 of these students (17+3+5) were not properly prepared by the AP Calculus course for their next math course.

As to (b), we noted above that 162 out of 332 students (48.8%) who took the AP Calculus exam received advanced placement. What happened to the other 170 students who did not receive advanced placement?

Although all of these students had on their high school transcripts the prerequisites for a first-semester calculus course, entry into Calculus 1 at Rutgers is permitted only to those who score sufficiently high on a reliable calculus-readiness placement test that has been administered by the Rutgers Department of Mathematics for almost 30 years. Of the 170 students who did not receive advanced placement, 131 placed into Calculus 1, but 30 were required to take a prerequisite course – precalculus or even Algebra 2; that is, 30 of these students were not ready for calculus even though they took an AP Calculus course in high school. These students were clearly disadvantaged by being permitted or even encouraged to take AP Calculus in high school.

Like most colleges and universities, Rutgers has two calculus tracks. One track begins with a "mainstream" course that leads to upper division courses in the mathematical sciences and that is taken by students majoring in math, physical sciences, and engineering; the AP Calculus AB course is intended to be comparable to the first semester of this track and the AP Calculus BC course in intended to be comparable to the first two semesters of this track. The other track is a two-semester terminal course, one or both semesters of which are typically taken by biology, business, economics, and psychology majors. Of the 131 students who were placed into Calculus 1, 40 began the mainstream track and 91 the non-mainstream track. It is striking that the first college mathematics course taken by all but 40 of the 170 students who were unsuccessful in gaining advanced placement was a lower level course than the AP Calculus course that they took in high school. Students who take advanced level courses in high school should not be taking lower level courses when they get to college, and high schools should not be expending their resources to prepare their students to take easier courses when they get to college.

It is true, however, that almost all of the 131 students who took Calculus 1 were successful in their respective courses, all but 5 receiving grades of C or better and all but 35 receiving grades of B or better. However, this is not an argument for acceleration, since it is equally true, for example, that students who repeat Algebra II would learn it better than those who take it just once. Moreover, it is not unexpected that after completing what is advertised as a higher level calculus course, students will do well on a lower level course. Finally, it should be noted that only 52 of these 131 students continued on to Calculus 2; the other 79 students successfully completed only one semester of calculus.

As to (c), our sample of 400 transcripts included 64 students who took the AP Calculus course but did not take the AP Calculus exam. What is most striking about this group is that 20 of these students took no math course in their first two years at college; it is as if AP Calculus functioned for them as a deterrent from math. Another 10 were placed into lower level courses. Only 33 took calculus, 15 the mainstream course, and 18 the terminal course. Most of them were successful in this course, all but 4 receiving grades of C or better. However, only 12 took Calculus 2 and only 7 of those received a grade of C or better. Once again, we can say that taking AP Calculus might have been of benefit for only a small percentage of students, 7 out of 64, in this category.

As to (d), our sample of 400 transcripts included 94 students who took a full-year non-AP calculus course in high school. Only 6 of the 94 took no math courses in college, a far lower percentage than the 20 out of 64 students who took the AP Calculus course but not the AP exam; although taking AP Calculus served as a deterrent to math for many students, taking non-AP calculus did not. A higher percentage of these students took Calculus 1 than the preceding group (68 out of 94, or 72.3% compared to 37 out of 64, or 57.8%) and a higher percentage successfully completed Calculus 2 (15 out of 94, or 16.0%, compared to 7 out of 64, or 10.9%).

From all of these data it is reasonably clear that the preponderance of the students who took AP Calculus but did not receive advanced placement did not really benefit from taking AP Calculus in high school and would have been better served by a non-AP Calculus course.

Why then do so many students take AP Calculus in high school? That question was one of the primary motivations for the second study that was conducted in the Spring of 2011 by both authors.

In the second study, we conducted an online survey with the cohort of undergraduate students who entered the School of Arts and Sciences of Rutgers University-New Brunswick in the fall of 2007 (and were therefore about to graduate) and whose scores on the AP Calculus exam had been reported to Rutgers.[6] In the survey we asked them a number of questions related to their reasons for taking AP Calculus, their undergraduate programs, and their intended careers. The Office of Institutional Research (OIR) at Rutgers provided us with the list of students in this cohort and their email addresses; OIR also provided us with the students' AP Calculus scores and a list of all the college math courses that they had taken over the past four years and their grades in these courses.

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[6] The survey did not include students in the College of Engineering. Perforrnance of engineering students on the AP Calculus tests is discussed in a later footnote.

The 478 students on the list who were still at Rutgers were invited to participate in an online survey of about 25 questions; they were given a week to respond to the survey and those that had not yet completed the survey received two reminders during that week to do so. To provide the students incentive to participate in the survey, those completing every question in the survey were to be entered into a raffle for an I-Pad 2. Altogether 194 of the 478 students actually completed the survey; the high participation rate (40.6%) was presumably due, at least in part, to the popular incentive we provided.

In the questionnaire, we asked students to provide the following information, and we report on some of their responses in this abbreviated article.

- Why they took the AP Calculus course and exam.
- Whether taking AP Calculus increased their interest in math and/or encouraged them to pursue studies leading to a career in math, science, or engineering.
- What math course they actually took in their first semester in college.
- Why they were unsuccessful in getting AP credit (if that was the case).
- How many math courses they took in college.
- Whether taking math courses in college increased their interest in math and/or encouraged them to pursue careers in math, science, or engineering.
- What were their majors and what were their intended careers.
- Whether they thought that they had benefited from taking AP Calculus in high school.

To analyze the survey responses, we divided the participants into three groups:

A. Group A consisted of the students who had earned credit for both Calculus 1 and 2 (score of 4 or 5 on the BC exam),

B. Group B consisted of the students that received credit for Calculus 1 (score of 4 or 5 on the AB exam or on the AB portion of the BC exam) but not for Calculus 2, and

C. Group C consisted of the students that did not receive college credit for Calculus 1 (score of 3 or below on the AB exam).

Of the 194 students who completed the survey, 37 (19.1%) were in Group A, 58 (29.9%) were in Group B and 99 (51.0%) were in Group C. Thus in this cohort, the percentage of students taking the AP Calculus exam who received advanced placement was 49.0%, in accordance with the third conjecture addressed in the first study.[7] [8]

To learn why students took the AP Calculus course, we provided the students with 15 reasons and asked them to rate each statement according to 5 possible choices: strongly agree, agree, somewhat agree, disagree, and strongly disagree. In the following table, the entries are the percentage of students who agreed or strongly agreed with the

[7] Actually, of all 478 students who took the AP Calculus exam, only 193, or 40.4%, received advanced placement; it is not surprising that the students who were more successful in AP Calculus were a bit more likely to participate in a survey related to AP Calculus than those who were unsuccessful.

[8] Of the 216 students in the College of Engineering, 134 received advanced placement. If they are combined with the students in the College of Arts and Sciences, 327 out of 694, or 47.1%, received advanced placement, which is still less than half of the students who took the AP test.

reason given. (Note that in the survey itself, the statements were not organized into categories, nor were they presented in the order that they appear in the table.)

Statement	**Group A N=37**	**Group B N=58**	**Group C N=99**
Intrinsic rationales			
I really liked math when I was in high school	94.6	89.6	67.7
I wanted to learn more higher level mathematics	83.8	79.4	45.4
I enjoy challenging math courses	89.2	79.0	52.0
Educational rationales			
I wanted to start off in college with a higher level course	83.8	84.5	74.7
I wanted to be better prepared for college courses	78.4	74.2	72.7
I planned to major in a math related subject	67.6	45.6	25.2
Pragmatic rationales			
AP Calculus looks good on college applications	78.4	82.7	80.8
Taking AP courses would enable me to save money by graduating from college in three years	21.6	20.7	24.2
Taking AP course would boost my GPA	45.9	32.8	30.3
Social rationales			
My friends were taking AP Calculus	81.1	53.5	69.7
My parent wanted me to take AP Calculus	51.4	39.7	46.5
It was expected of me	78.4	62.1	61.2
Default rationales			
I had to take some math course in my senior year	62.2	48.3	70.6
My math teachers or counselors suggested that I should take it	73.0	69.0	75.5
Negative rationales			
I wanted to avoid taking math in college	05.4	05.1	30.3

Why did students take AP Calculus in high school? There is clearly no simple answer. At least 2/3 of Group A students agreed or strongly agreed with 10 of the 15 reasons in the survey, and at least 2/3 of each of Group B and Group C students agreed with 7 of the 15 reasons.

Five reasons appeared on all three lists – that is, in all three groups, 2/3 of the students responded most positively to:

- I really liked math when I was in high school
- I wanted to start off in college with a higher level course
- I wanted to be better prepared for college courses
- AP Calculus looks good on college applications
- My teachers or counselors suggested that I take it.

If we look instead at the reasons with which students “strongly agreed,” we find that the top five reasons for students in Group A were

- I really liked math when I was in high school (83.8%)
- I wanted to start off in college with a higher level course (70.3%)
- I enjoy challenging math courses (64.9%)
- AP Calculus looks good on college applications (56.8%)
- I had to take some math course in my senior year (54.1%)

Whereas 20 of the 37 students (54.1%) in Group A gave the fifth response above, 19 students each “strongly agreed” that “I wanted to learn more higher level mathematics,” “I planned to major in a math related subject, “my friends were taking AP Calculus,” and “it was expected of me.”

The top five reasons for students in Group B were

- I wanted to start off in college with a higher level course (58.6%)
- I really liked math when I was in high school (51.7%)
- AP Calculus looks good on college applications (51.7%)
- I wanted to be better prepared for college courses (48.3%)
- I wanted to learn more high level mathematics (46.6%)

The top five reasons for students in Group C were

- AP Calculus looks good on college applications (48.5%)
- I wanted to start off in college with a higher level course (44.4%)
- I had to take some math course in my senior year (42.9%)
- I really liked math when I was in high school (41.4%)
- I wanted to be better prepared for college courses (39.4%)

What stands out in these data are:

- The difference between the enthusiastic response of Group A students to the intrinsic rationales for taking AP Calculus and the lukewarm response to those rationales among Group B and Group C students.
- The belief that AP Calculus looks good on college applications and the concomitant, though often undeclared, encouragement of math teachers and counselors, plays a prominent motivational role in students’ taking AP Calculus. These factors seem to play a greater role than pressure from parents or peers.
- The percentage of students who “strongly agreed” to any of the reasons decreased substantially from Group A to Group B to Group C.
- Not surprisingly, the students with the strongest performance on the AP Calculus exam (Group A) had the most positive attitudes towards mathematics. They

really liked math in high school, wanted to start off college with higher-level math course, wanted to learn higher-level math and enjoyed challenging math courses.

- Approximately 71% of Group C agreed or strongly agreed that they took AP calculus because they *had* to take some math in their senior year and 30% of Group C students who took the AP Calculus course wanted to avoid taking math in college.
- While a high percentage (25 out of 37, or 67.6%) of Group A students planned to major in a math related subject in college, fewer than half (26 out of 58, or 45.6%) in Group B and barely a quarter (25 out of 99, or 25.2%) in Group C planned to major in a math related subject. Thus a substantial percentage of the students in groups B and C took AP calculus even though they did not plan to major in a math related subject and were, therefore, not seeking to be "advanced" in mathematics. On the other hand, it is interesting that the number of students in each of the three groups that agreed or strongly agreed that they "planned to major in a math-related subject" was 25 or 26; that is, the potential number of math-related majors from each of the three groups is about the same, although the three groups are of different sizes.
- In all three groups more than half of the students said that it was expected of them to take AP calculus in high school. It is striking that so many students felt that they were "expected" to take the "advanced" math course. The exception has become the norm, that is, whereas AP Calculus was originally introduced to permit the best students to obtain "advanced placement" in mathematics, it was now expected that students should routinely take AP Calculus.
- None of the three groups thought that saving money (by spending fewer years in college) was an important reason for taking the course and fewer than half of the students in each group thought that taking the course would boost their GPA.

It is clear from looking at the observations above that the three groups had very different reasons for taking AP calculus in high school. While students in Group A were more inclined to take higher level math courses and pursue a math-related major, many students in Group C wanted to be done with math quickly and thought that AP Calculus would help them achieve that goal.

In the survey we also asked students why they took the AP Calculus *exam*, since in the previous study we had learned that only about half of the students who take the AP Calculus course take the AP Calculus exam. We offered students 10 possible reasons for taking the AP Calculus exam and offered the same options as possible responses. In this table also, the entries are the percentage of students who agreed or strongly agreed with the reason given.

Statement	**Group A N=37**	**Group B N=58**	**Group C N=99**
Intrinsic rationales			
I enjoyed studying math and taking math exams.	86.5	67.3	43.4
Educational rationales			
I wanted to start off at college with Calc 2 or Calc 3.	78.4	81.1	29.3

I thought that I would learn the course material better if I planned to take the exam.	59.4	41.4	52.5
Pragmatic rationales			
Colleges give extra consideration to applicants who are planning to take AP exams.	56.7	48.3	41.4
Taking the AP exam and getting advanced placement would enable me to save money by graduating from college in three years.	21.6	22.4	25.3
Social rationales			
My friends were taking the AP Calculus Exam.	62.1	48.3	48.4
My parents wanted me to take the AP Calculus Exam.	54.0	41.3	44.5
Default rationales			
My math teachers or counselors suggested that I should take it.	67.5	68.9	70.7
Negative rationales			
I wanted to take as few math courses as possible in college.	21.6	13.8	45.4
Requirements			
All students in AP Calculus were required to take the exam.	29.7	31.0	39.4

Why did students take the AP Calculus *exam*? Again the answer is not clear and differs among the three groups.

- About 80% of the students in Groups A and B (Group A more "strongly") wanted to start college with Calc 2 and Calc 3 but very few students in Group C wanted to do the same. This reflects, again, that many students in Group C were not looking for an "advanced placement" in math when they reached college.
- As expected, Group A students enjoyed studying math and taking math exams the most, followed by Group B and then Group C.
- Group A students were more likely than the other groups to agree that "Colleges give extra consideration to applicants who are planning to take AP exams." It is not clear that this belief corresponds to reality since colleges make decisions about applications before any AP exam scores are received.
- Significantly more students in Group C wanted to "take as few math courses as possible in college". This reflects that Group C students were less inclined towards a math-intensive major and perhaps viewed AP Calculus as a means to avoid future math courses. It is striking that, although 30.3% gave this as a reason for taking the AP *course* in the previous question, this percentage rose by half to total 45.4% when the students were asked to explain why they took the AP Calculus *exam*. Clearly the AP course served to discourage these 15 students from continuing to take math courses.
    - This data confirms a conjecture in the previous study that "the acceleration strategy has produced a lot of negative attitudes about mathematics," as 45 out

of the 99 students in Group C want to "take as few math courses as possible in college."

- The strongest reason by far for students in Group C to take the AP Calculus exam if that "my math teachers or counselors suggested that I take it." Although a similar percentage of students in Groups A and B agreed or strongly agreed with that reason, in both of those groups the first two reasons and, in particular, the intention to start off at college with Calc 2 or Calc 3 was more prominent.
- If we compare the responses in the second table to the statement "I wanted to start off at college with Calc 2 or Calc 3" with the responses in the first table to the comparable statement "I wanted to start off in college with a higher-level course," we find a major difference between both Groups A and B on the one hand and Group C on the other. About 80% of the students in both Groups A and B responded "agree" or "strongly agree" to both of those statements. However, although 72.7% of the students in Group C responded "agree" or "strongly agree" to the first statement (as to why they took the AP *course*), only 29.3% responded "agree" or "strongly agree" to the second statement (as to why they took the AP *exam*). That is, as a result of taking the AP course, about 60% of the Group C students who intended to start college with a higher-level math course had lowered their expectations so that by the end of the school year, when the AP calculus exam was given, they no longer expected that they would earn advanced placement.

We asked students to describe the extent to which taking AP calculus and its prerequisites in high school encouraged or discouraged them from pursuing studies leading to a career in math, science, or engineering; they had to choose one of the following five options – "They encouraged me a great deal toward such careers," "They encouraged me somewhat toward such careers," "Taking those courses didn't make much of a difference," "They discouraged me somewhat from such careers," and "They discouraged me a great deal from such careers." Students were offered the opportunity to provide a written explanation of their response to this question and a substantial number (139 out of 194, or 71.3%) responded.

- About 2/3 of the students in both Groups A and B responded that taking the AP Calculus course encouraged them toward careers in math, science, or engineering, and about 1/3 indicated that taking these courses made no difference. These fractions are reversed for Group C; that is, only 1/3 said that taking the AP Calculus course encouraged them toward careers in math, science, or engineering, and 2/3 indicated that taking these courses made no difference or discouraged them from such careers.
- For a great number of students in this survey (one third approximately in groups A and B and approximately half in Group C) taking the AP Calculus course made no difference toward their decisions to follow a STEM career path.
- Although many students indicated that taking the AP Calculus course reinforced their intentions to become STEM majors, there were essentially no students who indicated in their comments that they planned to become STEM majors because they took the AP Calculus course. This finding is significantly contrary to the

assumption of “Rising Above the Gathering Storm” that taking AP calculus course serves as a means to expand the STEM pipeline.

- In Group C almost 20% of the students who had taken a year of AP Calculus, were considered by Rutgers as being unprepared to take Calculus 1. Overall, 12% of all students (as Group C made up 60% of this cohort) who took the AP exam were unprepared for calculus at Rutgers even after taking a full-year AP Calculus course in high school. These students were not prepared for AP Calculus in the first place, and should have been steered away from AP Calculus.

As in the previous study, we examined the math courses taken by the students at college and their grades. Although for the first study Rosenstein only had data on math courses taken in the first two years, for the present study we had data on courses and grades for four years. For the present study, we also knew the subjects in which students had majored, since we asked them this question on the survey.

- In Group A, 86.4% were majors in math, science, or engineering, and 70.3% were majors in math-intensive subjects (including the physical sciences) that required a number of math courses.
- In Group B, 75.9% were majors in math, science, or engineering and 43.1% majored in math-intensive subjects. A high percentage of Group B students were inclined towards the biological sciences that required fewer math courses.
- In contrast, among students in Group C, only 35.4%, were majors in math, science, or engineering and only 11.1% had math-intensive majors; there was a sharp decline from the number of students who intended to major in a math-related subject to the number who actually did, presumably because a large percentage of these students were put off from math by their experience in the AP Calculus course.

In the survey, we asked students the “Yes-No” question “Are you planning a career in math, science, engineering, or technology?” and we also asked them to tell us what career they were planning to pursue. 174 students answered that question, and the remaining 22 wrote “undecided” or did not tell us what career they were planning to pursue – thus 52/174 in the bottom cell at the right means 52 of the 174 who actually responded with their intended career. The results are summarized in the following table:

	Group A (37 students)	Group B (58 students)	Group C (99 students)	Total (194 students)
Intends career in math, science, engineering	31 (83.8%)	47 (81.0%)	46 (46.5%)	124 (63.9%)
Listed career in math, science, engineering	26 (70.3%)	41/52 (78.8%)	25/83 (30.1%)	92/174 (52.9%)
Listed career in math-intensive subject	20 (54.1%)	20/52 (38.5%)	12/83 (14.5%)	52/174 (29.9%)

Summary of intended careers of all students in study

- There is clearly a discrepancy between what the students regard as "math, science, engineering, or technology" and what is commonly understood as falling under these categories. On the other hand, there seems no general agreement as to what falls under these categories. For example, there does not seem to be general agreement as to whether "physician" is a STEM career.
- The discrepancy between the percentage of Group C students who indicated that they were pursuing careers in math, science, or engineering (first row of table) and the percentage of Group C students who listed careers in math, science, or engineering (second row of the table) can be explained by observing that the health-related careers that many students indicated they were pursuing are usually not considered careers in math, science, or engineering (or STEM careers) since the science requirements for those careers are often very modest.
- Only 10 of the 194 students intend to be high school teachers (7 math and 3 science), areas in which the United States has a critical shortage.[9] Only a handful indicated that they intend to be engineers, but that is not surprising since the survey involved only students in the School of Arts and Sciences, and not those in the School of Engineering.
- Certainly increasing the number of students taking AP Calculus is not having the desired effect of increasing the number of students intending to teach in high school.

In the survey we asked students whether they benefitted from taking AP Calculus in high school and whether they would recommend it for future students, and most agreed with those conclusions. However, some of the students cited AP calculus as a vehicle that allowed them to take fewer math courses at college and avoid experiencing bad teaching at the college-level.

Recommendations

The two most important findings of these studies are that, first, a very small percentage of those who are accelerated throughout high school maintain that acceleration through their first year at college and, second, that there is no evidence that encouraging more students to take AP Calculus will expand the STEM pipeline.[10]

Yet the curriculum in many schools is organized to accelerate students into AP Calculus – beginning with the policy of expecting more (or even, all) students to take Algebra 1 in the 8th grade – often taught by middle school teachers who lack the appropriate mathematical background or credentials.

[9] According to an article by Christopher Drew in the New York Times of 11/4/11 entitled "Why Science Majors Change Their Minds": "The president and industry groups have called on colleges to graduate 10,000 more engineers a year and 100,000 new teachers with majors in STEM — science, technology, engineering and math."

[10] An important exception must be noted. There are many schools in the nation (perhaps even in New Jersey) in which AP Calculus is not offered at all, and we may lose from the STEM pipeline those students from such schools who would belong to Groups A and B.

Moreover, more and more students are encouraged to take AP Calculus, including those students who struggled to complete Algebra 2, Geometry, and Precalculus in the 9th, 10th, and 11th grades.

The driving force behind the practice, as seen from the students' responses and the recommendations of their teachers and counselors, is that colleges routinely favor students for admission if they have AP Calculus on their transcripts.

The basic recommendation is "STOP these practices."

Many detailed recommendations follow. But none of these will be implemented until colleges change their admissions policies and practices – that is, they should not "routinely" favor students with AP Calculus.

Many students, teachers, counselors, and principals believe that taking AP Calculus enhances the possibility of admission to college. As a result, many students take AP Calculus who do not benefit from that course. Ideally, colleges should announce that an application for admission will only be enhanced by taking AP courses if the students take the exams and score 4 or 5 on them. Unfortunately, that is impossible since the students are typically admitted to college well before the AP exams are given. However, we encourage colleges to adopt a policy of the following type in evaluating the AP credentials of applicants for admission.

Recommendation to Colleges on Admission Policy: With each student's application to admission to college, the high school must submit a document indicating, for each of the previous five years, the number of students who took an AP Calculus course, the number of students who took the AP Calculus exam, and the grades of those students on the AP Calculus exam. Each college should modify their admissions policy so that it gives extra weight for taking the AP Calculus course only to those students whose high schools can report that they treat the course seriously, not just as a means for enhancing college admissions. For example, a college might decide to give extra weight if 90% of the students taking the AP Calculus course in that high school took the AP Calculus exam, and that 75% of the students who took the exam received a grade of 4 or 5.

Recommendations on curriculum and entry to AP Calculus:

- Students should take Algebra 1 in 8th grade only if they have teachers who have the appropriate background and certification in teaching mathematics.[11]
- Students should be discouraged from taking any course if they did not achieve at least a B (and ideally a B+) in the previous course.[12]

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[11] The transcripts in the first study show that although Algebra 1 in the 8th grade benefits the best students by smoothing their path to AP Calculus, there is no evidence that it benefits average students. That is, for average students, the Rutgers course-taking patterns of those who took Algebra 1 in the 8th grade was not significantly different from those who didn't take Algebra 1 until the 9th grade. The most striking difference between the two groups is that 32 of the students who took Algebra 1 in the 8th grade did not take any mathematics course in their first two years at Rutgers.

[12] If parents insist on their child's taking the next course, as they may be able to do in many locations, then the student should be required to take an accompanying course, for as long as necessary, to remediate his or her deficiencies.

- Students should be encouraged to take an AP Calculus course only if they
  - received an A or B+ in the Precalculus course,
  - indicate that they like mathematics and wish to be challenged,
  - intend to take more advanced math courses in their first year in college, and
  - are strongly considering a career that requires a substantial number of college mathematics courses – i.e., mathematics, statistics, and the physical sciences.[13]

  Otherwise, AP Calculus is not the ideal course for them.
- School counselors should receive training that will enable them to identify students who should take the AP Calculus course and to actively and convincingly discourage students who should not be taking AP Calculus. This includes
  - students who are not prepared for AP Calculus – recall that at least 18% of the students who had completed a full-year AP Calculus course were placed at Rutgers into Intermediate Algebra or Precalculus
  - students who are intending to major in a subject that has limited mathematical prerequisites (including biology) – recall that only 76 of the 194 students agreed or strongly agreed that they took the AP Calculus course as "I planned to major in a math related subject".
  - students who will require extra help or private tutoring to keep up with the material in the course.
- Students and parents should be made aware that the AP Calculus course is challenging, intended for math-intensive majors and not necessarily going to lead to an advanced placement in mathematics or save them time in college.
- All students who take the AP Calculus course should be required to take the AP exam.
- AP Calculus should not be the "default" mathematics option for seniors, that is, the course that they take automatically after they complete Precalculus.[14] Since all college-bound students should take a math course in their senior year of high school, schools should design and offer other attractive and interesting courses that seniors can take after completing Precalculus. These courses should include material that reinforces algebra and geometry skills, should include probability,

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[13] One has to question on an individual level why many Group C students took the AP course when they already knew they were not going to be a STEM major down the road. Also, one has to ponder at a policy level if it is it worthwhile to spend resources teaching calculus to students similar to those in Group C.

[14] It appears that treating AP courses as "default" options is not unique to mathematics. Indeed, it seems that many students take several AP courses in their senior year because their high schools do not offer anything else. Although it is conceivable that it is appropriate for a student to take, simultaneously, AP courses in chemistry, mathematics, computer science, and English, it is rather unlikely that the student feels equally passionate about all four. It would make sense for high schools to require students to choose one subject (or at most two subjects) in which they take the AP courses.

statistics, and discrete mathematics, and should build on the Precalculus course (including some calculus topics) so that students completing these courses will be prepared for college math courses.

In this study we found no evidence that increasing the number of students taking AP Calculus expands the STEM pipeline. If that will not expand the STEM pipeline, what will? The problem is not that there are too few students in the STEM pipeline, but that the pipeline is too leaky, that is, students who were previously attracted to math (and similarly for science) have decided not to pursue careers in those areas. The national focus should not be on recruitment, but on retention of students already in the STEM pipeline; this applies at all grade levels, but perhaps particularly at the college level.[15] [16] [17]

But even at the high school level, there are important strategies for encouraging students to consider STEM careers:

- Implement appropriate and early interventions to cultivate and enhance younger students' interest in mathematics, science and engineering. For example, students can be exposed to field trips related to STEM careers or be given an opportunity to talk to real scientists, mathematicians and engineers to discuss the challenges and rewards of such careers.
- Interestingly, in all three groups in the second study a number of students claimed that the teachers were far more influential than the course itself in terms of motivating them to pursue or not pursue a STEM major. Teachers of science and mathematics, and particularly the teachers of AP Calculus, should be trained to discuss career options and real life applications of the ideas that students learn in the course.
- Ensure that students have positive experiences in their math classes by developing interesting courses with material that students see as relevant to their education, their careers, and their lives; that students get the assistance that they need to

[15] According to an article by Christopher Drew in the New York Times of 11/4/11 entitled "Why Science Majors Change Their Minds": "Studies have found that roughly 40 percent of students planning engineering and science majors end up switching to other subjects or failing to get any degree. That increases to as much as 60 percent when pre-medical students, who typically have the strongest SAT scores and high school science preparation, are included, according to new data from the University of California at Los Angeles."

[16] On the other hand, the current economic situation may have had a major impact on the number of students who are pursuing STEM careers. David Bressoud reports, based on data from the UCLA's Higher Education Research Institute, that: "There has been a strong upward trend toward mathematics, the sciences, and engineering over the past decade. It has recently accelerated. In the past five years, the number of students intending to major in Mathematics has risen by 31%, in the Physical Sciences by 37%, in Engineering by 44%, and in the Biological Sciences by 67%. Most of this growth has occurred in just the past three years, since 2007. This is most dramatic within Engineering, which went from 102,000 freshmen intending to major in this discipline in the Fall of 2007 to 156,000 in Fall 2010."

[17] It should also be noted that a substantial percentage of the students who participated in this survey, between 84.2% and 91.8%, of those who intended to major in math, science, or engineering actually did. This observation should not be taken to imply that taking AP Calculus results in staying in the STEM pipeline, but rather that those who intend to stay in the STEM pipeline typically take AP Calculus.

succeed in mathematics; and, that they are not pushed ahead at a pace that will cause them to avoid mathematics in the future.