

DIMACS Center  
Rutgers University

**DIMACS Research Experience for  
Undergraduates (REU) Program**

**Report**

November 2002

## **Ia. Participants from the program**

### **Participants:**

#### **DIMACS REU Domestic Program:**

Daniel Boylan, Harvey Mudd College, Claremont, CA  
Jakub Černý, Charles University, Prague, Czech Republic  
Zdeněk Dvořák, Charles University, Prague, Czech Republic  
Inessa Epstein, Rutgers University, New Brunswick, NJ  
Andrew Hodges, Manchester College, North Manchester, IN  
Vít Jelínek, Charles University, Prague, Czech Republic  
John McClain, Rutgers University, New Brunswick, NJ  
Hoa Nguyen, Rutgers University, New Brunswick, NJ  
Pavel Podbrdský, Charles University, Prague, Czech Republic  
Thomas Schneider, Rose-Hulman Institute of Technology, Terre Haute, IN

#### **DIMACS REU Prague Program:**

Paul Gross, Rose-Hulman Institute of Technology, Terre Haute, IN  
Steven Jasar, Rutgers University, New Brunswick, NJ  
Daniel Krasner, University of California at Berkeley, Berkeley, CA  
Yuki Saka, University of California at Berkeley, Berkeley, CA  
Ursula Whitcher, Swarthmore College, Swarthmore, PA

#### **VIGRE Program:**

Rahul Malhotra, Rutgers University, New Brunswick, NJ  
Michael Lesnick, Brown University, Providence, RI  
Stanislav Ostrovsky, Rutgers University, New Brunswick, NJ  
Samuel Stechmann, University of St. Thomas, St. Paul, MN  
Rupert Venzke, University of Pittsburgh, Pittsburgh, PA

#### **Organizers:**

Fred Roberts, Director, DIMACS, Rutgers University  
Janos Komlos, Dept. of Mathematics, Rutgers University  
Martin Mareš, Charles University, Prague, Czech Republic  
Jaroslav Nesetril, Director, DIMATIA, Charles University, Prague, Czech Republic  
Jose Torres, Graduate Student Assistant, Rutgers University  
Martin Mares, Graduate Student Assistant, Charles University

#### **Speakers:**

Stephanie Forrest, Professor of Computer Science, University of New Mexico  
Ira Schwartz, Naval Research Laboratory  
Eva Curry, Graduate Student, Rutgers University  
Stephen Greenfield, Director of the Mathematics Graduate Program, Rutgers University  
Jeff Kahn, Professor of Mathematics, Rutgers University  
Eric Allender, Professor of Computer Science, Rutgers University  
Rebecca Wright, Professor of Mathematical Sciences, Stevens Institute of Technology  
Robert E. Jamison, Professor of Mathematical Sciences, Clemson University  
Jozsef Beck, Professor of Mathematics, Rutgers University

#### **US Faculty Mentors:**

Eric Allender, Computer Science Department, Rutgers University  
Endre Boros, RUTCOR, Rutgers University  
Vladimir Gurvich, RUTCOR, Rutgers University  
Peter Hammer, RUTCOR, Rutgers University  
Leonid Khachiyan, Department of Computer Science, Rutgers University  
Friedrich Knop, Department of Mathematics, Rutgers University  
Janos Komlos, Department of Mathematics, Rutgers University  
Carlo Lancellotti, Department of Mathematics, Rutgers University  
Peter Landweber, Department of Mathematics, Rutgers University  
Matthew Leingang, Department of Mathematics, Rutgers University  
Brian Lins, Graduate Student, Department of Mathematics, Rutgers University  
David Madigan, Department of Statistics, Rutgers University  
Ofar Melnik, DIMACS, Rutgers University  
Kobbi Nissim, DIMACS, Rutgers University  
Wilma Olson, Department of Chemistry and Chemical Biology, Rutgers University  
Shawn Robinson, Department of Mathematics, Rutgers University  
Michael Rothkopf, RUTCOR, Rutgers University  
Avy Soffer, Department of Mathematics, Rutgers University  
Rebecca Wright, DIMACS, Rutgers University

**Faculty Members in Prague:**

Martin Klazar, Charles University  
Jan Kratochvil, Charles University  
Martin Loeb, Charles University  
Jiri Matousek, Charles University  
Jaroslav Nesetril, Charles University and Director of DIMATIA  
Pavel Valtr, Charles University

**Ib. Participating Organizations**

Telcordia Technologies: Facilities; Personnel Exchanges  
Partner organization of DIMACS. Individuals from the organization are participating in the program seminar series and hosting visiting students.

AT&T Labs - Research: Facilities; Personnel Exchanges  
Partner organization of DIMACS. Individuals from the organization are participating in the program seminar series and hosting visiting students.

NEC Research Institute Inc: Facilities; Personnel Exchanges  
Partner organization of DIMACS. Individuals from the organization are participating in the program seminar series and hosting visiting students.

Lucent Technologies, Bell Labs: Facilities; Personnel Exchanges  
Partner organization of DIMACS. Individuals from the organization are participating in the program seminar series and hosting visiting students.

Princeton University: Facilities; Personnel Exchanges  
Partner organization of DIMACS. Individuals from the organization are participating in the program planning and seminar series.

Charles University: Financial Support; Facilities; Collaborative Research; Personnel Exchanges  
DIMATIA is a major research center at Charles University in Prague, Czech Republic, and our partner in the program.

## **II. Project Activities**

The DIMACS/DIMATIA REU program had the following three parts: (i) A group of students from all across the U.S. participated in an 8-week REU program headquartered at DIMACS; (ii) A second group of five students from all across the U.S. participated in the 8-week domestic REU program and then spent 3 additional weeks at DIMATIA; (iii) a group of students from the Czech Republic participated in the 8-week domestic REU program and then acted as hosts at DIMATIA when the U.S. students went to Prague.

The overall goal of the program was to provide the participants with an exciting research experience that would help them decide on future educational and career paths. All students, including those participating only in the domestic program, got a taste of the international scientific enterprise. The U.S. students going to Prague got a more direct international experience and benefited from the scientific atmosphere at an international center of research, DIMATIA. The Czech students benefited similarly from exposure to their U.S. counterparts and, moreover, contributed to providing the global perspective that we sought. Our REU program is unique because it is run in the context of two major research centers with many scientific activities. The richness of the intellectual community and the international flavor contributed by the many foreign scientists participating in DIMACS and DIMATIA activities at the same time as the REU students added to the overall atmosphere.

The key to our REU program was the one-on-one research experience under the direction of a mentor. The domestic part of the program officially began when the students arrived on the Rutgers campus in mid-June, moved into campus housing (where the students were housed in a group), and received offices and computer and library accounts at DIMACS. They were met by a graduate student coordinator who organized activities aimed at getting the REU students to meet each other and introduced them to their mentors to begin a program of directed study and research, including regular student/mentor meetings. There were regular lunches and teas, to which all the mentors and students were invited, as well as our weekly DIMACS REU Seminar Series hosting both local speakers and renowned outside speakers. The seminars were preceded by a pizza lunch and followed by opportunities to interact with the speaker. One of these seminars was devoted to a presentation about careers and graduate school.

We introduced the students to industrial research by making a trip to AT&T Labs, where our students got a tour and there were technical presentations. A smaller group visited Bell Labs. We also introduced them to interdisciplinary research through a visit to the Nucleic Acids Database (NDB) at Rutgers and presentations of their work to undergraduate students at the NDB, and presentations from the NDB students. Students were encouraged to take advantage of all of the activities at DIMACS. Many of them attended lectures at our International Conference of Computational and Mathematical Epidemiology, at a Workshop on Graph Labelings, and of the DIMACS Connect Institute. A few attended the DIMACS Workshop and Tutorial on Data Mining for Homeland Security at the Institute for Defense Analysis in Princeton.

Students were asked to make several presentations about their projects during the course of the program. Near the beginning of the program, each student made a presentation describing their research problem. These short presentations, made before the entire REU group plus mentors, were aimed at introducing the whole group to the research topics of other members. This encouraged collaboration and discussion and

provided the opportunity, as appropriate, to work on multiple projects or even switch projects. Near the end of the domestic program, students made short presentations about their work to an audience consisting of REU students, mentors, and others in the DIMACS community. Students going to Prague repeated these presentations soon after their arrival at DIMATIA, in order to give the DIMATIA faculty and students an idea of the research activities of the visitors, and made further presentations of their work before they departed Prague. Several of our students were also selected to make presentations during the course of the summer to various DIMACS audiences.

In addition to oral presentations, students were asked to prepare personal websites. These websites described their problem area and were filled in during the program and after the program ended with results that the student obtained.

The DIMACS part of the program did not end with the end of the DIMACS stay. Our U.S. and Czech REU students have been encouraged to stay in touch with their mentors by email, prepare web pages with their work, and keep those pages up to date after they left. Some of them ended up working on a research paper and/or preparing a presentation at a scientific meeting, still under the guidance of their mentor, after they left DIMACS. Many of our students planned or presented talks back at their home institutions and others planned to make their REU project a major piece of their senior thesis or even eventually of a master's thesis.

The REU program was enhanced by joining forces with the Rutgers Mathematics Department's VIGRE REU program, which meant that we had a total of some 20 students on hand during the summer and there was outstanding cross-fertilization. We housed the VIGRE students in DIMACS offices with the DIMACS-DIMATIA students and put them up in the same dormitory, and they participated jointly in seminars and presentations.

The goals of the experience in Prague were different from those for the experience in the U.S., and centered around introducing the participating students to a wealth of open problems and questions and appropriate problem-solving techniques and strategies.

We did not assign students going to Prague to an individual DIMATIA faculty mentor in advance. Rather, Jaroslav Nesetril, director of DIMATIA, served at first as acting mentor for each of these students. The DIMATIA faculty were involved in mentoring when the students arrived in Prague.

Prior to leaving for Prague, students met several times with DIMACS staff to discuss the experience and the logistics. In addition, the Czech REU students and a Czech graduate student coordinator who accompanied the Czech students to DIMACS were available to offer pre-trip advice.

The Prague experience lasted three weeks. The U.S. graduate student coordinator accompanied the group and acted as the liaison person with the DIMATIA faculty and as a program coordinator while in the Czech Republic. The Czech students who participated in the DIMACS REU program in turn acted as hosts for the U.S. students in Prague. The arrival in Prague was timed to occur the week before the annual Prague Midsummer Combinatorics Workshop.

Students were met in Prague by the Prague student participants in our domestic program and moved into dormitory housing at Charles University that was provided by DIMATIA (through a companion Czech grant). Funds for meals in Prague were also provided by DIMATIA. During the first week, there were several days of tutorial presentations by DIMATIA faculty to give students another introduction to the scientific interests of the local faculty, to prepare them for the topics of the Midsummer Workshop, and to present potential research problems. Background material for these presentations was based on lecture

notes prepared for the annual DIMATIA “Spring School on Combinatorics.” This “School” is attended by some 40 undergraduates from throughout Europe.

The faculty members presenting tutorial lectures acted as mentors to the students, but we did not make formal assignments and we let the emphasis on problem solving lead to natural connections between students and mentors, often with more than one mentor per student. Students were encouraged to work on research problems as individuals or in groups or both. There were also presentations orienting the students to the culture and history of the city of Prague and the country, with special emphasis on the rich Czech mathematical tradition. The Czech REU students played a central role in this part of the program. The U.S. students also made informal presentations about their domestic REU research. Czech students were invited to attend the tutorials and presentations and joined the U.S. students for lunch and afternoon sightseeing. While at DIMATIA, the REU students participated in the Prague Midsummer Workshop, which has informal problem sessions each morning, aimed at identifying research topics. The Midsummer Workshop is attended by Czech students and faculty, and we involved the Czech students in afternoon sessions with the REU students to maximize the international exposure and contacts for the U.S. students. Several of the U.S. students also made presentations to the Midsummer Conference, describing their research accomplishments at DIMACS. In the afternoon, the U.S. and Czech coordinators/mentors led discussions on some of these topics, helping the students to focus on research topics identified. The group meetings emphasized approaches to unsolved problems, problem-solving strategies, and group attacks on problems. Students were encouraged to explore, in collaboration with Czech mentors and students, the research questions that arose during the visit to Prague, either through faculty lectures, the Midsummer Conference, or informal discussions, and to pursue the research project begun at DIMACS with Czech mentors.

After the one week Midsummer Workshop, the REU program concluded the next week with more intensive one-on-one sessions between students and mentors and group meetings for problem-solving and for presentations of research results. While the short visit in Prague didn't give students as much time as they had at DIMACS to get deeply into research in pre-defined areas with faculty mentors, the experience taught the students a good deal about how research questions are formulated and pursued. Moreover, with the background gained during the DIMACS portion of the program, the students were able to make a good start on some of the open problems and research topics that they could pursue after leaving Prague. They were encouraged to remain in continuing email contact with their Czech mentors after leaving the Czech Republic.

### **III. Project Findings**

During the 2002 DIMACS REU program, two undergraduates, Steven Jaslar (Rutgers) and Daniel Krasner (UC Berkeley) settled a conjecture about a generalization of the stable marriage problem. They were working under the mentorship of Professors Endre Boros and Vladimir Gurvich.

In 1962 Gale and Shapley proposed and solved what is called the Stable Marriage Problem. The problem considers a set of  $n$  men and  $n$  women, with each man given a preference ordering of all the women and each woman given a preference ordering of all the men. A matching of men and women is called stable if there does not exist a pair  $(m,w)$  such that man  $m$  prefers woman  $w$  to whomever he is matched with and  $w$  prefers  $m$  to whomever she is matched with. Gale and Shapley found a polynomial time algorithm that finds a stable matching for any set of preferences. A significant amount of work, relating to algorithms, lattice structure, strategic behavior, etc. was induced by this paper, as can be seen in the book *The Stable Marriage Problem*, by Dan Gusfield and Robert W. Irving. The problem is of serious practical interest, used for example in matching medical schools with applicants for residencies and in matching job applicants to employers conducting interviews at job fairs at professional society meetings. Jaslar and

Krasner were interested in whether the Stable Marriage Lemma could be generalized with families consisting of more than two people, which they called men, women, and cats.

In 1988, Alkan showed that the answer is negative in case of arbitrary preferences. He considered groups of  $n$  men,  $n$  women, and  $n$  cats, who form families consisting of one man, one woman, and one cat. Each man has a preference ordering of all woman-cat pairs; each woman has a preference ordering of all man-cat pairs; and each cat has a preference ordering of all man-woman pairs. Here  $(m,w,c)$  is a breaking triple if  $m$  prefers  $(w,c)$  to the pair he is with,  $w$  prefers  $(m,c)$  to the pair she is with, and  $c$  prefers  $(m,w)$  to the pair it is with. A matching is called stable if there are no breaking triples. Danilov showed that a stable matching in this sense does not always have to exist.

However, in 2001, Danilov proved that the answer is positive in the case of some "acyclic" preferences. The cases he considered are those in which each man cares primarily about the women and each woman cares primarily about the men (i.e. if man  $m$  prefers woman  $w_1$  to  $w_2$  then  $m$  prefers the pair  $(w_1, c_i)$  to  $(w_2, c_j)$  for every  $i,j$ , where  $c_i$  and  $c_j$  are cats), and each cat has a general preference ordering of all  $(m,w)$  pairs. Using an extension of the Gale-Shapley algorithm Danilov showed that, in such a case, a stable matching exists.

Danilov also introduced lexicographical-cyclic preferences, for which he left the question open. Jaslar and Krasner have proven the answer negative -- namely, there does not always exist a stable matching with this type of preference ordering. In lexicographic preference, each man cares primarily about women, each woman primarily about cats, and each cat primarily about men (i.e. if man  $m$  prefers woman  $w_1$  to  $w_2$  then  $m$  prefers the pair  $(w_1, c_i)$  to  $(w_2, c_j)$  for every  $i,j$ , etc.). A preference order that satisfies the above conditions is called lexicographical-cyclic.

Danilov's Conjecture was that for all  $n$  and all combinations of lexicographical-cyclic preference orders with  $n$  men, women, and cats, there exists a stable matching. Jaslar and Krasner found a counterexample for a combination of lexicographical-cyclic preference orders at the minimum  $n$ , namely when  $n=3$ . Furthermore, they proved that the counterexample at  $n=3$  can be extended to any  $n > 3$ .

Cyclic preference arises when every man has a preference over the women, every woman over the cats, and every cat over the men. Jaslar and Krasner showed that if there is a lexicographic-cyclic preference that does not admit a stable matching, then there is a cyclic preference that does not. Thus, their examples also show that for all  $n > 2$ , there is a cyclic preference that does not admit a stable matching.

These results were presented at the Prague Midsummer Combinatorics Conference in August 2002 and are being written up for publication.

#### References:

A. Alkan. Nonexistence of stable threesome matchings. *Mathematical Social Sciences*, 16: 207-209, 1988.

V.I. Danilov. Stable matchings in some three-sided systems. NATO Advanced Research Workshop on Mathematical Theory of Allocation of Discrete Resources: Equilibria, Matching, Mechanisms, Sabanci University, Istanbul, Turkey, December 16-19, 2001.

D. Gale and L.S. Shapley. College admissions and the stability of marriage. *American Mathematical Monthly*, 69:9-15, 1962.

D. Gusfield and R.W. Irving, *The Stable Marriage Problem*, MIT Press, Cambridge, Massachusetts, 1989.

#### **IV. Project Training/Development**

In a very real sense, the entire project was about training and development, focusing outstanding undergraduates on opportunities in a research career and exposing them to the international nature of the research enterprise. The following comments we received from students in the program reflect the impact the program had on training them in this sense:

“I learned something about Representation theory, more about linear algebra, and I learned how to use LaTeX. Plus, I learned more about myself and whether or not math is for me.”

“It gave me a better idea of what mathematics research is like.”

“Regarding international aspect of program] Great conference; good mathematicians.”

“The summer program was great in general. Thanks again for inviting me to be a part of the research group.”

“All of the workshops and seminars had extremely high (nutritional) value. Some of the research at the epidemiology conference was also interesting to me, although it had little to do with what I was working on.”

“The project was interesting to work on and the professor was very helpful. I actually got a lot of work done. The only complaint I have is that the conjecture I was working on didn't exactly turn out to be true. On a fortunate note, however, my results were pretty conclusive.”

“This helped me prepare to give talks in the future, which I am certain I will be doing quite a bit.”

“Hearing speakers from different fields definitely elucidated what it is that mathematicians do as well as allowed exposure to various fields of mathematics.”

“I learned a lot in this REU and I would say its evenly split between academic learning and social learning. This is certainly one of the most enjoyable experiences I've ever had.”

“My interests are more applied computer science concepts. The talk at AT&T Labs was my favorite because I felt they were doing really meaningful and interesting work. Some other theoretical research had meaning and interest, but not in the same way.”

“Not only did this open me up to more areas of research but it also offered opportunities to present. I consider such opportunities invaluable.”

#### **V. Outreach Activities**

A unique aspect of the program was its place within a vital and active research center with many other exciting programs which were made available to our REU students. The program coincided each summer with the DIMACS Connect Institute (DCI), which provides research experiences for high school teachers and connects them with active researchers in graph theory and its applications. The REU students interacted with participants in DCI and a number of them gave lectures to combined audiences of teachers and researchers at DCI.



The REU students also interacted with the DIMACS Reconnect Program, which is aimed at 2- and 4-year college faculty, and in particular aims to help them bring current research into the undergraduate program. Having a thriving REU program as an example was an important aspect of our summer Reconnect Program.

The students in the REU made connections with a variety of other DIMACS programs, including tutorials, workshops, and seminars, and we encouraged them to get involved with our industrial partners to expose them to industrial research as well as to our other partners such as the Institute for Advanced Study.

## **VI. Papers**

See attached PDF file for listing of publications.

## **VII. Other Products**

### Miscellaneous

One of the goals of our program was to emphasize student presentations of research. Students returning from our program made presentations in a variety of venues. For instance, Pavel Podbrdsky gave a short talk at the Mid-Summer Combinatorial Workshop 2002 in Prague. Rupert Venzke gave a talk on his project for the Algebra Group at University of Pittsburgh, and plans on giving a follow-up presentation in the near future for the Laboratory of Axiomatics Seminar in Geometry, also at University of Pittsburgh. Ursula Whitcher has submitted a poster/abstract to the student session of the Joint Mathematics Meetings and the PODC. Inessa Epstein is planning on giving a presentation at a conference in Nebraska in February 2003. Steven Jaslar and Daniel Krasner gave a presentation at the Mid Summer Combinatorial Workshop 2002 in Prague. And Jose Torres gave a survey talk on uses of Tutte's technique for proving that certain classes of graphs are Hamiltonian at the Mid Summer Combinatorial Workshop 2002 in Prague.

### Web Dissemination

<http://dimacs.rutgers.edu/REU/>

## **VIII. Contributions within Discipline**

Most of the research results obtained were in the fields of discrete mathematics and theoretical computer science, broadly defined. These are areas in which DIMACS is very strong and has a world-wide reputation. Still, it is remarkable that undergraduates could, in at least some of the cases, do cutting edge research and solve some open problems.

The most important contribution to the disciplines of discrete math and theoretical computer science in particular, and to the discipline of the mathematical sciences broadly speaking (including all areas of mathematics, computer science, operations research, statistics, etc.) was the opening up of new horizons to very bright undergraduates, some of whom we hope will become leaders in the field in the future.

## **IX. Contributions -- other Disciplines**

Throughout the program, we emphasized applications of discrete mathematics and theoretical computer science. Probably the biggest impact in terms of applications came from our molecular biology topics. One of our students worked with Professor Wilma Olson, Director of the Rutgers Center for Molecular Biophysics and Biophysical Chemistry. To understand how the genetic code is translated into three-dimensional structures used in biological processes, Professor Olson's group has been analyzing the known geometry of DNA bases in chemical structures and developing mathematical models to incorporate the sequence-dependent bending, twisting, and translation of known fragments in DNA molecules of varying lengths and chemical composition. The REU student was directly involved in the group's research. In addition, there was outreach to other mathematical biology research at Rutgers through connections with the international Nucleic Acid Database (NDB) and the DIMACS Special Focus on Computational Molecular Biology. A group of undergraduates working with the NDB visited our program to talk about their research and our students paid a return visit to the NDB. Some outstanding students received an exciting introduction to the interface between disciplines.

## **X. Contributions -- Human Resource Development**

### **XI. Contributions to Education and Human Resources**

US graduate student Jose Torres served as graduate student coordinator in this program. He went to Prague and ended up participating in the research program. In regard to his experience as graduate student coordinator, Jose stated: "Seeing the final presentations of the students was a highlight. I saw how many of them worked hard and improved with each practice session we had. I was proud of them and was very fulfilled to see them succeed and improve. I guess after all the time I dedicated to help them I felt as if it was my success as well. Overall the experience was great and I feel fortunate to have been given the chance to be the graduate REU coordinator." Perhaps as important as the specific research accomplishments and opportunities was the development of leadership and mentoring skills on the part of this graduate student.

The impact on the careers of our students will take awhile to determine. However, we do have some feedback already.: 100% of the respondents planned to go to graduate school, though one said that he was having reservations and the program helped him to understand what they were.

In our recent survey, we received some extremely thoughtful comments from past participants about the impact of our program on them. Here is a selection:

"I will try to get funding for theoretical CS research this summer on the strength of my REU project."

"Participating in the program allowed me to figure out that I really like doing mathematical research, through first hand experience."

"The research is a spectacular life experience that I was lucky to have. I wish that everyone could enjoy it. Even though I didn't accomplish my goals in research and did get frustrated with the work, I learned about research and what really goes into it. Both areas are fabulous learning experiences."

"After attending REU, I think going to graduate schools is a must to get specific knowledge at a deep level in some interesting field."

I got a better idea of what graduate school is like. I liked talking to the graduate students and professors classes, research, etc.”

“My goal has always been to become an active member of the research community and a teacher. The REU experience definitely proves to me that I’m heading in the right direction. Also being around young, ambitious, hardworking students motivates me to want to learn more and be able to compete with them. The atmosphere we created during the summer was stimulating and competitive, which I feel is healthy for us all.”