Crime Part 1: The Store

When an occurrence is reported to the police, forensic investigators are called to the scene to determine, through examination of forensic evidence, whether the incident involves a crime or if it is an accident, and whether there are any victims or suspects involved in the occurrence. The following is a conversation between two investigators concerning an incident that has occurred at a local store:

Frank: It was dark and rainy that night as I stood outside the storefront wishing I was home in bed dreaming of my retirement. Why do I always seem to be the one on duty during full moons? Clearly seniority means nothing in this town.

Barney: Hey Frank, I got your page, what’s up? Oh yeah, I heard a good one today. What can run but never walks, has a mouth but never talks, has a head but never weeps, and has a bed but never sleeps?

Frank: Really Barney? Maybe now isn’t the best time for riddles.

Barney: Whoa, look at that. There is a car in the middle of the store. How did that get there?

Frank: Perhaps someone drove it there Barney, but that’s just a guess.

Barney: But there is no one in the car?

Frank: Nice call, captain obvious, they seem to have fled.

Barney: Yikes, someone must’ve been a little angry at the store, eh?

Frank: It would certainly seem that way. Or angry at someone in the store.

Barney: Any ideas on who should be in our suspect list?

Frank: I have a few ideas on where to start, but we’ll need more info on who was driving the car.

Barney: Any evidence to suggest who that might be?

Frank: Not yet, but we are searching for evidence in the car. We’ve got a CSI team on it now. They should let us know by tomorrow.

Barney: What are they looking for?

The Investigation:
Every investigation begins with a general assessment of what constitutes and what does not constitute forensic evidence. After that assessment, the team begins collecting that evidence.

1. What kinds of information might the investigators be able to get from the car that could be used to identify the driver?

2. If investigators do find some evidence in the car, who should they compare it to? In other words, who do you think should be on the initial suspect list?
Crime Part II: The Evidence

Although television shows have popularized the method of “dusting” a print (using a fine brush along with a special powder), in fact this method is no longer the method of choice. Modern technologies have seen the use of digital imaging. Nevertheless, older methods are used when nothing else is available. For instance, it is possible to “lift” a print with clear tape.

It is important to preserve the details of a print as clearly as possible so that a good match can be made. The better the impression, the better it will be to clearly identify distinguishing characteristics. Digital imaging has vastly improved the ability to record fingerprint details clearly.

Collection methods also depend on the type of fingerprint. Some prints are latent, meaning they are not visible to the naked eye. A patent print is visible, being left in some medium such as blood, ink, grease, etc. Impressed prints (also called plastic prints) are indentations left in soft pliable surfaces such as wax, clay, paint, etc. As you might imagine, each of these types of prints has its own challenges and advantages.

When fingerprint evidence is collected at a crime scene, investigators’ first step is to search a national data base to see if there is a copy of that print on file, thus identifying the person associated with the print. The evidence may belong to the person responsible for committing the crime. The print may just belong to an innocent victim who was in the wrong place at the wrong time. Or the print may not have anything to do with the crime. Additional evidence may be needed to corroborate the investigators’ conclusions.

Back to the crime…

Barney: Hi Frank, did they find any evidence in the car?
Frank: Yes, they found some fingerprints.
Barney: Great, who do they belong to?
Frank: I’d like to be able to tell you but the lab says they are very backed up and our case isn’t exactly high on their to-do list.
Barney: Well it is high on our to-do list so what are we going to do?
Frank: I had a friend send over one of the partial prints they found so we can take a look.
Barney: I hope you have ‘brushed up’ on your fingerprint skills, ha, get it?
Frank: Very funny Barney, very funny. I know some people at a private lab who might be able to help us. I’ll send the print over to them. I’ve also put together a suspect list.

Frank sends the following print to your lab to see if you can help with the investigation.
Much of the mathematics in future lessons of this module deals with methods for searching a database for a fingerprint match. However, before you examine the search algorithms used by law enforcement you need to study the process of taking and recording fingerprints.

**How to take a set of your fingerprints**

**Materials:**
You will need
- A fingerprint recording sheet
- A number two pencil
- A 5 by 7 index card
- Clear wide scotch tape
- Several moist sanitary towelettes

**Instructions:**
To create your own set of prints, obtain the materials listed above.

1. Take the number 2 pencil and rub it on the index card until there is a solid smudge of lead large enough to cover a fingertip.
2. Rub your finger on the lead smudge firmly, but not too firmly, until the fingertip is completely covered. You must cover your finger from the tip to the first knuckle and from one side to the other side, so be sure to roll your finger in the pencil lead from side to side (nail to nail). Start with the right thumb.
3. Once your finger is sufficiently covered, you are ready to place your fingerprints in the appropriate boxes on the card. Locate the spot for your right thumb on the fingerprint card. Place one edge of your thumb in the box, press firmly and roll your finger to the opposite edge (nail to nail). Do NOT roll it back again or you will smudge the print. Be sure to keep your finger parallel to the table to get as much of your print as possible onto the card.
4. You may wish to place a piece of scotch tape over the print to protect it from damage (unless you used fingerprint ink rather than pencil lead)
5. Repeat this process for each finger until you have a complete set of fingerprint impressions.
6. At the bottom of the card there is a spot to press all four fingers at the same time as well as two spots for each thumb. This is a safety check to be sure that each finger was put in the correct box above.

**Homework:**
- Use your fingerprint cards to look for patterns
  - How many different ‘patterns’ do you see on your prints
    - For example, is the pattern the same on all 10 fingers for a particular individual?
    - How about on just one hand? Both index fingers?
- Come to class tomorrow ready to discuss your findings
<table>
<thead>
<tr>
<th>Applicant Information</th>
<th>Type or Print All Information in Black</th>
</tr>
</thead>
<tbody>
<tr>
<td>Last Name</td>
<td>NAM</td>
</tr>
<tr>
<td>First Name</td>
<td>MIDDLE NAME</td>
</tr>
<tr>
<td>FBI</td>
<td>LEAVE BLANK</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Signature of Person Fingerprinted</th>
<th>Aliases</th>
<th>ORI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residence of Person Fingerprinted</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Date</td>
<td>Signature of Official Taking Fingerprints</td>
<td></td>
</tr>
<tr>
<td>Employer and Address</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reason Fingerprinted</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date of Birth</th>
<th>DOB</th>
<th>Place of Birth</th>
<th>POB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Month</td>
<td>Day</td>
<td>Year</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Class</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fingerprint Details</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. R. Thumb</td>
<td>2. R. Index</td>
<td>3. R. Middle</td>
</tr>
<tr>
<td>4. R. Ring</td>
<td>5. R. Little</td>
<td></td>
</tr>
<tr>
<td>6. L. Thumb</td>
<td>7. L. Index</td>
<td>8. L. Middle</td>
</tr>
<tr>
<td>9. L. Ring</td>
<td>10. L. Little</td>
<td></td>
</tr>
</tbody>
</table>

Left four fingers taken simultaneously

<table>
<thead>
<tr>
<th>1. Thumb</th>
<th>R. Thumb</th>
</tr>
</thead>
</table>

Right four fingers taken simultaneously

The Suspects

The investigators were able to put together a list of possible suspects as to who may have driven the car into the store. The suspects include:

a. John McDonnell – the store owner who is struggling to keep up with the rent.  
   2. Christine Smith – a recently fired employee for smoking in the store with Frank Cooper.  
   3. Frank Cooper – also recently fired along with Christine Smith.  
   4. Lee Howe – an employee who was recently docked in pay for falling asleep in the dressing room.  
   5. Steven Riale – an employee recently accused of stealing candy from the store but they could not prove it. So now he is being watched ‘like a hawk’.  
   6. George Munson – a competing store owner who has recently threatened John McDonnell to stop telling people lies about George’s store, “or else!”  
   7. Caroline Fletcher – John McDonnell’s ex-wife who is angry about him being behind in alimony payments.  
   8. Kathy Sullivan – a recent customer who tried to return an item she just purchased the day before but lost her receipt. The store did not accept the return and she was overheard saying, “You’ll pay for this!”  
   9. Rick Dawson – the car owner who says his friend borrowed the car that night.  
  10. Darryl Wright – the friend who borrowed the car. He was in a bar that night and says when he came out the car had been stolen.  
  11. Jeremy Dawson – the 16 year old son of Rick who frequently sneaks out with the car.  
  12. Lin Huang – the employee of the month, for 14 straight months.  
  13. Linda Pierce – the runner up for employee of the month for 14 straight months.  
  14. Andy King – the 41 year old and longest employee (18 years) who was promised he would be store manager one day (16 years ago) and is still a stock ‘boy’.  
  15. Susan Tilly – a female employee whose health insurance was cut just one week before the birth of her triplets.  
  16. Ivan Carlson – the ex-boyfriend of Susan who recently got out of jail.  
  17. Donald French – an employee, the current boyfriend of Susan and probable father of the triplets.  
  18. Sadie Wiler – a local criminal who has a history of stealing cars.  
  20. Thomas Calvin – the high school jock who went out ‘partying’ last night and no one can seem to find.  
  21. Sarah Spencer – the local jock’s girlfriend, who is also currently missing.
Pattern Recognition

Now we will take a closer look at fingerprints and more clearly define the patterns found in fingerprint impressions.

Part I: What are deltas and cores?

Fingerprint patterns contain certain features that allow us to classify fingerprints easily into three primary categories. Let’s begin by defining the two features that are used to distinguish one major pattern type from another: deltas and cores. Deltas and cores are formed by ridge patterns. A fingerprint ridge is a slightly raised, long, narrow line or curve that appears on the surface of the skin.

**Definition:** A delta is triangular-shaped ridge pattern that appears at the center of three intersecting ridge flows.

**Definition:** A core is the termination of the inner-most ridge that appears in the interior of a concentric set of curved ridges.

**Examples of Deltas:**

![Examples of Deltas](image1)

**Examples of Cores:**

![Examples of Cores](image2)

**Discussion:**

Examine the three columns of fingerprints on the next page. Count the number of cores and deltas in each print and record the number in the corresponding column. What can you conclude from these numbers in relation to the fingerprint patterns you see?
<table>
<thead>
<tr>
<th>Whorls</th>
<th># of Cores</th>
<th># of Deltas</th>
<th>Loops</th>
<th># of Cores</th>
<th># of Deltas</th>
<th>Arches</th>
<th># of Cores</th>
<th># of Deltas</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Part II: The Pattern Ordered Pair (POP) Classification System

For the purposes of this module we will create our own system of categorizing fingerprints.\(^1\) We will use the following classification system, referred to here as the Pattern Ordered Pairs (POP). The POP system will use several features of prints in the classification process. In fact, the POP system is a multi-stage process, as are systems actually in use by law enforcement agencies. The first step, called the primary classification, is to write the ordered pair \((C, D)\), in which

- \(C\) represents the number of cores present in the pattern, and
- \(D\) represents the number of deltas present in the pattern.

In-Class Exercise:
Calculate the primary classification ordered pair, \((C, D)\), for each of your ten fingers on your own fingerprint card.

Homework:

Back to the crime...

We’d like to narrow down the list of suspects. The fingerprint database contains all the prints on file for the people from the community, including the possible suspects for the crime (but not limited to just those suspects). Calculate the primary classification ordered pair \((C, D)\) for the subset of prints you have been assigned from the database.

\(^1\) The reason for this is the limited database that we are using does not require the complete complexity of the Henry System, the extension of the Henry system, or recently employed systems by federal agencies such as ridge distribution systems. Although our system is simpler, it still reflects the process by which fingerprint experts would determine likely suspects through an algorithmic process of elimination.
Process of Elimination

The Pattern Ordered Pair System of Classification

Your homework for the previous lesson was to calculate the primary classification for each fingerprint in the crime database.

Activity:
Identifying a likely suspect from a database of fingerprints uses a process of elimination. The first step in the process of elimination is to compare the primary classification of fingerprint obtained at the scene with the primary classifications of the fingerprints in the database. Any suspect whose primary classification differs from the primary classification of the fingerprint obtained at the scene can be eliminated as a possible suspect.

1. Calculate the primary classification for the fingerprint obtained at the scene using the POP method.

2. Compare this primary classification with the primary classifications for each print in the database. Any fingerprint in the database that does not match the primary classification of the fingerprint found at the scene can be removed from consideration. How many prints from the database have the same primary classification as the fingerprint found at the scene? Which ones are they?

3. Which of the remaining fingerprints belong to individuals in our suspect list? Which suspects do not have the same primary classification as the fingerprint found at the scene? Can they now be eliminated as suspects?

Discussion:

1. What are some of the aspects of the fingerprint found at the scene that may help or hurt in finding a unique match from the database?

2. Is the primary classification enough to identify a single suspect? Explain.

3. Take a closer look at the 28 remaining prints. Although all of these prints have the same primary classification, and thus the same loop pattern, there are differences among the loops on each of the prints. Examine the loops from the 28 prints that remain from the database more closely. What are some differences that you notice among the loops?

4. At this point are you confident that those people who have been eliminated as suspects are indeed innocent?
Secondary Classification

It is clear that the primary classification is not enough to identify a suspect. Ideally a final comparison should be made between fingerprints found at a crime scene and as few prints from the database as possible. The next step is to reduce the database even more so that just a few prints remain as likely suspects. To do this it will be necessary to calculate the secondary classification. The secondary classification can be thought of as a way of measuring the difference in size of the general pattern and the direction (left or right) of the general pattern.

As you have seen, even though two prints may both be (1, 1) loops, they may still be visibly different. That is, the primary classification process of the POP system is not enough to distinguish among similar-looking prints. Therefore we introduce the secondary classification process of the POP system.

To carry out the secondary classification process, begin by drawing a straight line from the center of each core to the center of each delta. For a (1, 1) print, that is just one line, but for other primary classifications you will need to draw several lines. The center of a core is the center ridge ending at the heart of the core. The center of a delta is the ridge ending or the “fork” of a ridge that occurs within the delta.

For each line you drew, count the number of ridges that it crosses (DO NOT count the core itself or delta itself). If it happens to cross a point where a ridge splits into to ridges, count that as two crossings. Record your count as positive if the slope of the line is positive, and record your count as negative if the slope of the line is negative.

The secondary classification is now defined as the ordered pair \((M, m)\), where \(M\) represents the largest (absolute value) count among your lines and \(m\) represents the smallest (absolute value) count among your lines. Note that when calculating \(M\) and \(m\) for arches and loops, the maximum and minimum counts are the same, \(M = m\), because there is only one count. For example, for a loop if the line segment crosses 4 ridges, then \((M, m) = (4, 4)\) or \((-4, -4)\). For an arch, there is no delta. Therefore, the answer is always \((M, m) = (0, 0)\) for both plain and tented arches. Whorls are the only pattern where \(M\) and \(m\) could be of different values.

Here are two examples of secondary classification counting. Note that in Fingerprint 1 \(M\) and \(m\) are negative to indicate that the line segment has a negative slope and the number of ridges is counted as 9 instead of 8 because the line segment crosses a point at which a ridge line splits (a bifurcation). In Fingerprint 2 the maximum and minimum measurements are both positive slopes (the two line segments on the left-hand side). However, it is possible in other prints for one of \(M\) and \(m\) to be positive and the other to be negative.
Fingerprint 1: \((M, m) = (-9, -9)\)

Fingerprint 2: \((M, m) = (19, 8)\)
Homework:

*Back to the crime...*

Calculate the secondary classification for the fingerprint found at the crime scene and for each of the 28 remaining fingerprints from the database having the (1,1) primary classification. That is, calculate the second ordered pair (M, m) from the POP Classification System outlined above. A larger version of the crime scene print is provided below.
Zeroing In

Analyzing the Remaining Database Fingerprints

Your homework for the previous lesson was to calculate the secondary classification, the second POP ordered pair (M, m). The secondary classification identifies the “size” of the pattern (that is, the number of ridges between the core and the delta) and the “direction” of the pattern (that is, whether the delta is placed to the right or the left of the core).

Activity: Ridge Characteristics

Now that the number of prints from the database has been narrowed down to a few choices, the final stage is to determine an exact match between the fingerprint found at the crime scene and the remaining suspects. To do this a more refined examination of the finer ridge characteristics of fingerprints will be examined. To begin the study of ridge characteristics you will do a simple in-class activity.

Materials:
You will need
- A number two pencil
- A 5 by 7 index card
- A small to medium size balloon
- A moist sanitary towelette

To examine the ridge details of your thumb more closely, you will create a print of your thumb on the balloon, and then fill the balloon with air. To create an image of your thumb print on the balloon complete the following instructions:

1. Take the number 2 pencil and rub it on the index card until there is a solid smudge of lead large enough to cover a fingertip.

2. Rub your thumb on the lead smudge firmly, but not too firmly, until the thumb tip is completely covered.

3. Firmly place your thumb on the side of the balloon so that the entire print from tip to knuckle appears on the balloon.

4. Once your thumb print is on the balloon blow up the balloon so that it is about ½ to ¾ full or until the ridge details of your thumb are clearly visible.
**Discussion:**

Look closely at your thumb print on the balloon. You will notice many characteristics among the ridges. A list of some of the more common ones is given below:

<table>
<thead>
<tr>
<th>Ridge Ending</th>
<th>Lake (Enclosure)</th>
<th>Trifurcation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bifurcation</td>
<td>Ridge Ending</td>
<td>Opposed Bifurcation</td>
</tr>
<tr>
<td>Dot</td>
<td>Bridge</td>
<td>Ridge Crossing</td>
</tr>
<tr>
<td>Island (Short Ridge)</td>
<td>Double Bifurcation</td>
<td>Opposed Bifurcation/Ridge Ending</td>
</tr>
</tbody>
</table>

Study the ridge details on your balloon thumb print and answer the following questions:

1. Which of the ridge characteristics listed above are evident in your thumb print? How frequent is each of these characteristics on your thumb print? Are most of the characteristics present in your thumb print? Do you think that is the case for everyone?

2. Compare the characteristics of your thumb print with the characteristics of your classmates’ thumb prints. Do these characteristics appear to occur in any pattern? That is, are they concentrated in a particular area of the thumb print? Is one characteristic more common than any others?

3. Do you think the frequency and placement of ridge characteristics for your other nine fingers will be any different?
Introducing Graph Theory

The final classification to determine a match between the fingerprint found at the scene and the remaining suspects is to compare ridge characteristics among the prints.

“Graphs” will be used to further refine the search for a suspect. A graph is a commonly-used diagram that represents problems in mathematics, biology, and other sciences. Graphs can be represented visually by diagrams that use dots and arcs (or curved line segments). By studying the properties of a graph, scientists can understand the nature of the problem it represents, giving them a better opportunity to find a solution. A simple example of a graph is given by:

![Graph Diagram]

In order to study the properties of a graph, a more formal definition will be needed.

A graph (or network) $G$ is a set of at least one vertex, $V(G) = \{v_1, v_2, \ldots, v_n\}$, commonly represented as points, and a set of edges $E(G) = \{e_1, e_2, \ldots, e_m\}$, commonly represented as a line or curve having a vertex at each end. The edge connecting vertices $u$ and $v$ is written $uv$. Mathematically we represent this as $e = uv \in E(G)$. This notation is read “the edge $e = uv$ is an element of the edge set of $G$, $E(G)$.” In that case, we say that vertices $u$ and $v$ are adjacent, $u$ and $v$ are called the endpoints of $uv$, and $u$ and $v$ are said to be incident to edge $e = uv$. Vertices are also known as nodes.

Because the vertex-edge connections do not depend on position, a graph has many visually different representations.
For example, in the figure below all three diagrams represent equivalent graphs since connections in the one graph correspond to connections in the other graphs. (‘a’ is connected to ‘b’ and ‘e’, ‘d’ is connected to ‘c’ and ‘e’ etc…) 

Graphs can be used to identify and connect ridge point characteristics. The resulting graph could then be used to match the suspect’s fingerprint with the fingerprint from the crime scene.

The nodes or vertices of the graph will be ridge-point characteristics (a dot, the tip of a ridge ending, the vertex of a bifurcation, etc.) in the fingerprint. To distinguish one type of point characteristic from another the nodes will be color-coded. This is referred to as vertex coloring. For simplicity, this activity will concentrate on identifying only three point characteristics: bifurcations (red), ridge endings (blue), and dots or delta centers (green). The diagram below shows the fingerprint from the crime scene with several nodes already identified. It is usually suggested to identify at least 8 to 12 point characteristics for better match identification.
In general, it is best to choose point characteristics that are spread across the fingerprint. Look for nodes that are not concentrated in one area.

The next step is to add edges joining the vertices. Not all vertices will be joined by an edge. The rule for joining two vertices with an edge is this: A pair of vertices will be joined (be adjacent) if the edge drawn will “cross” the ridges in a nearly perpendicular fashion. If the edge appears to be almost “parallel” (that is, follows the general flow of the ridges), then it will not be added to the graph. The reason for this is that you will need to count the number of ridges that cross the edge. The edge will then be “labeled” with this number (count). This number is referred to as the weight of the edge. Enough edges should also be added so that the resulting graph is connected. Weighted edges have been added to the vertices of the fingerprint from the crime scene to obtain:

Note that the graph added to the above fingerprint is just one example of a possible graph on this print. There are many other ridge characteristics that could have been used to create a graph associated with the fingerprint. As practice, try creating a different graph for the fingerprint above. Here are some suggestions when creating your graph:

1. Try to select point characteristics that are clearly visible so there is no ambiguity as to what they are.

2. Try to choose point characteristics that cover as much of the print as possible.

3. Try to choose point characteristics that can be joined to at least one other node by an edge so that a connected graph can be constructed.
Activity: Matching with Graph Theory

Reprinted below are the two suspect prints from the database. On one of the prints it will be possible to construct a graph that has identical connections, with corresponding colors and weights, as the graph on the fingerprint from the crime scene above. When two graphs can be matched in such a way that all connections, colors, and weights are identical, the two graphs are said to be isomorphic. Construct a graph on each of the suspect prints attempting to construct a graph that is isomorphic to the graph on the print from the crime scene. The one on which an isomorphic graph can be constructed will be the matching print.

Z (1,1) (8,8)

CC (1,1) (8,8)
Activity: Numerical Representations of Graphs

Comparing graph diagrams is not the way experts determine if two graphs are isomorphic. It is much more efficient to represent graphs numerically. Representing the graphs numerically will allow the process to be more precise and to be able to encode the information into a computer. One way to represent graphs numerically is with an adjacency matrix. An adjacency matrix for a graph has rows and columns that represent the graph’s edges with entries of 1 and 0. A 1 is placed in a row and column if the vertices represented by that row and column are joined by an edge (i.e., are adjacent). A 0 is placed in a row and column if the vertices represented by that row and column are not joined by an edge (i.e., are not adjacent). The adjacency matrix below represents the graph constructed for the fingerprint found at the crime scene. Note, for example, the 1 at row c and column a means that vertices a and c are adjacent.

\[
\begin{array}{cccccccc}
\text{a} & \text{b} & \text{c} & \text{d} & \text{e} & \text{f} & \text{g} & \text{h} & \text{i} \\
a & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
b & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
c & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
d & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
e & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
f & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
g & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
i & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
\end{array}
\]

This matrix can be altered to include some additional information. For example, the 1’s in the matrix can be replaced with the edge weights for the corresponding edges to produce this new matrix, indicating that vertices a and c have 18 ridges between them:

\[
\begin{array}{cccccccc}
\text{a} & \text{b} & \text{c} & \text{d} & \text{e} & \text{f} & \text{g} & \text{h} & \text{i} \\
a & 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 \\
b & 0 & 0 & 12 & 0 & 0 & 0 & 0 & 3 \\
c & 18 & 12 & 0 & 0 & 0 & 0 & 0 & 0 \\
d & 0 & 0 & 6 & 0 & 4 & 0 & 0 & 0 \\
e & 0 & 0 & 0 & 4 & 0 & 2 & 7 & 0 \\
f & 0 & 0 & 0 & 0 & 2 & 0 & 3 & 0 \\
g & 0 & 0 & 0 & 0 & 7 & 3 & 0 & 0 \\
h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\
i & 0 & 3 & 0 & 0 & 0 & 0 & 3 & 0 \\
\end{array}
\]

It is even possible to represent the vertex colorings numerically in the matrix. Let the color red be represented by 1, blue represented by 2, and green represented by 3. Each vertex will be labeled with a 1, 2, or 3 to identify its color. Each matrix entry will be an ordered triple with the first coordinate the edge weight, the second coordinate the row vertex color and the third coordinate the column vertex color. This new matrix will be
called the *expanded adjacency matrix* for the graph. The new matrix shows that vertices a and c have 18 ridges between them, and that vertex c is red and vertex a is green:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>i</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0</td>
<td>0</td>
<td>(18, 3, 1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>0</td>
<td>0</td>
<td>(12, 2, 1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(3, 2, 2)</td>
</tr>
<tr>
<td>c</td>
<td>(18, 1, 3)</td>
<td>(12, 1, 2)</td>
<td>0</td>
<td>(6, 1, 1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>d</td>
<td>0</td>
<td>0</td>
<td>(6, 1, 1)</td>
<td>0</td>
<td>(4, 1, 1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>e</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(4, 1, 1)</td>
<td>0</td>
<td>(2, 1, 1)</td>
<td>(7, 1, 1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>f</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(2, 1, 1)</td>
<td>0</td>
<td>(3, 1, 1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>g</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(7, 1, 1)</td>
<td>(3, 1, 1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>h</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(3, 2, 2)</td>
</tr>
<tr>
<td>i</td>
<td>0</td>
<td>(3, 2, 2)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(3, 2, 2)</td>
<td>0</td>
</tr>
</tbody>
</table>

It is now possible to compare the two expanded adjacency matrices for two fingerprint graphs numerically. If the matrices have identical numerical values in their rows and columns, the corresponding graphs can be said to be isomorphic. Note that row/column operations (interchanging rows/columns with one another) may need to be performed in order for corresponding vertices to be “lined up” to make the matrices identical.

1. Construct the expanded adjacency matrix for each of the graphs you produced for the suspects’ fingerprints.

2. Using row operations, if necessary, compare the expanded adjacency matrices to determine which suspect print matches the fingerprint at the crime scene.

3. What is your conclusion?
The End… or is it?

**Barney**: Wow, those lab friends of yours sure did a great job!
**Frank**: I’m glad we got them involved. The crime lab is still backed up.
**Barney**: Well, at least the guilty person has been caught and admitted to the crime.
**Frank**: There sure are some crazy people in this world. But luckily, one of them is now off the streets.
**Barney**: Unfortunately, I’m sure we’ll meet another one tomorrow.
**Frank**: Why wait until tomorrow; we have another case right now.
**Barney**: Why isn’t that a surprise? We really need to get out of this town.
**Frank**: At least it keeps us busy. By the way, it is a river.
**Barney**: Huh? What is a river?
**Frank**: The answer. It’s a river.
**Barney**: Oh wow, I almost forgot. Good one Frank, I’ll have another one for you tomorrow. Now where is this new case of ours?

A crime was committed at a local bank and the following partial fingerprint was lifted from the scene. It may or may not be in the correct orientation. First determine the **primary** and **secondary classifications** of the print. Then create a graph and associated matrix for this print. Use the graph and its associated matrix to prove this print matches one of the prints in the assessment database.
Assessment Database

A.  

B.  

C.  

D.