Applied Combinatorics

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Answers to Selected Exercises[1]

Chapter 12

Section 12.1.

1(a-c). $\{a, \alpha\}, \{b, \beta\}, \{c, \gamma\}, \{e, \delta\}$;
3(a). $\{a, b\}, \{c, d\}, \{e, f\}$;
5. find a minimum weight matching;
7(b). put a very small weight on all edges not in the graph.

Section 12.2.

1(a). {Cutting, Shaping, Polishing, Packaging};
1(b). $\{1, 2, 3\}$;
1(c). $\{a, b, c, d\}$;
2(a). {Smith, Jones, Black};
2(b). {White, Cutting, Shaping, Gluing, Packaging};
2(c). $\{a, b, c, d, f, i\}$;
3. (a): no; (b): yes; (c): no; (d): no;
5(a). (a, a, a, a, b, d, a);
6(a). no SDR;
6(b). no SDR;
6(c). (a_1, a_3, a_2, a_4);
6(d). (b_5, b_1, b_3, b_4);
6(e). no SDR;
6(f). no SDR;
8(a). two;
8(b). two;
8(c). 2^5;
8(d). 2^n;
9. no;
14(a). yes;
14(b). three: (c, d, a, b, e), (d, c, a, b, e), (d, e, a, b, c);
16(a). yes: $\begin{pmatrix} 6 \\ 7 \\ 4 \\ 2 \\ 1 \end{pmatrix}$;

[1]More solutions to come. Comments/Corrections would be appreciated and should be sent to: Barry Tesman (tesman@dickinson.edu) or Fred Roberts (froberts@dimacs.rutgers.edu).

16(b). no;
18(a). n even;
18(b). n even;
18(c). n odd;
20. yes: show each likes exactly p;
22(a). since an SDR exists, either x is used or it is not. If not and S_i contains x, replace S_i's representative in the SDR with x;
22(b). no, e.g., let $S_1 = \{x\}$, $S_2 = \{a, x\}$, and $S_3 = \{b, x\}$;
23(a). for example, $(2, 3, 4, \ldots, n-1, n, 1)$;
24. no, consider the vertex/vertices of degree one.

Section 12.3.
1. yes;
2(a). yes;
2(b). yes;
2(c). no;
2(d). no;
2(e). yes;
2(f). no;
2(g). yes;
2(h). yes.

Section 12.4.
1(a). $\{1, 2, 3, 4, 5\}$;
1(b). $\{1, 2, 4, 5\}$;
2(b). {Smith, Jones, Brown, Black, White};
6(a). minimum $\{2, 6\}, \{3, 5\}, \{1, 4\}$;
6(b). minimum $\{1, 2\}, \{3, 4\}, \{5, 6\}$;
6(c). minimum $\{1, 3\}, \{2, 4\}, \{5, 8\}, \{6, 7\}$;
6(c). minimum $\{a, \alpha\}, \{b, \beta\}, \{b, \delta\}, \{c, \gamma\}$;
7(a). no;
7(b). $\left\lceil \dfrac{n}{2} \right\rceil$;
9. minimum edge covering;
13. If I is independent, $V - I$ is a vertex cover and if K is a vertex cover, $V - K$ is independent;
15. $|M^*| \leq |K^*|$ implies $|I| \leq |I^*| \leq |F^*| \leq |F|$.

Section 12.5.
1(a). $8, \{8, 9\}, 9, \{9, 6\}, 6$;
1(b). $8, \{8, 9\}, 9, \{9, 6\}, 6, \{6, 3\}, 3$;
1(c). add edges $\{8, 9\}$ and $\{6, 3\}$, delete edge $\{9, 6\}$;
6(a). pick vertex 8, place edge $\{5, 8\}$ in T, note 8 is unsaturated, and note $5, \{5, 8\}, 8$ is an M-augmenting chain;
9(a). use edges $\{6, 7\}, \{7, 11\}, \{11, 9\}$.

Section 12.6.
1. (a): $\delta(G) = 1, m(G) = 3$; (b): $\delta(G) = 0, m(G) = 3$; (c): $\delta(G) = 1, m(G) = 4$; (d): $\delta(G) = 1, m(G) = 4$;
2(b). $2p \leq 3|N(S)|$;
2(c). $|S| - |N(S)| \leq p - \frac{2}{3}p$;
2(d). $m(G) = |X| - \delta(G) \geq 9 - \frac{1}{3}(9) = 6$.

Section 12.7.
2(a). worker 1 to job 2, 2 to 3, 3 to 1, and 4 to 4;
2(b). worker 1 to job 2, 2 to 3, 3 to 1, and 4 to 4;
2(c). worker 1 to job 2, 2 to 5, 3 to 4, 4 to 1, and 5 to 3;
3. worker 1 to job 4, 2 to 1, 3 to 5, 4 to 2, and 5 to 3;
4. machine 1 to location 2, 2 to 1, 3 to 5, 4 to 4, and 5 to 3;
5(a). speaker 1 with speaker 3, 2 with 5, 3 with 1, 4 with 6, 5 with 2, and 6 with 4;
6. no, there are three solutions: $B_1 - H_1$, $B_4 - H_2$, $B_3 - H_3$ or $B_1 - H_2$, $B_2 - H_1$, $B_3 - H_3$ or $B_2 - H_1$, $B_4 - H_2$, $B_3 - H_3$.

Section 12.8.
1. $(n!)^{2n}$ since there are $n!$ choices for the preference list for each of the n men and n women;
2. $n!$: there are n choices for the first man, then $n-1$ choices for the second man, $\ldots$, and finally, 1 choice for the n^{th} man;
4. m_3 has w_4 higher on his preference list and w_4 has m_3 higher on her preference list;
5(a). $m_1 - w_1$, $m_2 - w_2$, $m_3 - w_3$, $m_4 - w_4$;
5(b). $m_1 - w_4$, $m_2 - w_3$, $m_3 - w_2$, $m_4 - w_1$;
11(a). there are 3 possible matchings each with a blocking pair: $p_1 - p_2, p_3 - p_4$ with blocking pair p_2, p_3; and $p_1 - p_3, p_2 - p_4$ with blocking pair p_1, p_2; and $p_1 - p_4, p_2 - p_3$ with blocking pair p_1, p_3;
11(b). no.