Applied Combinatorics

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Answers to Selected Exercises[1]

Chapter 6

Section 6.1.

1. $a_5 = 1535$, $a_6 = 6143$,;

2. $S_4 = 9000$;

3. $S_4 = 10,368$;

4(b). 528;

4(c). 00, 11, 12, 13, 21, 22, 23, 31, 32, 33;

9. $b_4 = F_5 = 8$: XOOX, XOXO, OXOX, XOOO, OXOO, OOXO, OOOX, OOOO;

11. 4123, 4312, 4321, 3142, 3412, 3421, 2143, 2341, 2413;

14(a). $7! - D_7$;

14(b). 1;

19(a). $(D_4)^2$;

19(b). $(4!)^2$;

24. $f(n+1) = (2n+1)f(n)$;

31. $F_{n+2} - 1$;

34. $f(n+1) = f(n) + 2n, n \geq 1, f(1) = 2$;

38(a). $n!$;

40. $C_3 = 4$.

Section 6.2.

1(a). linear;

1(e). not linear;

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[1]More solutions to come. Comments/Corrections would be appreciated and should be sent to: Barry Tesman (tesman@dickinson.edu) or Fred Roberts (froberts@dimacs.rutgers.edu).

2(a). not homogeneous;

2(e). homogeneous;

3(a). has constant coefficients;

3(f). does not have constant coefficients;

5(a). $x^2 + 2x + 1 = 0$;

5(f). $x^2 - 2x - 3 = 0$;

6(a). -1, -1;

6(i). 1, 2, -2;

10(a). a solution;

11(c). not a solution;

12(a). $a_n = 2 \cdot (-1)^n - 4n \cdot (-1)^n$;

12(e). $h_n = \frac{7}{3} \cdot 3^n + \frac{5}{3} \cdot (-3)^n$;

17(c). yes;

17(d). $a_n = (-\frac{i}{2})(i)^n + (\frac{i}{2})(-i)^n$;

22. 3 is a characteristic root of multiplicity 3;

26(a). $a_n = \frac{8}{9}n(3^n) + (-\frac{5}{9})n^2(3^n)$;

27(a). $a_n = 5^n - \frac{3}{5}n5^n$.

Section 6.3.

2(a). $a_k = 3k + 1$;

3(a). $a_n = 2 \cdot (-1)^n - 4n \cdot (-1)^n$;

3(e). $h_n = \frac{7}{3} \cdot 3^n + \frac{5}{3} \cdot (-3)^n$;

4(a). $-\frac{1}{2} + \frac{3}{2} \cdot 3^k$;

4(e). $\left(\frac{1}{2}\right)^k - \left(\frac{1}{3}\right)^k$;

9. $G_n = \left(\frac{3+\sqrt{5}}{2}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{3-\sqrt{5}}{2}\right)\left(\frac{1-\sqrt{5}}{2}\right)^n$;

15. $G(x) = \frac{2(x-2)}{(x-1)(x^2-2x+2)}$;

19. $G(x) = \frac{x^3}{(1-2x)(x^2+1)}$;

27. $1 + 7x + 13x^2 + 6x^3$.

Section 6.4.

2. $u_5 = 42$;

4. $R_4 = 4$;

5. $h_4 = 36$;

8(a). $C(x) - x^2 = A(x) + B(x) - x$;

8(b). $A(x) = 4xC(x)$;

8(c). $B(x) = x\{[C(x)]^2 + 1\}$;

8(d). $C(x) - x^2 = 4xC(x) + x[C(x)]^2$;

15(a). $q_3 = 5$;

19(i). $B(x) = x[U(x)]^2$;

22(c). $A(x)B(x) = \frac{x^3}{1-2x}$.