IN DISCRETE MATHEMATICS

Using Discrete Mathematics in the Classroom

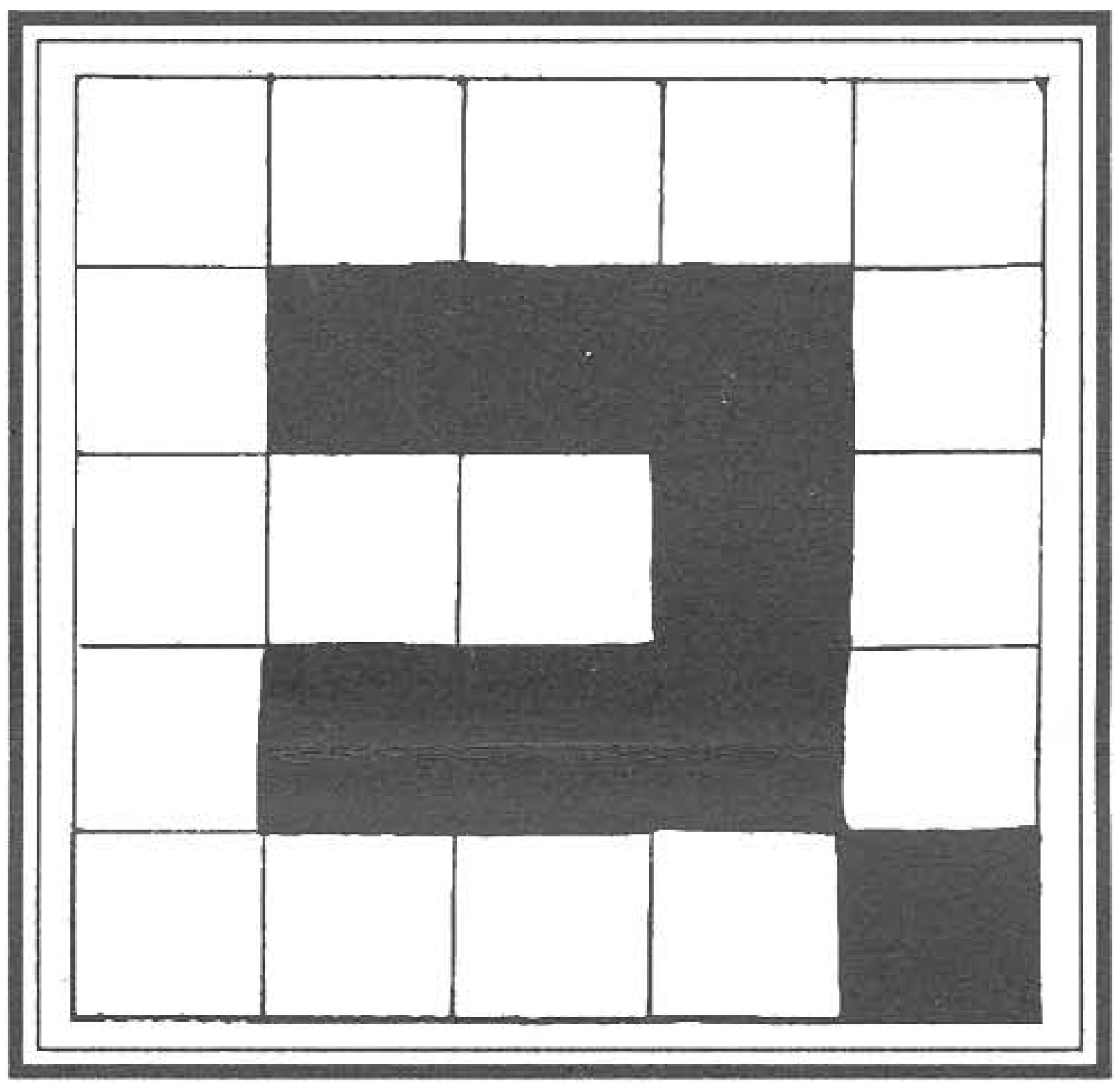
Issue #3 *August 1993*

Speaking discretely...

by Joseph G. Rosenstein

This is the third issue -- it took longer to produce than we intended, but here it is, and we hope that again the Newsletter meets your expectations!

Featured in this issue is a discussion of codes, including the ''Have-you-seen ... '' article at the right and the ''Mini-bibliography'' on page 9, both by Joseph Malkevitch; the picture at the right represents the problem, discussed on page 9, of receiving and interpreting data from space that is distorted by ''noise''. Also featured is fair division; the cartoon on page 12 and the accompanying article on page 10 address the problem of fairly dividing a cake, and the ''Dear Ann Landers'' article on page 2 addresses the problem of fairly dividing an estate. Also included in this issue are articles dealing with Pascal's triangle, the NBA draft lottery, and collecting for Goodwill.



You are invited to use these pages to share with us your thoughts about discrete mathematics, your classroom activities and experiences, your students' response to a new topic, etc. A one page summary of an interesting lesson would be valuable to all of your colleagues.

You may be looking for an opportunity to help get your colleagues interested in and excited by discrete mathematics. Participants in the *Rutgers Leadership Program in Discrete Mathematics* are available to conduct one-day workshops in over half the fifty states. Further information is provided on page 5, which can also serve as a flyer for our ''workshops in your district'' program; you are welcome to make copies of the flyer and distribute them at conferences.

Send us your comments for the next issue -- and enjoy this one!

In This Issue

Have-you-seen...

by Joseph Malkevitch

...the many articles in recent years dealing with codes and their applications.

Codes are a part of our daily lives and our daily vocabulary. We see a zipcode on nearly every letter that we receive. Every item that we purchase has a universal product code on it. Hardly a day goes by when the newspapers do not tell of some new aspect of the genetic code being discovered. Bar codes are used to track books at our libraries and our luggage at the airports. Codes published in the newspaper make it easier for us to program our VCR's.

Codes are also at work in less obvious ways. They are making it possible to have videophones, enjoy music from our compact disc players, get a picture back from Saturn, transfer money between banks, and keep our embassies in foreign countries advised on political developments. Codes continue to be used by spies and the military. (Study of secret codes allows for interesting joint lessons between social science and mathematics. Codes played an important role in the World Wars and, during World War II, in both the Atlantic and Pacific theaters.) Yet how many people realize that mathematics is at work in designing and implementing nearly all of these codes?

Why do codes exist? Thinking about the codes described above, it turns out that codes often serve different functions in different contexts.

Sometimes codes exist to hide information. This is true in the case of diplomatic and military codes. Yet hiding information is not the monopoly of the military and the State Department. As modern business practices have grown, the need for exchanging information between businesses in secret has grown.

(Continued on page 8)

Dear Ann Landers:

by Janice Ricks

Your recent column [on the right], in which **Faithful Reader in New York** wrote to request a solution to the age-old problem of how to amicably divide up an estate, prompted me to write this letter. Instead of suggesting that the family rely upon the advice of a lawyer, you should have recommended that they send for a mathematician!

Any student or teacher of discrete mathematics could tell you that not only is it possible to find an equitable way to divide an estate, but it is also possible for each person to end up with MORE than what they perceive to be their fair share. One way of doing this is called the Steinhaus Method [see box below].

So, Ann, the next time someone asks you **"How can we divide this fairly?"**, you might suggest that they find a mathematician to provide them with a method for doing the job neatly. It will undoubtedly be far less costly than letting a lawyer handle it. Mathematicians have been researching many areas of interest to the general public and you owe it to your readers to keep them better informed.

-- Discrete Reader in Philadelphia

The Steinhaus Method. In this method, each person bids on the items in question according to his or her own assessment of its worth, knowing that the highest bidder on any item will be awarded that item at the bid price. For example, suppose that Amy, Bart, and Carl are heirs to an estate that includes a painting, a car, a NY Yankee season ticket, and $5000 in cash. The following amounts are bid:

	Amy	Bart	Carl
painting	2000	5000	3000
car	4000	2000	3000
ticket	400	300	100

The total value of the estate to each person will be determined by the sum of that person's bids plus the $5000 cash. A "fair share" for each person can easily be determined by taking the total value of the estate to that person and dividing by the number of people bidding.

total value	11,400	12,300	11,100
fair share	3,800	4,100	3,700

The highest bidder on each item is then awarded that item, and cash adjustments are made so that each person receives a fair share by his or her own assessment. Thus Amy receives the car and the ticket, which to her are worth $4,400, and must return $600 to the cash pool. Bart receives the painting, which to him is worth $5,000, and must return $900 to the cash pool. Carl receives none of the items, and must receive $3,700 from the cash pool. Thus everyone has received what he or she feels is a fair share of the total. However, the cash pool now has a total of $5,000 + $600 + $900 - $3,700 = $2,800 which is divided equally so that each of the three heirs actually receives $2,800/3 more than what they consider to be their fair share.

There are, of course, a few basic assumptions that must be met in order for this method to work. However, so long as the parties agree to the method, everyone will come away satisfied.

Dear Ann Landers:

I am one of five children. Our mother passed away seven years ago and Dad has been living in the same home with a housekeeper. Dad is doing well, but soon he will be 92, and we have started to talk about what to do with the possessions that he and Mom accumulated over a lifetime.

My brothers and sisters all own their own homes, and all but one sister lives in this state. Most of us are well off financially.

The problem is, how do we divide the contents of Dad's house? My brother's wife has already asked for a painting she's always admired. My older sister has said on several occasions that she wants the wing chair, even though I also would like it. One of my sisters received jewelry from my mother before she died and the other two sisters did not. Needless to say, the list goes on and on, and what's worse, some of the better items have already started to appear in my siblings' homes.

How can we be fair about this? I think the wives and husbands of the siblings should not be involved or should get only token items. Should we number the valuable items and draw lots? Should we select according to age?

I hope you can come up with an amicable solution. I don't want the family to come apart over this.

-- Faithful Reader in New York

Dear New York:

Does your dad have a lawyer? I think he'd better step in and say, "Nothing goes out of this house as long as your father is alive."

After your father dies, the children (no spouses) should be permitted to select the items they want, with the eldest getting the first pick (one item only) and so on down the line. There's nothing like dividing an inheritance to bring out the worst (and best) in people.

TEACHING BRIEFS... Goodwill Tours

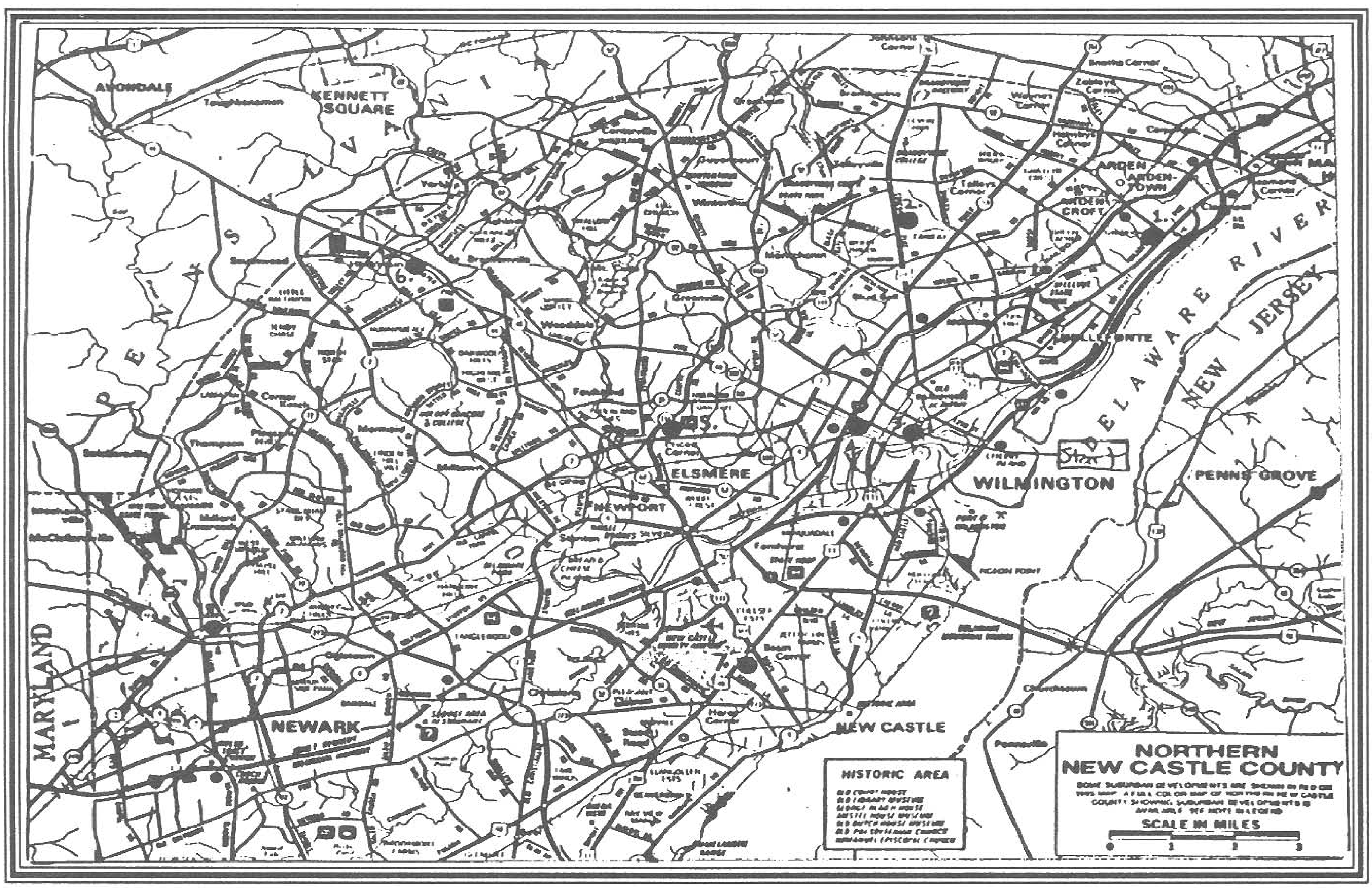
by Chuck Biehl

A radio commercial advertised that Goodwill Industries of Delaware had just opened a new collection center in the Wilmington area. The people in the commercial were trying to figure out how the new center would affect where they could drop off their goods for donation. Although the "mathematics" used in the commercial consisted of meaningless jargon and formulas typical of the media's portrayal of mathematics, the situation did lead to a Traveling Salesperson Problem (TSP) that evolved into a project for my consumer math class -- find the shortest circuit of the collection centers, beginning and ending at the downtown office.

The project was to take a map of the metropolitan area [see map below] and locate the total of eight Goodwill collection centers, including the central collection office downtown. (The director of Goodwill Industries provided the class with brochures listing all the locations and a tape of the radio commercial.) Using the map's scale, the students generated mileage charts showing the actual driving distance (not straight line distance) from any one center to all others. Using calculators and algorithms discussed prior to the project (nearest neighbor and cheapest link) the groups competed **to find the shortest circuit.** The final step was to take the results and use them to write directions for the driver, which turned out to be the hardest part!

In completing this project, the students used a wide variety of concepts and skills, including: measuring, map-reading, modeling (a graph was used based on the street map), estimation, rounding (distances were to be to the nearest tenth of a mile), making and reading tables, TSP algorithms, and writing. The cooperative learning environment was very effective in the sharing of the large tasks like generating the distance charts and directions from place to place, and the competitive nature of the final product kept class members actively engaged.

The most interesting aspect of the project was the follow-up discussion, in which the strengths and weaknesses of each model were examined. Although the ultimate solution could be useful to Goodwill Industries of Delaware, the students observed that it was impossible to account for factors such as time of day for collections, traffic patterns, construction along the route, and imbalance in the quantities of goods collected at each center. Generally speaking, however, the project was a great success, especially because the students felt that they had been able to use mathematics in a real situation that could have a positive impact on an organization which was helping the community.



RESOURCES...Decision Mathematics

by Alistair Carr

The standard network problems -- shortest path, network inspection, shortest connections, traveling salesperson -- and problems of choosing a team or a sequence of actions to optimize a quality -- these are all challenging problems, yet are easily within the grasp of any secondary school student with an inquiring mind and rudimentary arithmetical skills.

With this in mind, The Spode Group (Britain) was motivated to create and publish *The Decision Maths Pack*, worksheets for teachers, and *Decision Mathematics*, an A-level textbook for an Oxford University examination, both in 1986.

Later, Prof. Peter Galbraith and I adapted the material for Australian students. The texts have now been used for introductory graph theory with senior secondary school students as well as general problem solving classes on the middle school level, and to introduce some of these topics in a college-level Operations Research course.

The Decisions Maths Pack begins with problems that can be solved by trial and error, then progresses to more complicated problems where it is hoped that students will search for general methods, strategies, and perhaps a polished algorithm. (See the box at the right for an example.)

Teaching notes in the pack outline some theory, but more important, the notes suggest possibilities for adaptation of local examples, e.g., milk delivery, a postal carrier route, gas, telephone or cable TV networks, etc. Our text deliberately avoids a didactic approach, instead focusing on motivating mathematical learning via applications. It is a method which many students respond well to, and one that many teachers enjoy doing.

Project PAM (Practical Applications of Mathematics) has published a companion volume to *The Decision Maths Pack*, called *The Problem-Solving Pack, Australian Edition*. It was created for younger students, aged 11-14, and includes network and packing problems, as well as other discrete math topics. It has been used by a variety of students at levels from grades 6-10.

Inquiries about *The Decision Maths Pack* (UK Edition) may be made to Prof. David Burghes, Centre for Innovation in Mathematics Teaching, School of Education, University of Exeter, St. Luke's, Exeter EX1 2LU, Devon, England. Inquiries about Project PAM for secondary school students, may be made to Prof. Alistair Carr, School of Applied Science, Monash University College -- Gippsland, Switchback Road, Churchill, Victoria 3842, Australia.

Stamp Books

With Christmas approaching, Australia Post decides to try selling booklets of stamps. Ordinary letter stamps cost 36 cents and Christmas card stamps cost 30 cents.

1. Design a book of stamps costing \$3.00. It must contain both 36 cent and 30 cent stamps and the total value of the stamps must be \$3.00. What is the total number of stamps in the book?
2. Design a book of stamps costing \$6.00, containing 36 cent and 30 cent stamps.
3. Find other designs for a \$6.00 book of stamps.
4. Which of these books is likely to be the most practical? Why?
5. Suppose postage increases to 40 cents for letters and 35 cents for cards. Design a book of 20 stamps costing between \$7.00 and \$8.00 that has more ordinary stamps than card stamps and at least one of each. How many such books are possible?

(See "Solutions..." on page 6.)

RESOURCES... Drawing Pictures With One Line: Exploring Graph Theory; and Ethnomathematics: A Multicultural View of Mathematical Ideas...

reviewed by Susan Picker

Drawing Pictures With One Line: Exploring Graph Theory by Darrah Chavey, HistoMap Module #21, COMAP. This new module in the HistoMap series takes teachers and students through the historical beginnings of graph theory as recreational puzzles, to the array of applications for which graph theory is used today. Included are multicultural aspects of graph theory as it exists in cultures in Africa and Oceania as part of a heritage of sophisticated story-telling.

The historical sections of Chavey's module are excellent, providing a richer background on graph theory than I have seen before in a text meant for the secondary school level. In addition, many exercises are provided (although without solutions) along with the continuous relation of theorems and their proofs to real life problem solving. The module should prove a valuable resource for teachers and can introduce educators to the fascinating presence of graph theory in non-western mathematics.

The many examples of non-western graph theory in *Drawing Pictures With One Line* come from Chapter 2 of a new text by Marcia Ascher, *Ethnomathematics: A Multicultural View of Mathematical Ideas*, published by Brooks/Cole, 1991. In this chapter titled "Tracing Graphs in the Sand," the principles of graph theory are introduced in

(Continued on page 11)

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RUTGERS UNIVERSITY

LEADERSHIP PROGRAM IN DISCRETE MATHEMATICS

WORKSHOPS IN YOUR DISTRICT

WHAT? The Leadership Program in Discrete Mathematics will sponsor one or more full-day workshops in your school or district covering contemporary topics in discrete mathematics and offering ways of introducing these concepts into the classroom and into the curriculum.

WHO? Teachers at all levels should be encouraged to participate in the workshops, since many of the concepts can be introduced to and discussed with students at all levels. Teachers will leave the workshop with complete in-service packages which can be used both by workshop participants and, with their assistance, by other teachers in the district.

WHEN? Workshops will be scheduled during the 1993-94 academic year on an individual basis at the request of the participating district. Workshops may also be available for scheduling during the summer. Every effort will be made to accommodate dates requested by interested districts.

BY WHOM? The workshops will be conducted by experienced teachers who have participated in the summer institutes of the Leadership Program in Discrete Mathematics at Rutgers University and in a seven-day training program on preparing and presenting such workshops.

COST? Funding from the National Science Foundation will pay for all administrative costs of the discrete mathematics workshops. The district will be expected to pay only direct costs -- expenses and honoraria for the workshop leaders and instructional materials for the participants.

WHERE? Workshop leaders live in or near the following states: AZ, CA, CO, CT, DC, DE, FL, GA, KY, MA, MD, ME, NH, NJ, NM, NV, NY, OH, OR, PA, RI, TN, UT, VA, VT, WA, WV. For workshops in other states, there will be additional travel expenses.

FURTHER INFORMATION? Write to Discrete Mathematics Workshops, P.O. Box 10867, New Brunswick, NJ 08906 or contact Michelle Bartley-Taylor, Center Coordinator, at 908/932-4065. Requests may be made at any time. The number of workshops that can be offered is limited, so early requests are encouraged.

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Encouraging words...

This is *your* Newsletter -- that means that its success will be dependent on the willingness of you the readers to share your discrete thoughts and classroom experiences. So while you are going about your way in discrete mathematics, keep the Newsletter in mind, and if you notice something that might be of interest, write a few paragraphs to submit to the Newsletter. You will be thanked profusely by the other readers of *IN DISCRETE MATHEMATICS... Using Discrete Mathematics in the Classroom.*

Subscriptions...

Please send us the name, address, phone number, and school of any teacher who should receive a copy of this Newsletter, and we will include him/her on our mailing list.

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Using Discrete Mathematics in the Classroom

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RESOURCES ... COMAP is continuing to add to its Video Applications Library with the release of *Geometry: New Tools for New Technology*. This five-part video illustrates the geometry of the 20th century: motion-planning, error-correcting codes, Euler circuits, vertex coloring, and tomography. Additional videos include one showing the applications of calculus to medicine and engineering, with special emphasis given to modeling the AIDS epidemic, and another on historical applications of coding. These videos join the existing Video Applications Library which includes *Statistics: Decisions Through Data*, an introductory-level high school statistics course, and *Math TV* a light-hearted look at mathematics in the real world. For information call COMAP at 1-800-772-6627.

Credits...

The preparation of this Newsletter is a project of the *Leadership Program in Discrete Mathematics* at Rutgers University, New Brunswick, New Jersey. Funding for the Newsletter is provided by the National Science Foundation (NSF).

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DIMACS is a national Science and Technology Center (STC) founded by the National Science Foundation (NSF); it was formed in 1989 as a consortium of four institutions -- Rutgers University, Princeton University, AT&T Bell Laboratories, and Bell Communications Research.

Solutions... *(Continued from page 4)*

1. The book must contain five 36 cent stamps and four 30 cent stamps.

2. There are ten 36 cent stamps and eight 30 cent stamps in the book.

3. Other designs are possible: five 36 cent stamps and fourteen 30 cent stamps; fifteen 36 cent stamps and two 30 cent stamps.

4. The design with ten 36 cent stamps is probably best as it allows for a more even balance between numbers of letters and cards. If mainly cards (or letters) are needed, the respective designs in 3 above are probably best.

5. One possible book has fifteen 40 cent and five 35 cent stamps. There are nine possible books containing from 11 to 19 ordinary stamps.

TEACHING BRIEFS...The NBA Draft Lottery

by Martin Levinton

Not long ago the National Basketball Association had a simple system to help improve the teams which won the fewest games the previous year. The team with the worst record would get the first choice of the college basketball players eligible to play professional ball, the team with the next to worst record would get the second choice, and so on. But under that system the poorer teams might actually gain an advantage by losing more games. Realizing this, and wanting to maintain public confidence, the NBA changed the system of selecting new players.

At present, eleven teams in the NBA do not reach the championship playoffs each year. The other teams still choose their new players in reverse order according to their records the previous year (after the eleven non-playoff teams choose theirs), but the system for those eleven teams is as follows:

1. The eleven teams are ranked in reverse order and each is given a different number of ping-pong balls with its logo; the worst team gets 11 ping-pong balls, the next to worst, 10, and so on.
2. The first three picks are done by lottery and the fourth through eleventh picks proceed inversely by record. This insures that the very worst teams are still guaranteed relatively good picks.
3. Ping-pong balls are not replaced after chosen, and if a ball designating a team already selected in the lottery is drawn, it is disregarded.

Here is a list in order of the eleven teams with the worst records last year; #1 was worst.

1. Minnesota Timberwolves
2. Orlando Magic
3. Dallas Mavericks
4. Denver Nuggets
5. Washington Bullets
6. Sacramento Kings
7. Milwaukee Bucks
8. Charlotte Hornets
9. Philadelphia 76ers
10. Atlanta Hawks
11. Houston Rockets

Under the new system, if Denver (#4) had gotten lucky and obtained the first pick, Sacramento (#6) the second pick, and Minnesota (#1) the third pick then the other eight teams would have chosen basketball players in the following order: #2, #3, #5, #7, #8, #9, #10, #11.

(Continued on page 9)

TEACHING BRIEFS...The 12 Days OF XMAS

by Georgeanna Fernandez

Every December I put up a bulletin board which includes Pascal's triangle and the old song "The Twelve Days of Christmas." The major question I ask is Question 1: *"How many gifts were given altogether?"* I clarify that a "partridge in a pear tree" is one gift. I also ask Question 2: *"How many gifts were given on the fifth day?"* If you remember, the song says "five golden rings, four calling birds, three french hens, two turtle doves and a partridge in a pear tree." This is not the same as Question 3: *"How many gifts had been received by the fifth day?"* You can also ask Question 4: *"Which day had the most gifts and how many?"* or Question 5: *"How many golden rings?"*

The answer to most of these questions are in the diagonals of Pascal's triangle (see below), where each entry is the sum of the two entries immediately above it. Construct Pascal's triangle up to at least the row that starts 1, 14, 91, 364 ... Next to the "ones" diagonal is the "counting number" diagonal which represents the days or the number of new gifts for any particular day. For example, day 7 is 7 swans a-swimming. The next diagonal represents the total of gifts received in one particular day. For example, the answer to Question 2 is $5+4+3+2+1=15$, which is the number below and to the left of 5 if you are on the right side of the triangle. Also, for Question 4, the best day is 12, with 78 gifts, again below and to the left of 12.

The diagonal that contains 1, 4, 10, 20 ... is the cumulative total. The answer to Question 3 is 35, the fifth entry in that diagonal. In simpler terms, at the end of day 2 you had 1 old plus $3=(2+1)$ new for a total of 4, at the end of day 3 you had 4 old plus $6=(3+2+1)$ new for a total of 10, and

(Continued on page 11)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 56 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 .... 715 286 78 13 1
1 14 91 364 .... 364 91 14 1

Have-you-seen...

(Continued from page 1)

Banks want to transfer sums of money without the money being diverted by intruders. The users of an ATM machine want assurance that their transactions are private.

On the other hand, in the case of the (machine readable) zipcode or barcodes, the goal for the code is not to hide information but to speed or track information. Thus, the universal product code serves the dual function of allowing the customer to get out of a supermarket faster, and the store that sold the item to keep inventory on what it sells. This makes it more likely that your favorite store will have your favorite cereal in stock.

The codes used for the information in pictures from Saturn or for the information on a compact disc serve yet another function. These codes are there to correct or detect errors which might occur due to a solar flare during transmission of a picture or due to a speck of dust on the compact disc. (See the example discussed at the right.)

Another recent use of code technology has been in compressing sounds, text (files), and pictures. When television pictures are sent through a cable or a text file is sent electronically from one place to another, the information is represented by a gigantic number of 0's and 1's. Since English has great redundancy and since pictures have a limited number of gray levels, it becomes feasible to use a code that allows faster transmission but that also allows perfect reconstruction of the original.

Finally, codes are being used to synchronize information. Thus, sound and image, recorded separately at a live concert, must be combined for natural viewing.

One of the major tools in the design of new codes for secrecy is the theory of numbers, the subject that the pacifist number theorist G.H. Hardy prided himself for being a researcher in because he felt it could never be put to use. Today number theory is the key to dramatic improvements in what crypt-systems can accomplish. More than ever before the hand-in-hand relationship between mathematics being developed for no specific purpose and applicable mathematics is being demonstrated.

Error-Correction

How can one send a picture back from a distant planet? Imagine that the image to be sent (see box on page 1) has been divided into a grid involving, for simplicity, five rows and five columns. The gray level of each of the 25 cells can now be recorded and a binary code used to represent each gray level.

For simplicity, we have used only two gray levels, black and white. If black is represented by the code word 1 and white by the code word 0, the original image on page 1 can be transmitted as the following sequence of 25 zeros and ones:

0000001110000100111000001

However, if any noise occurs during transmission, which results in zeros and ones being interchanged, the result is that the original image is reconstructed improperly. Richard Hamming pioneered the development of codes to correct errors. A simple example of such a code is to use the code word 111 for black and the code word 000 for white. If no more than one error is made in the transmission of each code word then the code allows accurate restoration of the original image. For example, although a number of errors have been made in transmission, the following sequence can be decoded as the original picture on page 1.

0000001000100000001111111
1100010001000011100010000
1111111010100000001001000011

The basic idea that Hamming had was that if a code word can not be transformed into another code word by fewer than 2s+1 digit changes, then the code will be able to correct s errors. In our example, the code requires 3 changes to transform one word to another so 1 error can be corrected.

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2. *Fisher, L.,* ***Yes, CD Sound Is "Perfect".*** *... NY Times, Oct. 25, 1992, v.142 p. F10(N).* Description of a new data compression technique for audio compact discs.
3. *Fleisschmann, M.,* ***More-accurate CDs;*** *Normile, D.,* ***Dazzling Shows Debut New Audio and Video,*** *Popular Science, (v. 242, no. 2), February 1993, pg. 28-29.* An account of how new advances in compression techniques will make possible better quality sound and images in the near future.
4. *Healey, B.,* ***How To Decipher Those Rather Simple Symbols.*** *NY Times, Jan. 28, 1990, v. 139, p. 22(n).* Bar codes in philately (stamps) column.
5. *Markov, J.,* ***A Public Battle Over Secret Codes,*** *NY Times, May 7, 1992, v. 141 p. C1 (N).* An account of feuding between the government and business over the regulation of encryption techniques.

(Continued on page 9)

Mini-Bibliography... Codes

by Joseph Malkevitch

This mini-bibliography addresses the major applications of codes. In the future, some of the specific ways codes are used in each of these areas below will be explored in more detail.

1. CODES FOR SECRECY

*Chaum, D., **Achieving Electronic Privacy,** Scientific American, Aug. 1992, p. 96-100.* This article documents dramatic new uses for ideas related to public-key cryptography.

*Kahn, D., **The Code-Breakers,** Macmillan, New York, 1967.* Kahn treats in detail the history of the use of secret codes and writes in such an exciting manner that this long book reads like a novel.

*Kahn, D., **Kahn On Codes,** Macmillan, New York, 1983.* This is a series of short essays which details the saga of secret codes during the cold war era.

*Sinkov, A., **Elementary Cryptanalysis,** Mathematical Association of America, Washington, 1968.* This book serves as a primer concerning the design of substitution ciphers (both monoalphabetic and polyalphabetic) and how to break such ciphers.

*Hellman, M., **The Mathematics of Public-key Cryptography,** Scientific American, Aug. 1979, p. 146-157.* Hellman gives an account of the pioneering concept he helped to develop, now called public-key cryptography. Public-key systems are now being used in the commercial sector of the economy and offer great promise for exciting new possibilities such as "smart" credit cards.

2. ERROR DETECTING AND CORRECTING CODES

*Malkevitch, J., and G. Froelich, **Codes Galore,** COMAP, Lexington, 1991.* An account of some of the ways error detecting and correcting codes are designed and are being used. Also, historical background on secret codes.

*Gallian, J. and S. Winters, **Modular Arithmetic In The Market Place,** Amer. Math. Monthly, 91 (1988) 548-551.* An account of a great variety of error-detection schemes.

*Hill, R., **A First Course In Coding Theory,** Oxford U. Press, New York, 1988.* A lucid but technical account of the theory of error correcting codes.

*Truxel, J., **The Age of Electronic Messages,** Mc Graw Hill, 1990.* An expository account of the revolution wrought by information theory, the branch of mathematics pioneered by Claude Shannon.

3. DATA COMPRESSION CODES

*Malkevitch, J., and G. Froelich, **Loads of Codes,** COMAP, Lexington, 1993.* This volume (which complements ***Codes Galore***) provides brief descriptions of the use of substitution and transposition ciphers. It includes material on the use of matrices to design codes. Finally, a discussion of the need for and construction of codes to compress data is given.

*Meyer, W., **Huffman Codes and Data Compression,** UMAP Journal, 5 (1984) 277-297.* A clear and detailed account of various points of view about the data compression codes developed by David Huffman.

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(Continued from page 8)

6. *Powell, R., **Digitizing TV Into Obsolescence,** NY Times, Oct. 20, 1991, v. 141 p. F11(N).* An account of how data compression technology is causing rethinking about how television technology can be advanced.
7. *Shapiro, E., **Self-Serve At The Checkout Lane; Bar Code Scanning In Its Ultimate Test,** NY Times, Jan. 6, 1991, v. 140 p. C1(N).* This article describes experiments to show the feasibility of using bar codes to allow self-service in stores.
8. *Stix, G., **Encoding The "Neatness" Of Ones and Zeroes,** Scientific American, Sept. 1991, v. 265 p. 54.* A biographical account of David Huffman, a mathematical engineer, who developed codes for data compression.
9. *Weber, J., **New Electronics To Pack More In Less Space,** LA Times, Jan. 15, 1991 p. D6.* An account of new products that are being developed based on data compression techniques.

TEACHING BRIEFS...The NBA Draft Lottery

(Continued from page 7)

Here are examples of questions which might be asked concerning this lottery; for "Solutions..." see page 11.

a. What is the probability that Dallas gets the first pick?
b. What is the probability that Minnesota does not get the first pick?
c. What is the probability that at least one of the first two ping-pong balls drawn had Minnesota's logo on it? (Note: The first two balls drawn could both have Minnesota's logo on them but by rule 3 the second ball would not be counted towards the lottery.)
d. Give an example of a draft lottery sequence where Washington gets the seventh pick.
e. Explain why it is not possible for Minnesota to get a draft pick worse than fourth.
f. What is the probability that Washington gets the fourth pick?

What Do Mathematicians Do? *(See cartoon on page 12)*

If one asks the "person on the street" what plumbers, electricians, chemists, or geologists do, they are likely to give you a reasonable answer. Put in more dramatic terms, when home-owners see water cascading through the ceiling, they do not call a carpenter or a mathematician. But few people on the street know when to call a mathematician.

One thing that we can do about mathematics' image problem is to discuss how mathematics affects people's lives, even if we cannot always do proper justice to the mathematics involved. For example, we can say that mathematicians (not chemists or plumbers) study waiting lines, and show that this can be applied at banks, airports, and in computers. Or, we can say that mathematicians find shortest paths and networks, and show that this can be applied to travel arrangements and telephone connections.

The above paragraphs are adapted from "Mathematics' Image Problem" by Joseph Malkevitch (see address on page 11). The cartoon on page 12 was drawn by Joe Pipari one of whose colleagues at Thomas McKean High School (Wilmington, Delaware) participated in a discussion with Malkevitch at the 1990 Leadership Program in Discrete Mathematics. The situation presented in the cartoon is discussed below. ■

Calling that mathematician... by Ethel Breuche

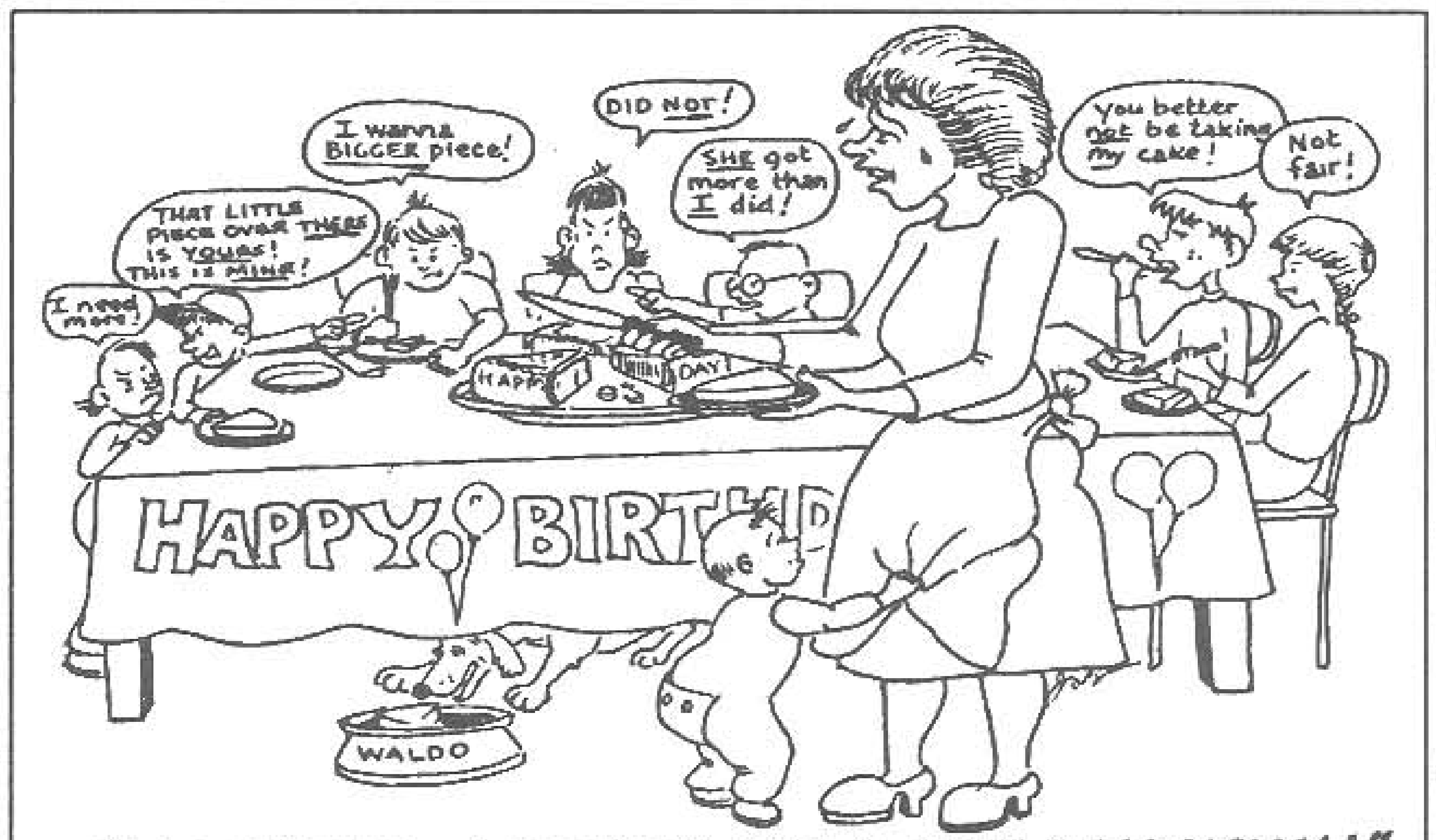
(see cartoon on page 12)

The situation depicted in the cartoon points out vividly the problem of how to divide a cake fairly. (The standard answer "just divide it into thirds" would work only if we were able to do that perfectly; in real life, it is hard to divide things perfectly -- see page 2 for another example.)

In discrete mathematics, *fair division* provides a rich source of real life problems which include dividing such things as cake, candy, pizza, the air waves, property, and estates.

I have taught fair division problems in quick one day lessons, as well as an extended four day unit; this was great fun, and motivating for both the students and teacher. Cooperative learning was the teaching strategy I used most frequently. Working in small groups, my students actually performed fair division on brownies using each of the methods described in the box at the right, and several others.

Brownies are easy to bake, cheap to make, and easy to push back together for further division (especially if you don't make them too chewy). Roles were rotated in the groups so that each person "taught" one method of fair division to the rest. Time was provided for students to discuss the fairness of the method used. Variations were added to the basic problem by introducing another player or by having certain players have particular preferences. Follow up problems for homework can be created at all levels of difficulty, and groups can be challenged to come up with their own problems.



"M-O-M-M-M-Y... WE BETTER CALL A MAFF-A-MAGICIAN!"

A Sampler of Fair Division Methods

Divider Chooser Method: This method involves two players. One player divides the item (property, cake, land, etc.) into what he/she considers two equal shares. The other person chooses.

Lone Divider Method: This method involves three players (or more with variation). One player (chosen randomly) divides the item into what he/she considers to be three equal parts. Each of the other two players (the choosers) declares independently (usually by writing it down on a slip of paper) which of the pieces he/she believes to be a fair share (i.e. worth at least one-third) and therefore acceptable. More than one answer is admissible. Only one situation requires extra steps taken, when both choosers select the same one piece. In this case, the divider picks one of the other two pieces and the remaining two pieces are put together and the two choosers then use the ***Divider Chooser Method***.

Lone Chooser Method: This method involves three players. One player, chosen randomly, will be the chooser of the three players. The other two players will cut the item into two pieces using the divider chooser method (i.e. one will cut the item in half and the other will choose the piece he/she will work with.) Now these two same players divide their piece into three pieces. There are six pieces altogether. The chooser picks one piece from each of the dividers and then the dividers keep the remaining two pieces.

Last Diminisher Method: This method is good with five or more players.

Round 1. Using an uncut dish of brownies (or a large batch of wrapped candy), player 1 cuts a section that he/she believes to be a fair share (one-fifth) of the whole. Player 2 now has the right to pass or to play. If player 2 thinks that player 1's claim is a bad choice (worth less than 1/5th), then player 2 passes and remains in contention for a fair share of the rest of the goodies. On the other hand if player 2 thinks that player 1's claim is a good one, then player 2 can make a claim on player 1's claim by

(Continued on page 11)

A Sampler of Fair Division Methods

(Continued from page 10)

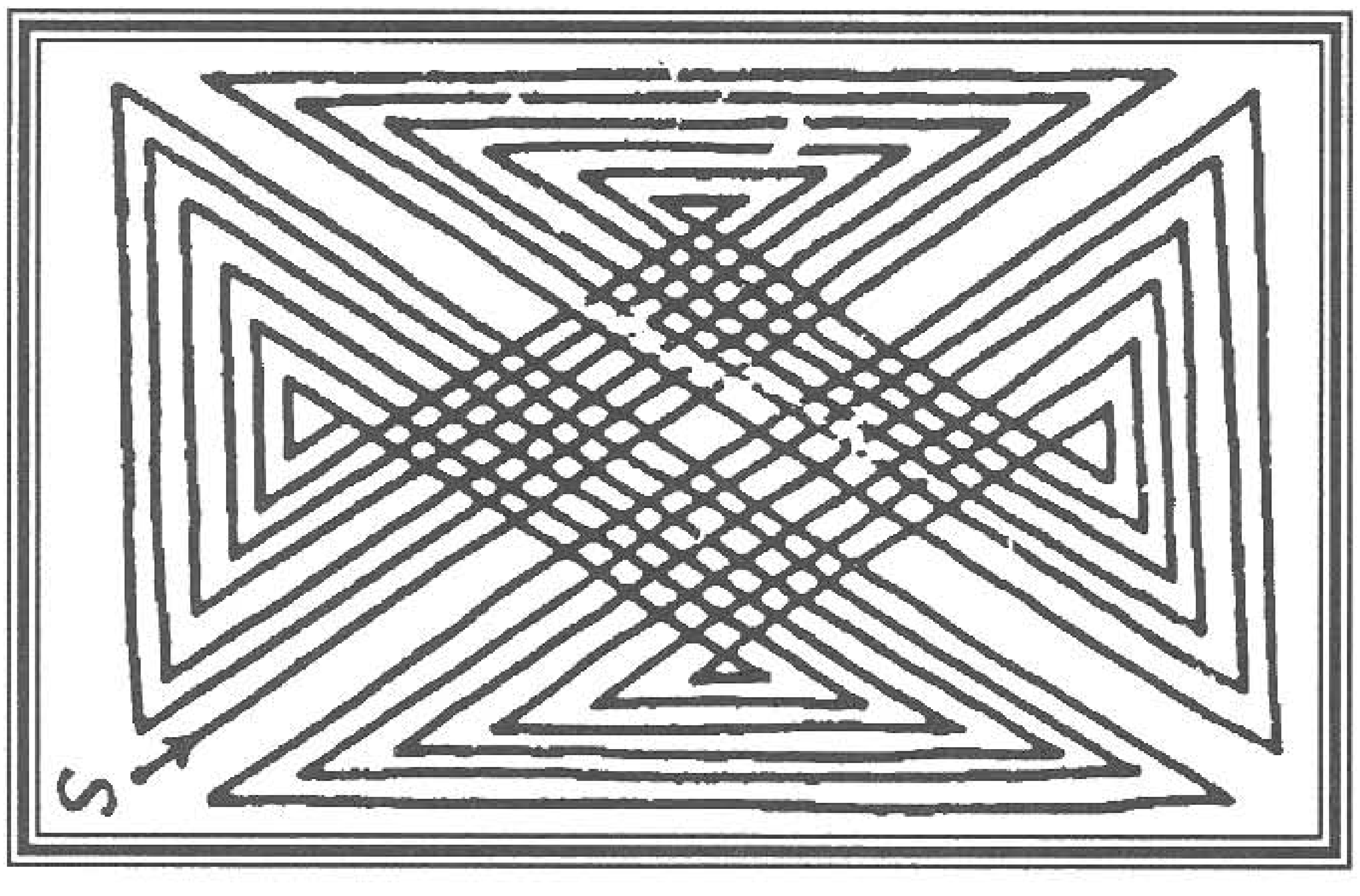
staking out a subpiece of it of diminished size, in which case player 2 is called a **diminisher**. If player 2 becomes a diminisher, then the difference between player 1's claim and player 2's claim is added to the remainder of the goodies and player 1 returns to the group of players contending for that remainder. It is now player 3's turn. Player 3 has a choice to pass or be a diminisher. Similarly player 4 and 5 have the same choices. After all the players have had a chance to play, the player whose claim is current (the **last diminisher**) gets that portion of the goodies and departs the game. **Rounds 2 and 3** are played in exactly the same way. **Round 4**. There are now two players left and the left-over goodies can be divided using the **Divider-Chooser Method**.

RESOURCES...

(Continued from page 4)

both the context with which we are familiar (e.g., the Konigsberg Bridge problem and Euler's Theorem) and in the context of the cultures of peoples in Africa and the Pacific Islands. Here for example, we find the concept of an Eulerian Path utilized not for the purpose of finding an efficient delivery route, but used instead to fulfill a different need in the South Pacific island of Malekula, from which it comes; here Eulerian paths are used in the telling of complex stories passed down in an oral tradition -- see, for example the graph of the *rame'* bird's nest below.

But *Ethnomathematics* explores much more than the topic of graph theory as it presents the mathematical ideas of number, kin relations, games of chance and strategy, perception and use of space, and symmetric strip decorations in Native South and North America cultures. It provides a comprehensive look at the meaning and use of similar mathematical ideas in different cultures, illuminating both the mathematics and the culture in which it appears, and through this showing the value of the study of mathematics in a multicultural setting.



TEACHING BRIEFS...The 12 Days OF XMAS

(Continued from page 7)

at the end of day 4 you had 10 old plus 10 = (4 + 3 + 2 + 1) new for a total of 20. Then at the end of day 5 you had 20 old plus 15 new or 35. To answer the main question, look at the 12th entry in this diagonal for 364. This means there was one gift for every day of the year, except your birthday. The answer to question 5 is 40, which has nothing to do with Pascal's triangle.

For those of you who due to certain court rulings prefer a more discrete mathematical song, I offer the following version of Verse 12:

On the twelfth day of math class,
my teacher gave to me
twelve spanning trees
eleven one-way streets
ten Steiner points
nine algorithms
eight combinations
seven Sierpinski triangles
six permutations
five fascinating fractals
four matrices
three Euler circuits
two calculators
and a book titled Geometry.

SOLUTIONS...

(Continued from page 9)

1. 9/66 = 3/22.
2. 1 - 11/66 = 55/66 = 5/6
3. P(Minn,Minn) + P(Minn,notMinn) + P(notMinn,Minn) = (11/66)(10/65) + (11/66)(55/65) + (55/66)(11/65) = 2/78 + 11/78 + 11/78 = 24/78 = 4/13. It could also have been calculated using 1 - P(notMinn,notMinn).
4. Two teams with numbers higher than 5 must get two of the first three picks and the last eight numbers must be arranged in ascending order -- ex. #6, #3, #10, #1, #2, #4, #5, #7, #8, #9, #11.
5. If Minnesota does not get one of the first three picks it would have the lowest ranking of the remaining teams and therefore pick fourth.
6. Zero. It is impossible to create a sequence with Washington fourth.

"M-O-M-M-M-Y... WE BETTER CALL A MAFF-A-MAGICIAN!"

[See article on page 10]

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