IN DISCRETE MATHEMATICS

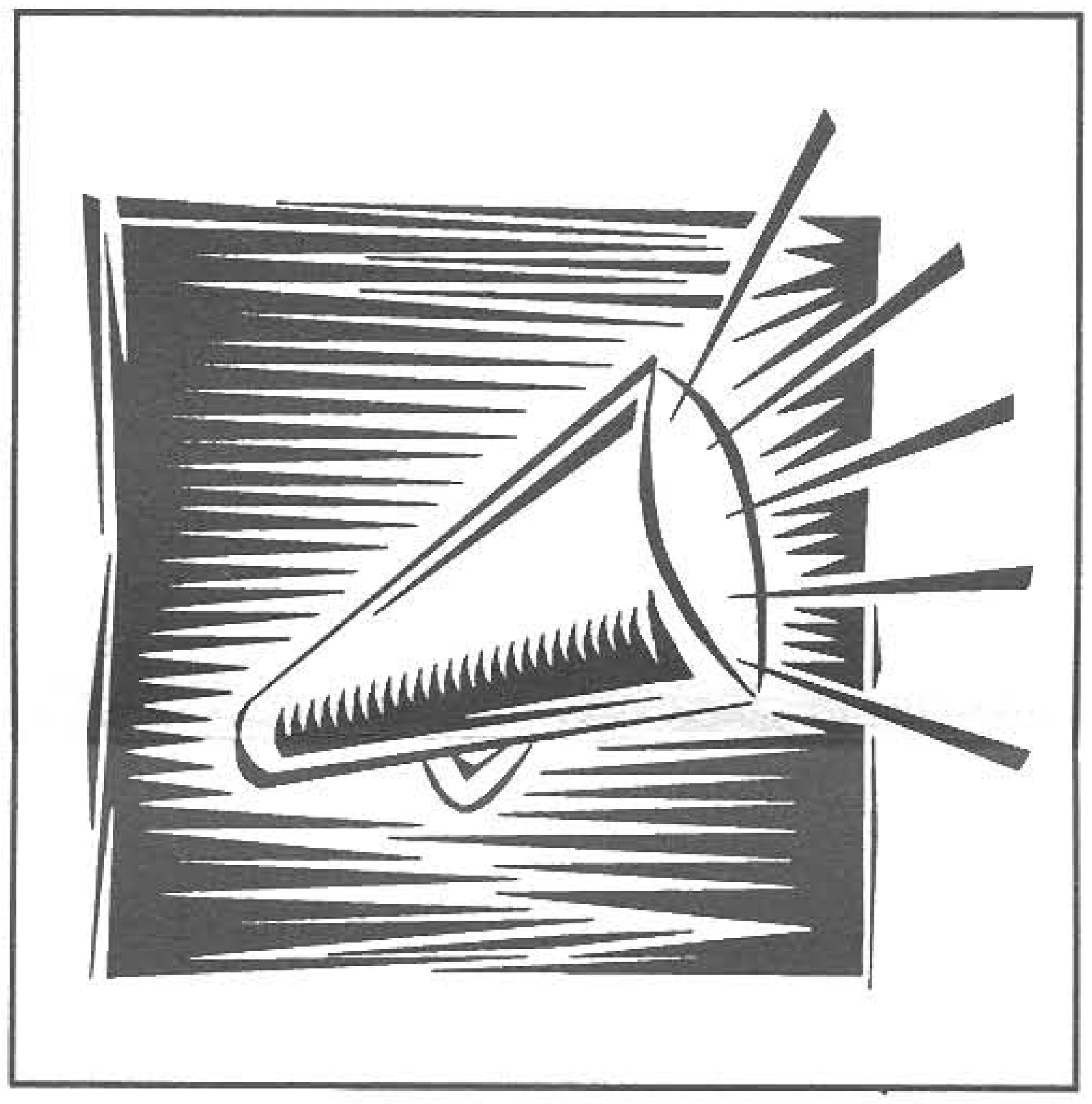
Using Discrete Mathematics in the Classroom

Issue #7 *Fall/Winter 1995*

Speaking Discretely . . .

by Deborah S. Franzblau

I first want to welcome a new group of readers, the first group of elementary school teachers to join the Leadership Program in Discrete Mathematics. There is still time to apply for this summer (see the flyer on p. 11). If you are a high school teacher, this issue has information on two new summer activities (see p. 10). (If you are interested but the deadlines have passed, please call us anyway—there may still be spaces available.)



Eric Simonian's article, "The Tangram Magicians," (p. 3) is perhaps an ideal start for expanding our scope to the elementary level; he describes a project involving 2nd and 4th graders and high school students working together. Linda Dodge's article, "The Venn Diagram Game" (p. 5) describes a popular activity that can be adapted to many grade levels.

Our lead article in this issue, by Anne Carroll, is a practical guide to publicizing in the local news the interesting things you and your students are doing in the classroom. As you will see, this is easier than you might think. (And when you do get into the news, don't forget to send us a copy of the clipping!)

There are a variety of topics in this issue. For a great story about students addressing a real problem at their school, see "In Case of Fire" by Melissa Kennedy (p. 2). Chuck Tiberio shares an interesting number puzzle that can be used as an introduction to dynamical systems and chaos (p. 5). And, Carol Price describes her students' success in discovering their own algorithms for solving graph problems (p. 4). ❖

In This Issue

Your Classroom in the News

by Anne Carroll

Mathematics is making more headlines these days than ever before, and we as mathematics educators can create a renewed awareness and appreciation for the "Queen of the Sciences." Believe it or not, the press is anxious for submissions about education, and they rely on school personnel to provide the information. For example, free newspapers are hungry for well-written stories. Often, they will even come to your school to cover an event.

I was recently successful in getting one of my classroom activities in graph theory published [1]. The "hook" was the great costuming my students did for a production of "The Case of the Stolen Diamonds" [2], which made for terrific press photos. (It also helped that I am the publicity director for my school district—when news from other sources is slow, I can fill in with my own.) Chuck Biehl's use of discrete mathematics in teaching problem-solving skills was touted in a front-page article [3] that featured his students with their wall-sized Sierpinski Triangle and discussed their attempts to solve traffic conflicts at Delaware intersections. More recently, Chuck was interviewed in the same paper, and discussed his appointment as Dean of the Academy of Mathematics and Sciences.

Even if you haven't had much experience with media relations, I hope that this article will help you make the events in *your* classroom tomorrow's headline. First, if you have a publicity director in your school or district, contact him or her about what is needed to get items published. If you are on your own, get the name of the education editor or reporter(s) at weekly and daily local newspapers which cover your school district. (In most papers, editors are listed on page two, or the editorial page; look for reporters' bylines with articles.) Direct all correspondence (including

(Continued on page 8)

In Case of Fire

by Melissa Kennedy

West Islip High School was built in 1958, and since that time, new wings were added and changes were made in the fire exits, creating some problems with the old fire drill plan. I became aware of this problem when my classroom was changed—during the first fire drill, it seemed that my classes had to walk farther than any other class in the building!

At the time I could do nothing but a slow burn, but the next year, after my discrete math class had its first fire drill, the students were easily convinced that a study of the fire exits was due. The class, which consists of six above-average students, came up with three objectives for this project:

1. Shorten the distances to exits where possible.
2. Cut down on the congestion at exits.
3. Assign paths to exits so that no two paths cross.

I copied floor plans of the school from the student handbook and fire exit assignments from the faculty handbook (see picture). Each student was assigned several exits along with the job of finding all the rooms that used those particular exits. Next, we drew a chart with the exits and the rooms that used them, and color-coded a floor plan to show this information. Most of this was accomplished during one class period, and finished as homework the same night. Next, the students decided that the following were the most important things to measure:

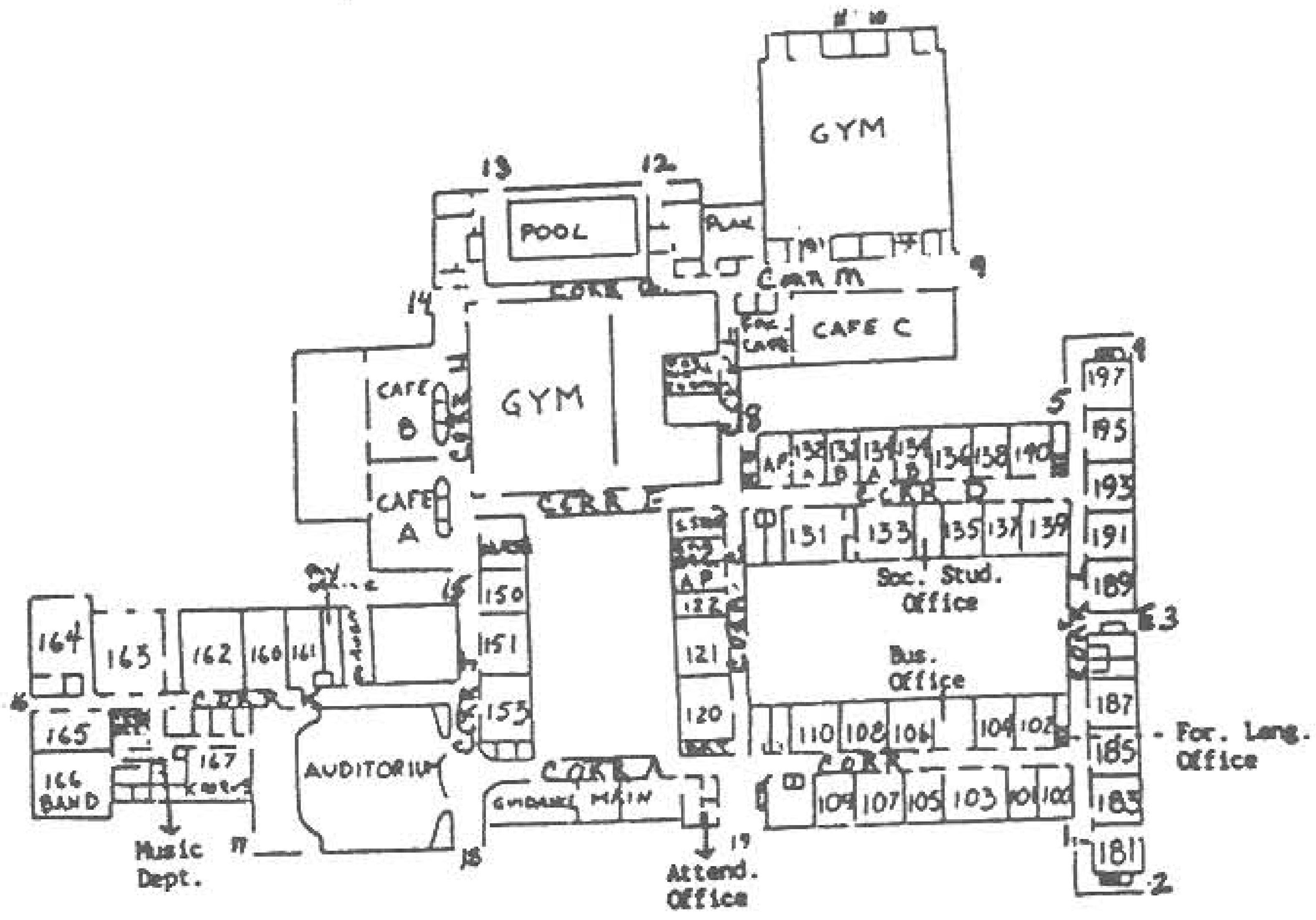
1. The maximum distance to each exit among the rooms assigned to it.
2. The number of doors at each exit.
3. The number of people using each exit.

Using these figures, we could then find the maximum distance from any room to its exit, compute the number of people per door (a measure of congestion), and thus estimate the maximum time necessary for all rooms to clear their exits.

Working in pairs and using their floor plans as a guide, the students measured the distance to the room farthest from its exit for each of twelve exits. They used a 50-foot tape measure (donated by the physical education department) and measured distances to the nearest foot. In order to estimate the number of people using an exit, the students decided to look at a worst-case scenario. They assumed that each classroom was filled to capacity with 30 students, each cafeteria with 250 students, and each gym with 60 students. Various offices—the nurse's office, guidance office, and main office were assigned appropriate numbers. From these figures the maximum number of people using an exit was calculated.

After gathering the information and studying it, the students wrote up their findings and recommendations and presented their report to the building principals. The report included a list of doors that were not operational or inaccessible, as well as those that were underused or overused. It presented a plan to re-route some of the paths to use the exits more efficiently. The students also suggested that the old fire drill signs in the classrooms be replaced with new ones that showed floor plans with arrows indicating the evacuation routes. This would help both students and teachers follow the assigned routes and alleviate congestion caused by classes using the wrong route.

In the course of studying the fire drill exits, the students discovered that there were good reasons for having certain rooms use certain exits. They found that our classroom was indeed the farthest in the school from its assigned exit, but found a way to re-route this classroom and others in the same wing to a closer, underused exit. They made positive suggestions that would improve both the safety and efficiency of a school evacuation. The students were especially pleased that the principals took their project seriously and used their report the next time they made changes in the fire exit procedures. Not only did this project provide a natural setting to develop skill in optimization, measurement, and estimation, but it gave the students an excellent model for making improvements in their community. ❖



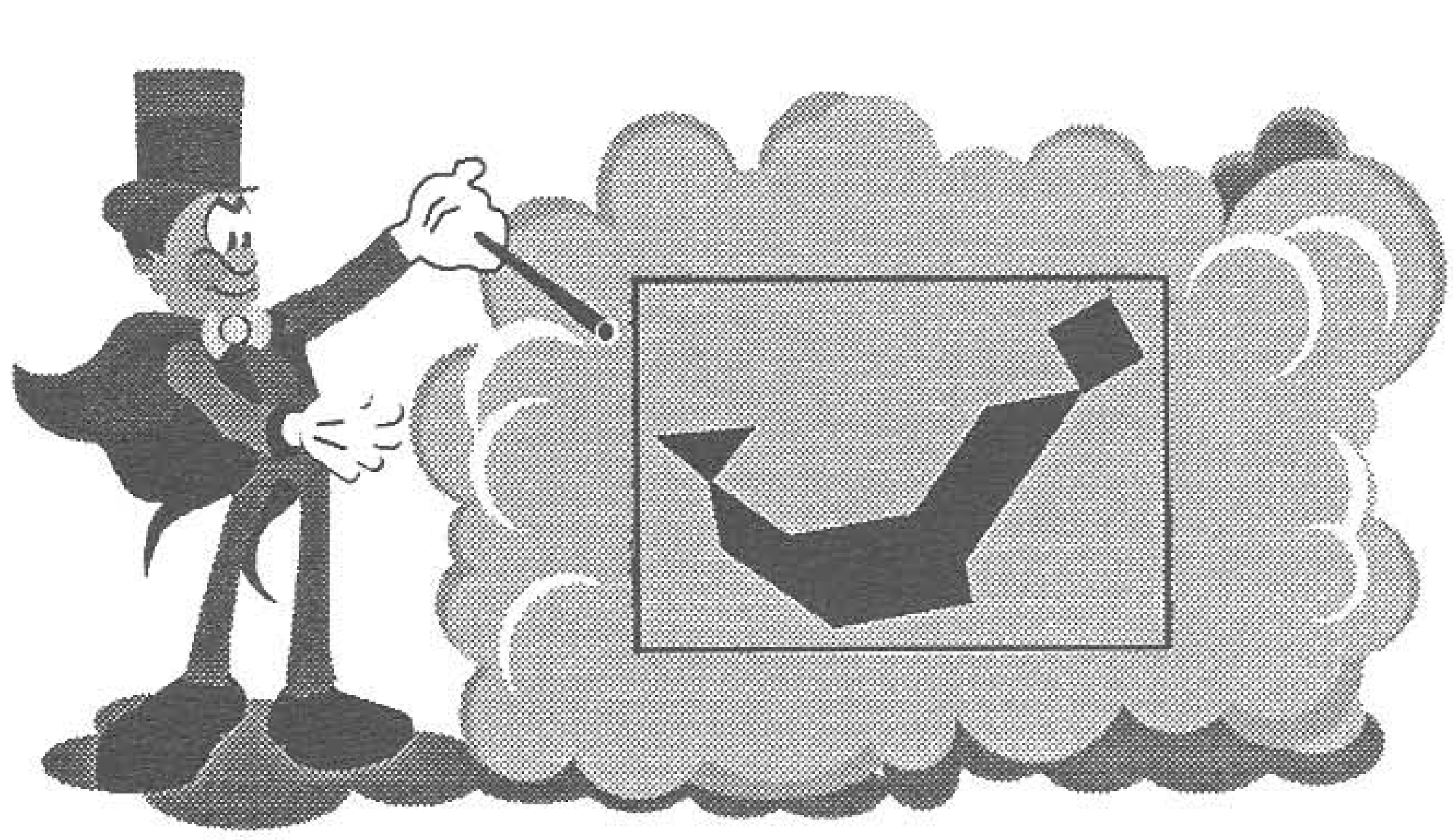
First Floor

AP - Assistant Principal
E - Elevator
Exits - Numbered
Corridors - Lettered

FLOOR PLAN

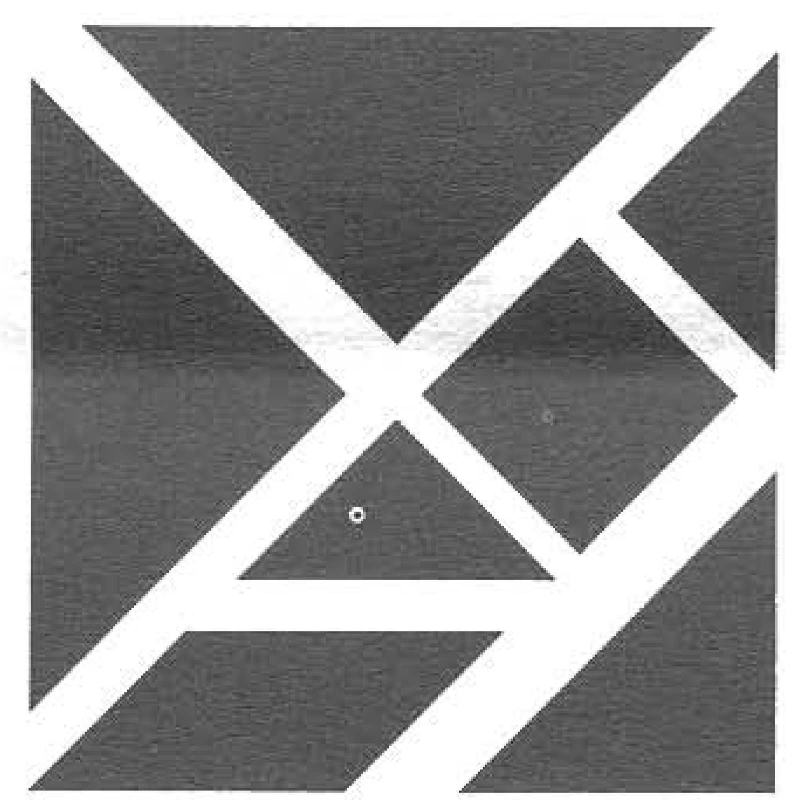
The Tangram Magicians

by Eric Simonian



During a Rhode Island College course on Alternative Assessment in the fall of 1994, I was paired with a second and a fourth grade teacher to plan a group project. I was teaching a geometry course for 10-12th grade students at the time. Somehow, we began talking about using tangrams (which the other teachers had worked with for years). The more we talked, the more nervous I got—I had never worked with tangrams before. But I was willing to give it a shot. I suggested getting together with them so that I could learn more. Then, one of the elementary school teachers had a brainstorm. She suggested that we put our students together—she wanted to see how different age groups interacted on an unfamiliar problem.

I was to read the students a story, *The Tangram Magician*[1], about a magician who could become anything or anyone he wanted to become, illustrated with tangrams. The last page of the story is left blank: it was up to the students to figure out what the magician would become next and create it with the tangram pieces. We decided to assign the students randomly to groups of six, each with two high school students, two second-graders, and two fourth-graders. We held the "event" at my school's library (we were all in different districts).



Set of Tangrams

The high school students didn't like the activity at first: their initial comments were "I can't do this," or "It's too hard to think"! But after seeing the younger students dive right in, and getting the idea, they started to have fun, and became much more enthusiastic. Interestingly, there were no significant differences in the types of designs the students made, although the older students were in fact more inhibited and slower to create their designs. (See p. 12 for one of the students' designs.)

Since my students knew as little about tangrams as I did, in order to prepare for the visit, they spent three days working in pairs solving puzzles with tangrams. In addition, I had them keep a journal recording their day-to-day thoughts, before and after the visit. The results were fascinating. Many of my students were not looking forward to the visit, but changed their minds quickly afterwards. Some of them reported that during the activity they felt that the elementary school students were smarter than they were, since they were able to solve the puzzles so quickly. Eventually, however, they realized that the younger students were simply much freer in using their imaginations, and how useful this was in solving problems.

My hope was that the activity would help my students in our work with proofs. It did—though indirectly. As I realized later, visual imagination is very important in creating geometry proofs: you need to "see" which triangles might be similar or congruent before you try to prove such facts using "rules." This is exactly the type of imagination you need to work with tangrams, the ability to "see" how you might construct a figure before you try to put the pieces together.

I had also noticed that my geometry students had a difficult time writing down the steps for proofs. They did not see the need for the steps that they had left out—even after these were pointed out to them. I decided to try another tangram activity, hoping that I could get them to see how important writing and following instructions can be. The students were seated across from one another but separated so that they could not see each others' work space. Each student was to design a tangram picture and write a series of (seven) steps that, if followed correctly, would enable the other student to replicate the design. Students were assessed both on how well the steps were written and how well they were followed. Again, I had them keep a journal on the activity. I was pleased with the results: they seemed to realize quite soon that writing and listening were extremely important skills and that terminology was equally important (some of them took quite a bit of time to get "left" and "right" worked out while sitting opposite each other!).

These tangram activities worked exceptionally well for teaching visualization and communication. For doing mathematics, students need many more such activities—in which they must both use their imaginations and think analytically. ❖

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[1]by Lisa Campbell Ernst and Lee Ernst, Harry N. Abrams, NY ISBN 0-8109-3851-0

Algorithms — Before or After?

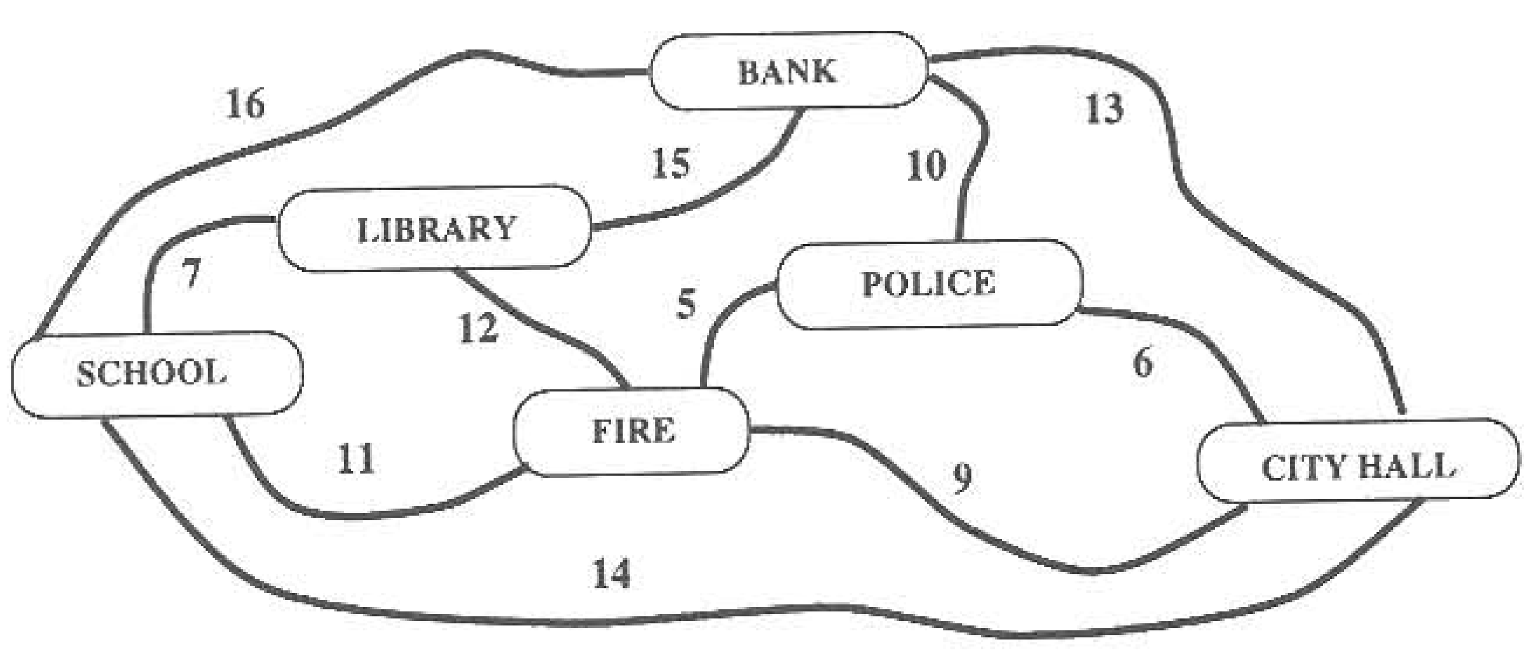
by Carol Price

I am a mathematics teacher at Robert E. Lee High School, an urban and ethnically diverse school in Baton Rouge, Louisiana. I teach Algebra 2 and Advanced Math (both regular and honors versions) to mostly 10 - 12th graders. One day, upon committing the sin of missing school for a math workshop, I left a poor unsuspecting substitute teacher to start a unit on weighted graphs. I prepared the students for my absence by letting them know that they would be starting another "discrete math" unit. That was all that they needed to hear—they loved the first unit I gave them on graph coloring and felt confident about their ability to handle anything labeled "discrete."

I left the substitute armed with numerous worksheets on minimum spanning tree problems. The directions for this first sheet were to connect all of a set of cities (directly or indirectly) to minimize the cost. The students were also told to avoid cycles because they waste money. That was it. They were left to contemplate other problems involving installation of water pipes, electrical wires, telephone wires, roads, etc., with only the help of their partner. The students had never heard the term *minimum spanning tree*, and they were sent off to battle without even a hint of an algorithm for protection.

I am a firm believer in "not giving the students what they don't need." If I tell the students how to solve a problem, not only am I telling them that they can't do it without me, I am taking away their chance to feel mathematically powerful. On the other hand, I was not sure how far the students would be able to go in this case.

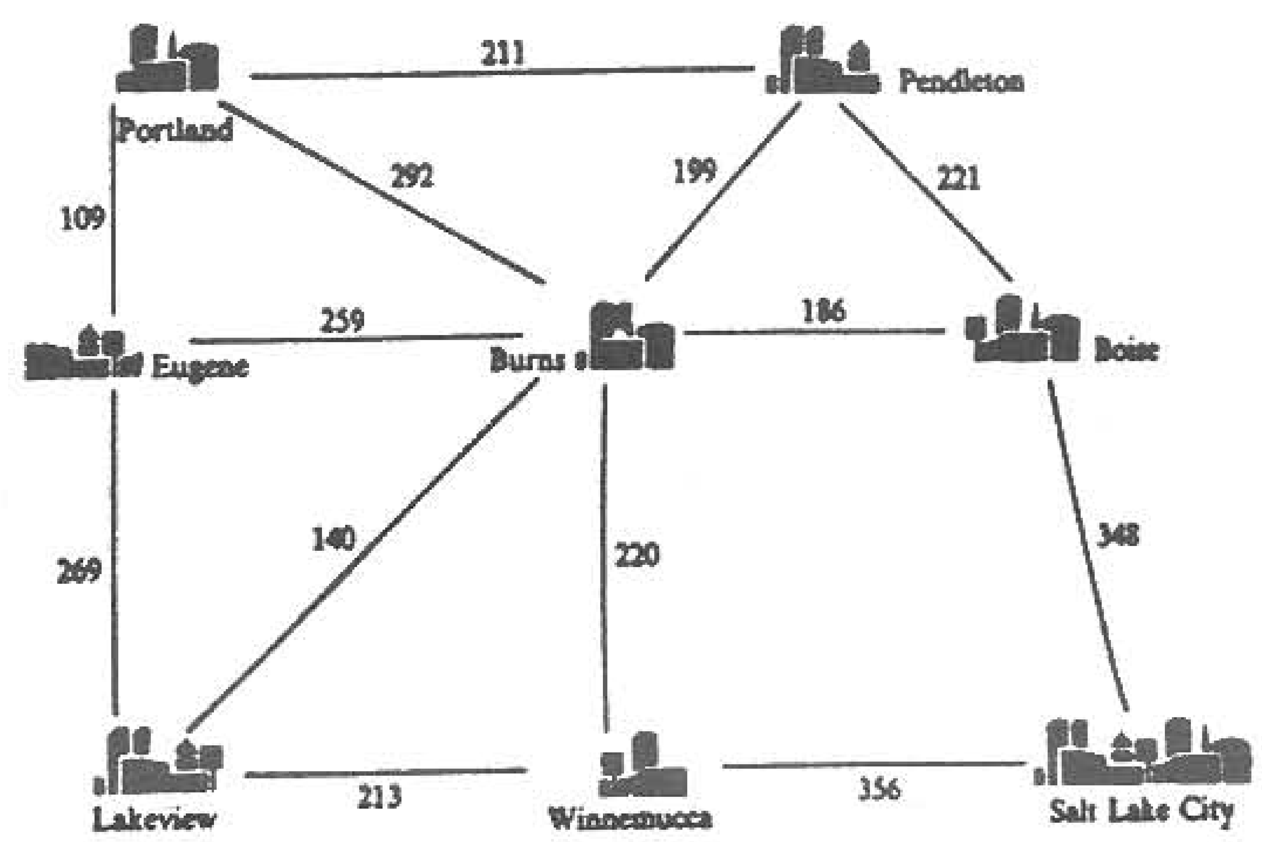
But, I was overwhelmed when I read the substitute's report. The majority of students breezed through worksheet after worksheet, and even helped the ones that were struggling. Almost everyone solved the problems correctly!



A Minimum Spanning-Tree Problem
The Muddy City Council wants to pave roads joining the six major buildings at minimum cost. The cost of paving each of the possible roads is given (in $100K); which roads should be paved? [3]

The next morning, I praised the students for their outstanding behavior and performance, and handed back their papers. It was at that time that I introduced Kruskal's algorithm[1] ([1], pp. 43-47) and Prim's algorithm ([2], pp. 529-530), both of which the students had discovered on their own. (A problem on water pipes led the students right to Prim's algorithm because they realized that the pipes had to stay connected.)

Having mastered the minimum spanning-tree problem, we moved on to the shortest-path problem. Again, I sent them diving into the problem without so much as a hint.



A Shortest-Path Problem
A family wants to drive from Portland to Salt Lake City. Distances are given in miles—what is the shortest route they can take? [4]

Their only directions were to get from *A* to *B* using the shortest path possible. As I walked around the room, I saw a few groups jotting down paths and comparing their lengths, some listing paths with tree diagrams, and a few actually discovering Dijkstra's algorithm ([2], Sec. 13.2). Of course, the latter two approaches generated a shortest path; the first approach generated a response more quickly, but did not always give the shortest path. After having some of the students explain their approaches and answers to the class, and discussing the advantages and disadvantages of their methods, I then introduced Dijkstra's algorithm.

On the third and final day of our weighted graphs unit, we discussed the famous Traveling Salesperson Problem (TSP). For the final time, I sent them diving into the problem without any help. After discussing their various approaches and solutions, we discussed algorithms and computer solutions. They were stunned that there currently isn't any algorithm that can solve this solve this type of problem quickly for a large number of cities. One could hear loud

(Continued on page 9)

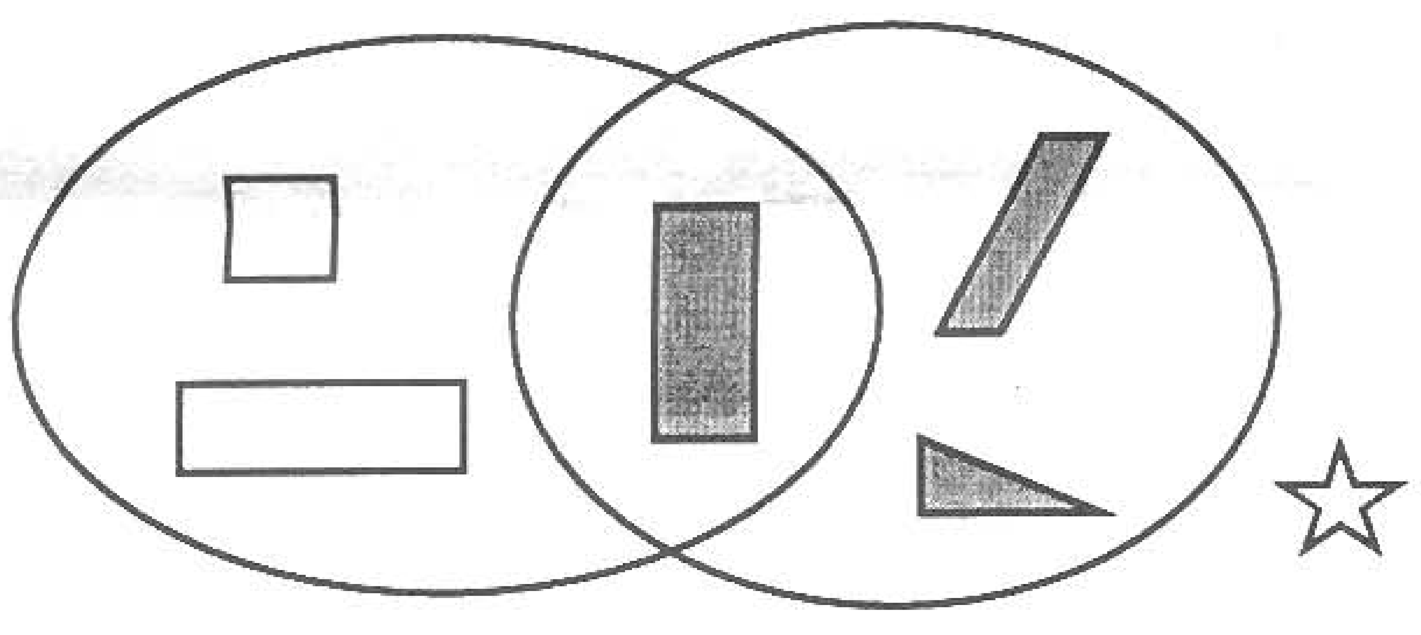
[1] In the spirit of this article, no algorithms are described here! References are given, however.

The Venn Diagram Game

by Linda Dodge

This is a popular activity that I have used to illustrate set concepts and reasoning. I have the class stand around two large circles of rope placed on the floor. I like to use two different colors of rope, let's say yellow and blue. To be sure that everyone is familiar with the idea of a Venn diagram (see figure), I will say something like, "Everyone who is wearing a white shirt please stand in the yellow circle." I allow time for them to sort themselves out, then I say, "Everyone wearing black sneakers should stand in the blue circle . . . Oh look—Josh is wearing black sneakers *and* a white shirt, where should he stand?" Then I make a *big* deal of the "complement," the set of people outside the two circles which "completes" the diagram. (Not only is the complement a useful concept, but I've found that otherwise younger children feel left out if they are not inside the circles.)

After we repeat this exercise a few times with different sets of rules, I make the game more interesting. I sort the



Venn Diagram
The left circle contains rectangles; the right circle contains filled-in shapes. The star (in the complement) is neither filled in nor a rectangle.

people—but I don't tell them the rules I am using. They have to make observations and use inductive reasoning to guess the rules. This part of the dialogue might sound something like this:

> Mary, you stand in the yellow circle but not in the intersection. Raj, you belong in the blue circle. Consuelo, you have both traits—go stand in the intersection. Jed, you are part of the complement. . . .
>
> Does anybody think they know the rule yet? If you do, raise your hand and tell me where you think you belong, but don't say the rule out loud.

After each student tells me where they think they belong, I verify whether they are right or wrong; if they're right, they can go stand in the correct part of the diagram, if not they wait where they are.

(Continued on page 8)

What's So Special About 6174?

by Ronald S. Tiberio

I am always intrigued by good number patterns; here's one that that I've used with my students which requires only basic arithmetic. For reasons that I'll explain below, I'll call this "Kaprekar's game."

1. Choose any four-digit number where not all the digits are the same (not divisible by 1111). (The number can start with 0.)
2. Rearrange the digits from largest to smallest and then from smallest to largest.
3. Subtract the smaller rearrangement from the larger.
4. Take your answer from step 3 and go back to step 2.

For example, if you start with 4997, this is what happens:

9974	7551	9954	5553	9981	8820	8532	7641
-4799	-1557	-4599	-3555	-1899	-0288	-2358	-1467
5175	5994	5355	1998	8082	8532	6174	6174

Now ask your students to try to figure out the maximum number of steps you can play the game before repeating a number. The amazing answer is that if you play Kaprekar's game on *any* four-digit number (other than multiples of 1111) you will reach 6174 in at most seven steps! (A sketch of a proof using only elementary algebra appears at the end of this article, on page 9.)

I first saw this property of 6174 as an elementary problem in the *American Mathematical Monthly* [1]. The original source was a short note in an obscure journal by D.R. Kaprekar of Devlali, India [2]; since that time 6174 has been referred to as *Kaprekar's constant*.

Of course there's no need to stop with four digits. What happens if you perform this operation with a two-, three- or five-digit number? Is there always a fixed point? Can there be more than one fixed point (not counting 0, which is always a fixed point)? Such questions are the basis for the concepts of *iteration* and *dynamical systems*, and can be introduced to students with very little background. ❖

References

[1] Elementary problem E2222, *American Mathematical Monthly*, March 1970, p. 307.

[2] D. R. Kaprekar, "Another Solitaire Game", *Scripta Mathematica*, September 1949, p. 244-5.

Editor's notes

(1) This article first appeared as a posting to the Leadership Program electronic mailing list, where teachers share information and discuss issues related to teaching

(Continued on page 9)

Credits...

This Newsletter is a project of the *Leadership Program in Discrete Mathematics* (LP). The LP is funded by the NSF and is co-sponsored by the Rutgers University Center for Mathematics, Science and Computer Education (CMSCE) and the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS).

Joseph G. Rosenstein is Director of the LP and Founding Editor of this Newsletter. **Valerie DeBellis** is Associate Director of the LP.

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Editors...

The Newsletter is edited by **Deborah S. Franzblau**, Research Assistant Professor and Education Coordinator at DIMACS. She has taught at Vassar College and in the *Leadership Program*. The editor of the column, "The Discrete Reviewer", is **Janice Kowalczyk** (LP '92). She is an Educational Mathematics Specialist at Rutgers and on the staff of the LP. She is Director of the Rhode Island School of the Future project, and teaches part-time at Lesley College (MA).

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Carol Price (LP `94) teaches at Robert E. Lee High School in Louisiana. She taught discrete mathematics workshops during Summer, 1995.

Eric Simonian (LP `94) teaches at Toll Gate High School in Rhode Island. He was a lead teacher for the LP commuter program in Summer, 1995, and is one of the organizers for the commuter institute in Rhode Island for summer 1996.

Ronald S. (Chuck) Tiberio (LP `92) teaches at Wellesley High School in Massachusetts.

Coming Soon...

We are starting a new column on Internet and World-Wide Web resources, edited by **Judy Brown**, our long-time Web consultant. Please send your suggestions to her at judyann@dimacs.rutgers.edu.

Submissions...

In Discrete Mathematics welcomes contributions from its readers, especially short articles about their experiences while using discrete mathematics in the classroom.

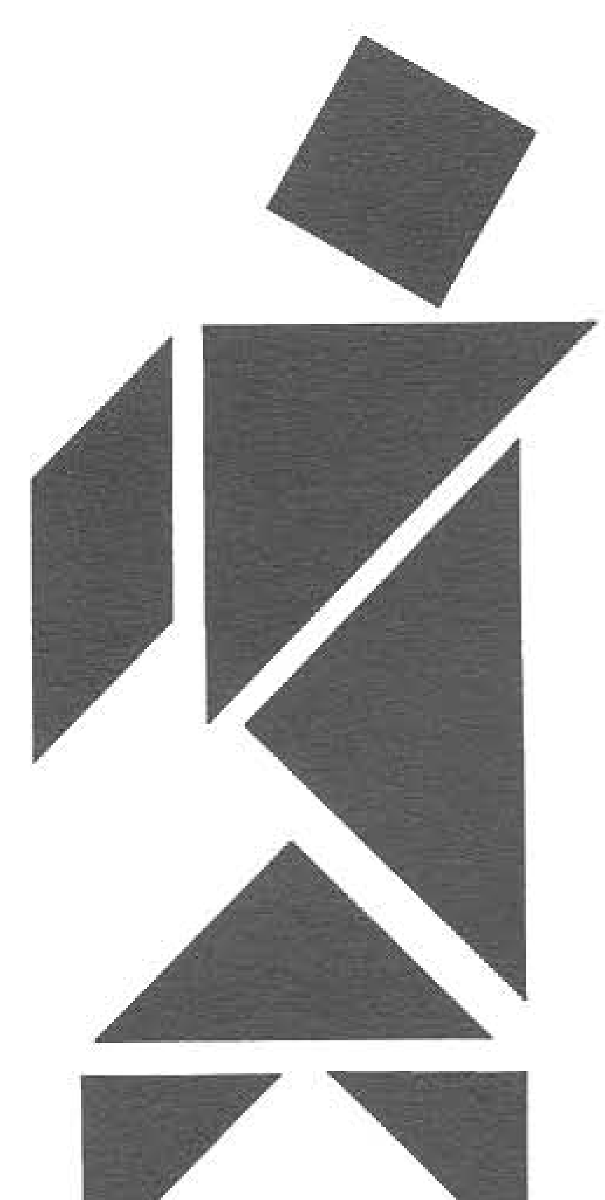
Send descriptions and reviews of resources (books, articles, software, videos, etc.) to Janice Kowalczyk, jkowalcz@k12.brown.edu. All other contributions should be sent to the Editor, Deborah Franzblau, by email at franzbla@dimacs.rutgers.edu. Or write us at the address on this page.

Solutions...

★ The top-rated math education software in CLIME's survey is the **Geometer's Sketchpad** (see p. 7).

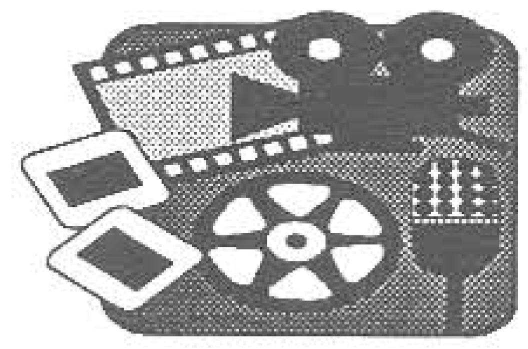
★**Venn Diagram puzzle** (p. 12)
Our answer: The lower circle contains creatures with hats; the upper circle contains creatures that live in the sea.

★**Tangram puzzle** (p. 12)
This design, entitled "Tango," was created by David C. Chase, a student at Tollgate High School (see article on p. 3). The solution is given below. The design on page 3, called the "Talented Seal," was also designed by a student in the class.



The Discrete Reviewer

by Janice Kowalczyk



The Discrete Reviewer provides recommendations and suggestions from teachers for discrete mathematics resources for the classroom. The material in this issue comes mostly from the recent summer institutes of the *Leadership Program in Discrete Mathematics*, which were geared towards K-8 teachers for the first time. We would greatly appreciate your comments and recommendations for this column. Your feedback on any resources that you try as a result of this column is also very useful. In a future column we plan to focus on discrete mathematics in literature: please send your suggestions and ideas.

ACTIVITY BOOKS

(K-8) **INsides, OUTsides, LOOPS AND LINES**, by Herbert Kohl,
W.H. Freeman, 41 Madison Ave, New York, NY, 10010
Cost: $12.95
ISBN: 0-7167-6586-1

If I had to recommend one resource book to elementary and middle grade teachers in discrete mathematics this would be it. Discovered in the children's section of the Border's book store this past summer by Leadership Program participants, this book provides playful introductory activities in a number of areas of discrete mathematics. The five chapter titles give you the best flavor for its contents: *Lost in the Garden* — simple closed curves; *Map Coloring* — figuring out the rules; *Tracings* — simple beginnings lead to complicated patterns; *Stretching, Bending and Twisting* — a new way to look at shapes; and *Mobius Strips* — some thoughts on doing mathematics, with a twist.

(4-8) **Let's Investigate Series**, by Marion Smoothey,
Marshal Cavendish Corp., 2415 Jerusalem Ave.,
P.O. Box 587, North Bellmore, NY, 11710
Cost: $16.95 each
Let's Investigate Number Patterns: ISBN: 1-85435-458-2
. . . *Codes and Sequences*: ISBN: 1-85435-774-3
. . . *Shape Patterns*: ISBN: 1-85435-465-5

"This three-book series is an excellent reference set to have in your classroom or school library. A wide variety of math topics are covered, and there are enough activities to pique everyone's interest."
—Judy Ann Brown, Pleasant Valley Middle School, Brodheadsville, PA

READINGS ON MODERN MATHEMATICS

Nature's Numbers, by Ian Stewart
Basic Books, 10 East 53rd St., New York, NY 10022-5299
Cost: $20.00
ISBN: 0-465-07273-9

A 1995 publication, this book is intended for a broad audience and is wonderful reading for anyone interested in the connections between mathematics and nature. "*Nature's Numbers* will equip you with a mathematician's eyes. It will take you sightseeing in a mathematical universe. And it will change the way you view your own world." (From the preface.)

Five Golden Rules: Great Theories of 20th-Century Mathematics — and Why They Matter, by John L. Casti,
Wiley & Sons, 605 Third Ave., New York, NY 10158-0012
Cost: $24.95
ISBN: 0-471-00261-5

Five Golden Rules is the engaging story of five great mathematical breakthroughs of this century and how they are shaping our lives and our thinking. Discrete mathematics is a theme throughout the book. The author's ideas are illustrated with a resplendent array of real world problems that have been solved by these mathematical discoveries. Fascinating reading.

NEWSLETTER ON CLASSROOM TECHNOLOGY

CLIME Connections, Ihor Charischak,
Stevens Institute of Technology - CIESE Center,
Hoboken, NJ 07030
`icharisc@stevens-tech.edu`
Hardcopy: $2.00/Issue, or $10/Yr.
On-Line version: Free.

CLIME stands for the *Council for Logo & Technology in Mathematics Education*. It is affiliated with the NCTM; its purpose is to help teachers use technology effectively in their classrooms. I have been a member for a number of years, and have worked in the past with the founder, Ihor Charischak. Over the years I have used many of their materials. The latest issue of the newsletter has a list of the top ten mathematics software products, based on a survey of 40 teachers in a NSF-funded mentorship project at Stevens. Can you guess what their top-rated software is? (See page 6.)

News *(Continued from page 1)*

Time-tested trategies for getting news coverage

1. For "minor" classroom activities which have interesting photo opportunities, take pictures yourself. Most newspapers these days accept color photos, provided that they are clear and bright. Submit the photos along with a short article in which you clearly explain the activity and identify people in the photo.[1]

2. For interesting "major" events or activities that will present many photo opportunities, invite a reporter to your classroom. Do this by mailing or faxing a press release with a brief description of the event about a week ahead of time. Follow it up with a phone call a few days in advance.

3. If something "big" is happening at your school, like a day devoted to mathematics, with assemblies or panel discussions, invite the electronic media to cover it along with the print media. Network affiliates as well as local public television stations may be interested. And don't forget cable networks in your area—broadcasting educational events fulfills some of their community programming requirements. Mail or fax a printed summary of the event at least two weeks in advance. Follow up your written release with a phone call.

faxes or electronic mail) to these individuals.

A good article or press release is constructed around the who, what, when, where, and why. Avoid judgmental words and editorial phrases like "an outstanding project." Follow the "Dragnet" advice: "the facts, ma'am, just the facts." Try starting your story with an interesting fact or question like "How many combinations of pizza can your local pizzeria really deliver?" Then, follow it up with the facts your students discovered. Be accurate, write in clear and grammatically correct language, and (above all) be concise. It's a good idea to have a colleague read and edit your story before you send it out. Don't despair if an article is not printed; it may be because of space limitations, or other factors beyond your control. Keep trying.

Remember also, it's not just classroom activities and our students' work that are newsworthy. Publicizing our own accomplishments is extremely important in these days of tight funding for education. It is vital that the taxpayers know that teaching is not just a ten-month, eight-to-three occupation. Newspapers frequently devote a column to professional development and advancement. Send in a photo of yourself with a brief press release any time you participate in a conference—for example by introducing the speakers in a session, or making a presentation. If you complete a degree, are elected to an office in your regional CTM, or receive an award or other recognition, let your community know: parents want to know that their children's teachers are recognized as leaders in their profession. Make sure that you clear all news releases with the appropriate school personnel, such as your building administrator or principal. (In fact, it is good public relations to invite administrators to such events; they like to get into the act, too.) If your school does not have a standard news release form, you should create one; be sure to check it with your administration before you send it out. And *do send it out*! You and your students will be on your way to fame—if not fortune—and the whole mathematics community will benefit from your efforts. ❖

Acknowledgment. I thank Joe Rosenstein for suggesting that I write this article, as a follow-up to a public relations panel discussion in Summer, 1994.

References

[1] "Kennett Mathematicians Solve a Mystery", *Community Courier*, 2-15-95.

[2] Neil Goldstein, "The Case of the Stolen Diamonds." Performed regularly in the *Leadership Program in Discrete Mathematics* —the plot involves Euler paths.

[3] "Applying Their Skills", *Wilmington News Jourrnal*, "Crossroads" Sec., 5-4-94, p. 1.

[1]Include first and last names. Add titles and job descriptions if applicable, or grade levels for students.

Venn Diagram Game *(Continued from page 5)*

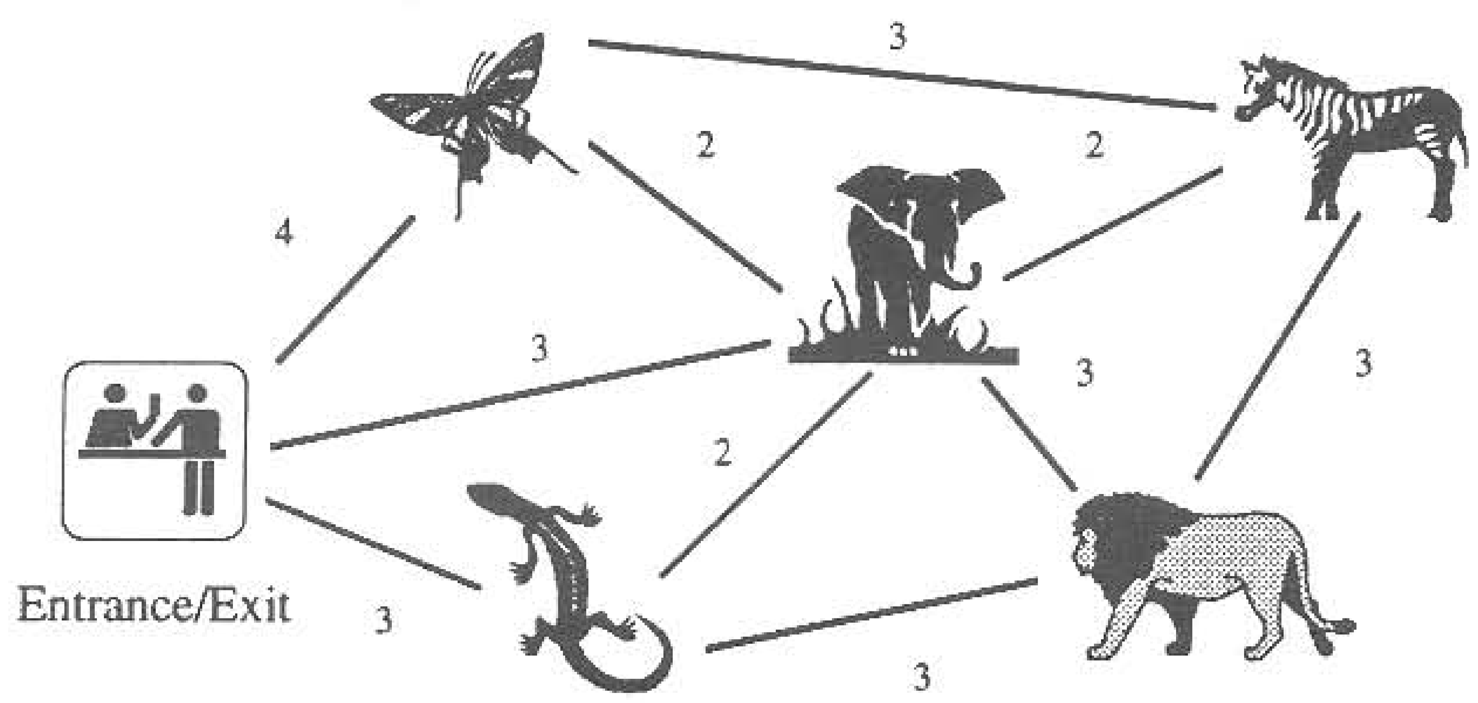
Once all the students seem to be guessing correctly where they belong, I ask them to tell me what they think the rule is. Some students will give incorrect rules; I help them see why the rule is incorrect by asking them whether certain students in the diagram are in the right place, according to the (wrong) rule. On the other hand, sometimes a student will come up with a rule that works for everyone in the diagram, even though it is not the rule I had in mind; I'll point out that their rule is also correct—based on the evidence so far. After I have done the "rule-making" a couple times, I ask pairs of students to take over running the game. (I advise them not to make the rule too complicated, and usually have them tell me the rule in advance.)

I have used this activity very successfully with adults in teacher inservice training as well as with 4th-graders. My 5/6th-grade math club used to beg me to play the game! I have found it an excellent way to model the process of inductive reasoning, i.e., learning from examples. When you only have a few examples to work with, it's hard to guess the rule correctly— you have to keep trying. The more examples you have, the fewer the possibilities, and the easier it becomes to find the rule. ❖

Algorithms *(Continued from page 4)*

cries of "No way!" when the students saw on the overhead that it would take a computer over 19 million years to solve a TSP with 25 cities by exhaustive search. This led to a discussion of saving time and money by using algorithms that could generate a close approximation of the optimal answer. We discussed how researchers at Bell Laboratories have studied this type of problem for years and have come up with various algorithms that are up to 98% accurate and very fast. (See [1], pp. 38-43.) The unit ended with a take-home quiz.

Overall the unit was a success. To improve it, I would give the students another day on TSP problems and a fifth day for making up their own problems.



A Traveling Salesperson Problem
Judy and Jim want to see all the animals at the zoo.
Each path is labeled with the number of minutes it takes to walk it.
What's the shortest circuit they can take? [5]

As I had hoped, the students clearly did not "need" the algorithms to solve the problems—they came up with their own! Isn't that what math is all about?! What they *do* need is the experience of discovering their own methods for solving problems, to prove to themselves that they really are mathematicians, and don't need to be told what to do. They need to be exposed eventually to the "classic" algorithms, because they are useful and are part of the vocabulary of mathematics, but they don't need them right away.

I am still hoping to get the students' enthusiasm and confidence to permeate into what they perceive as "real math." And I am still trying to get them to see that the discrete units *are* real mathematics—wish me luck! ❖

References

[1] COMAP, *For All Practical Purposes*, 3rd ed., W.H. Freeman, NY, 1994.

[2] Roberts, Fred, *Applied Combinatorics*, Prentice Hall, Englewood Cliffs, NJ, 1984.

[3] Problem by Joe Rosenstein, from a story by Mike Fellows.

[4] From Cozzens, Margaret B. and Richard Porter, *Problem Solving Using Graphs*, HiMAP Module #6, COMAP, Arlington, MA, 1987.

[5] Based on a problem by Alistair Carr and Susan Picker.

What's so Special . . .? *(Continued from page 5)*

discrete mathematics topics.

(2) In the October 1994 *Mathematics Teacher* (p. 488) is a letter entitled "birthday converse", from John Koker. You might enjoy figuring out how his letter is related to this problem.

Proving that Kaprekar's Game Works

Here is a sketch of a proof that playing Kaprekar's game (see p. 5) on any four-digit number that is not a multiple of 1111 will yield 6174 in at most seven steps.

To warm up, start by playing Kaprekar's game with two-digit numbers. For example: 61 − 16 = 45, 54 − 45 = 09, 90 − 09 = 81, 81 − 18 = 63, 63 − 36 = 27, 72 − 27 = 45. This time, instead of being stuck at one number, we're stuck in a loop (45 → 09 → 81 → 63 → 27 → 45).

In the example, you also notice that once you get a difference that is a multiple of 9, after at most one more step you are either stuck at 0 or in this loop. (One step is needed to get into the loop from 18, 36, 54, 72, or 90.) However, after one step of Kaprekar's game, you will *always* get a multiple of 9; the reason is that, given a two-digit number with digits **a** and **b**,

$$10\mathbf{a} + \mathbf{b} - (10\mathbf{b} + \mathbf{a}) = (10 - 1)\mathbf{a} - (10 - 1)\mathbf{b} = 9(\mathbf{a} - \mathbf{b}).$$

Since $\mathbf{a} \geq \mathbf{b}$, $\mathbf{a} - \mathbf{b}$ is a whole number between 0 and 9. Thus, for any two-digit number, after at most two steps of Kaprekar's game, if you're not at the fixed point 0 (which occurs only for multiples of 11), then you are stuck in the "9-loop" of length five.

Now let's look at four-digit numbers, using "**abcd**" as a shorthand for $1000\mathbf{a} + 100\mathbf{b} + 10\mathbf{c} + \mathbf{d}$. This time you find that (after doing a bit of algebra),

$$\mathbf{abcd} - \mathbf{dcba} = 999(\mathbf{a} - \mathbf{d}) + 90(\mathbf{b} - \mathbf{c}).$$

For example,

$$7641 - 1467 = (999 * 6) + (90 * 2) = 6174.$$

There are at most ten possibilites for $\mathbf{a} - \mathbf{d}$ and $\mathbf{b} - \mathbf{c}$, so you need to check at most 10 * 10 = 100 numbers to check whether you always reach 6174 in at most 7 steps. If you start to fill in a table of these 100 numbers, you will find that there is a lot of symmetry, and that you really only have to check 30 numbers.

Unfortunately, this "brute-force" approach doesn't really explain "why" 6174 is the only fixed point; better anwers are welcome. Also, I'll let you find out for yourself what happens with three-digit numbers. I don't know what happens for numbers with five or more digits; if you or your class find any results on these, please write to us. —Editor

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How does the shape of an airplane's wings affect its flight?
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Participants: Mathematics/CS teachers or supervisors of grades 9-12, and researchers at all levels.

Activities: Teachers will participate in two workshops:
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There will also be seminars for researchers, and an all-institute program.

Support: Lodging and meals (weekdays, on campus), a travel allowance, and a stipend ($900) will be provided for the three weeks. *Funding is provided by the National Science Foundation* (NSF).

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(Please call if this notice reaches you too late to meet the deadline.)
Preference will be given to applicants already using the Internet.

To Receive an Application: Please contact Elaine Foley, *e-mail*: drei@dimacs.rutgers.edu
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DIMACS Symposium on

TEACHING LOGIC AND REASONING IN AN ILLOGICAL WORLD

Rutgers University — 25-26 July 1996

Wanted: Presentations or Demonstration High School Lessons On Innovative Approaches to Teaching Logic

Deadline for Abstracts: April 1, 1996
(Please call if the deadline has passed—we may still be accepting abstracts)

This symposium will explore the teaching of introductory logic and logical thinking, with a primary focus on the college level and a secondary focus on the high school level. The symposium will be interdisciplinary, emphasizing and contrasting approaches used in mathematics, computer science, natural sciences, and engineering. The symposium seeks a sharing of ideas, rather than consensus, on how to teach logic. Our aim is that participants are better able to motivate the importance of logic and to convey the foundations of logical reasoning to students.

For further information, contact Peter Henderson, Department of Computer Science, SUNY Stony Brook, Stony Brook, NY 11794-4400 USA. (516) 632-8463, pbh@cs.sunysb.edu

Organizers: Susanna Epp (DePaul University), David Gries (Cornell University), Peter Henderson (SUNY Stony Brook), Ann Yasuhara (Rutgers University).

Sponsored by the DIMACS Special Year on Logic and Algorithms and the Association for Symbolic Logic

LEADERSHIP PROGRAM IN DISCRETE MATHEMATICS

SUMMER INSTITUTES FOR K-8 TEACHERS . . . IN DISCRETE MATHEMATICS

RUTGERS UNIVERSITY

WHAT? Two-week residential institutes at Rutgers University and two-week commuter institutes at Rutgers (New Jersey), in Rhode Island, and Virginia.

WHO? For teachers of K-8 students, as well as mathematics supervisors or specialists.

WHEN? Residential workshops will run July 8 - 19. Commuter institutes will run June 24 - July 9 (Rutgers), and August 5 - 16 (RI and VA).

STIPENDS Funding from the National Science Foundation provides a $600 stipend for the two weeks; meals and weeknight lodging (double occupancy) are provided for residential institutes.

PARTICIPANTS . . . are expected to
- *introduce discrete mathematics into their classrooms
- *develop classroom materials for other teachers
- *present workshops on institute topics

APPLICATIONS . . . are due by April 1, 1996 (please call if you cannot meet the deadline).

QUESTIONS? For further information, or to receive an application, call Stephanie Micale at 908/445-4065, send email to micale@dimacs.rutgers.edu, or write to the address below.

DISCRETE MATHEMATICS . . .WORKSHOPS IN YOUR DISTRICT

WHAT? Full-day workshops can be scheduled in your district, for teachers of all grades, on topics in discrete mathematics which can be introduced into K-12 classrooms and curricula.

WHEN? Workshops will be scheduled during the school year (or during the summer) on an individual basis at the request of the participating district.

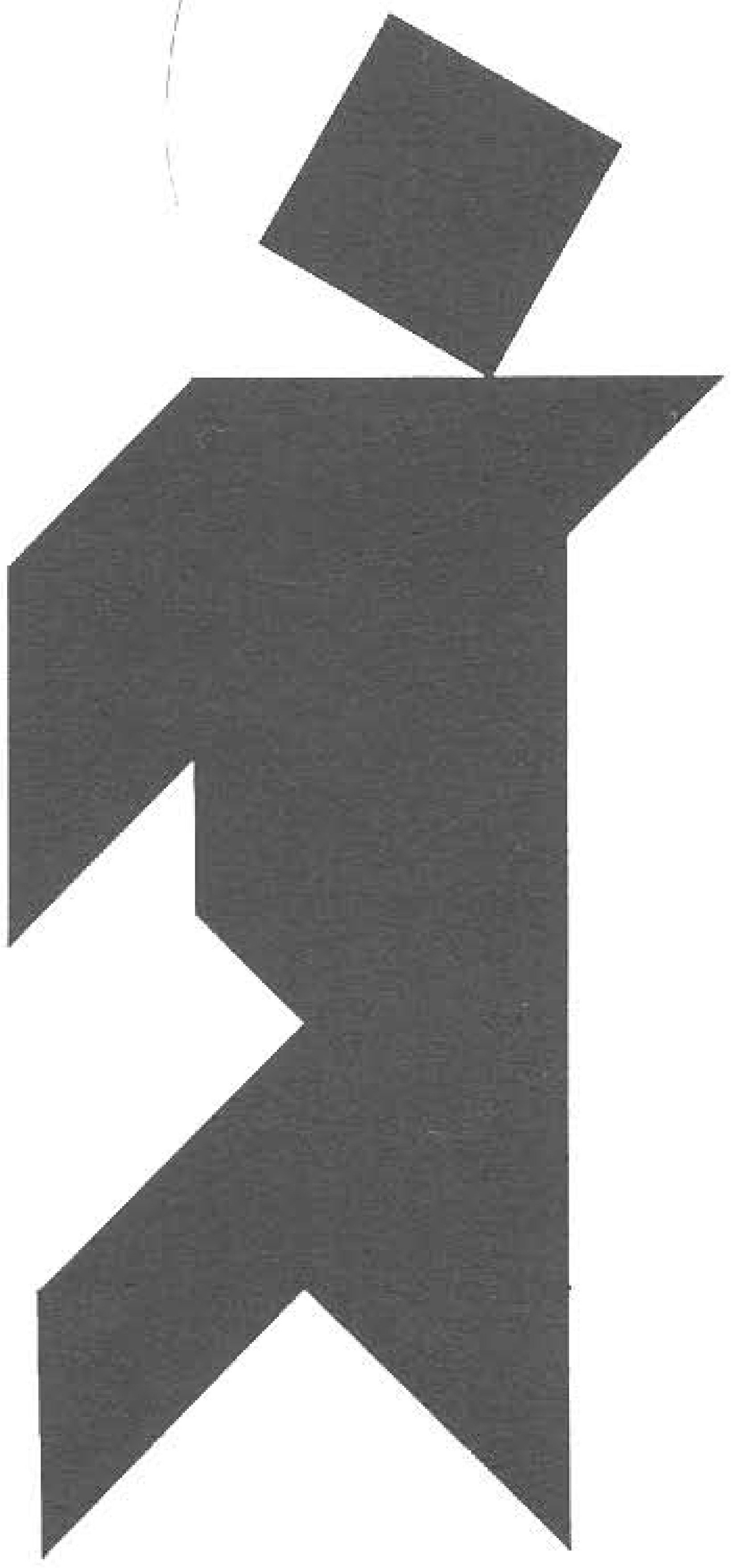
BY WHOM? Experienced teachers from the Leadership Program in Discrete Mathematics, who have participated in a special training program, prepare and present the workshops.

COST The district will be expected to pay only direct costs, that is, expenses and honoraria for the workshop leaders and instructional materials for the participants.

QUESTIONS? For further information, call Lisa Estler at 908/445-4065, send email to estler@dimacs.rutgers.edu, or write to the address below.

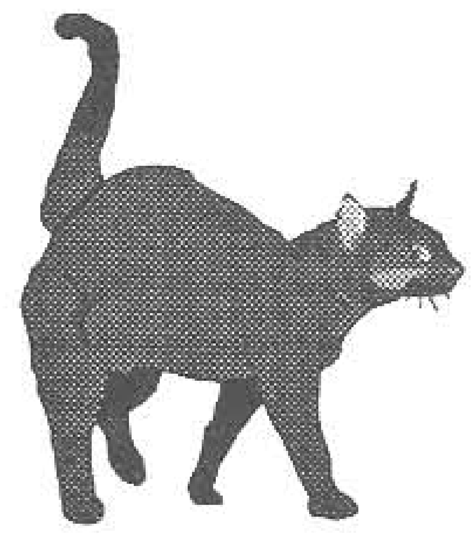
ADDRESS

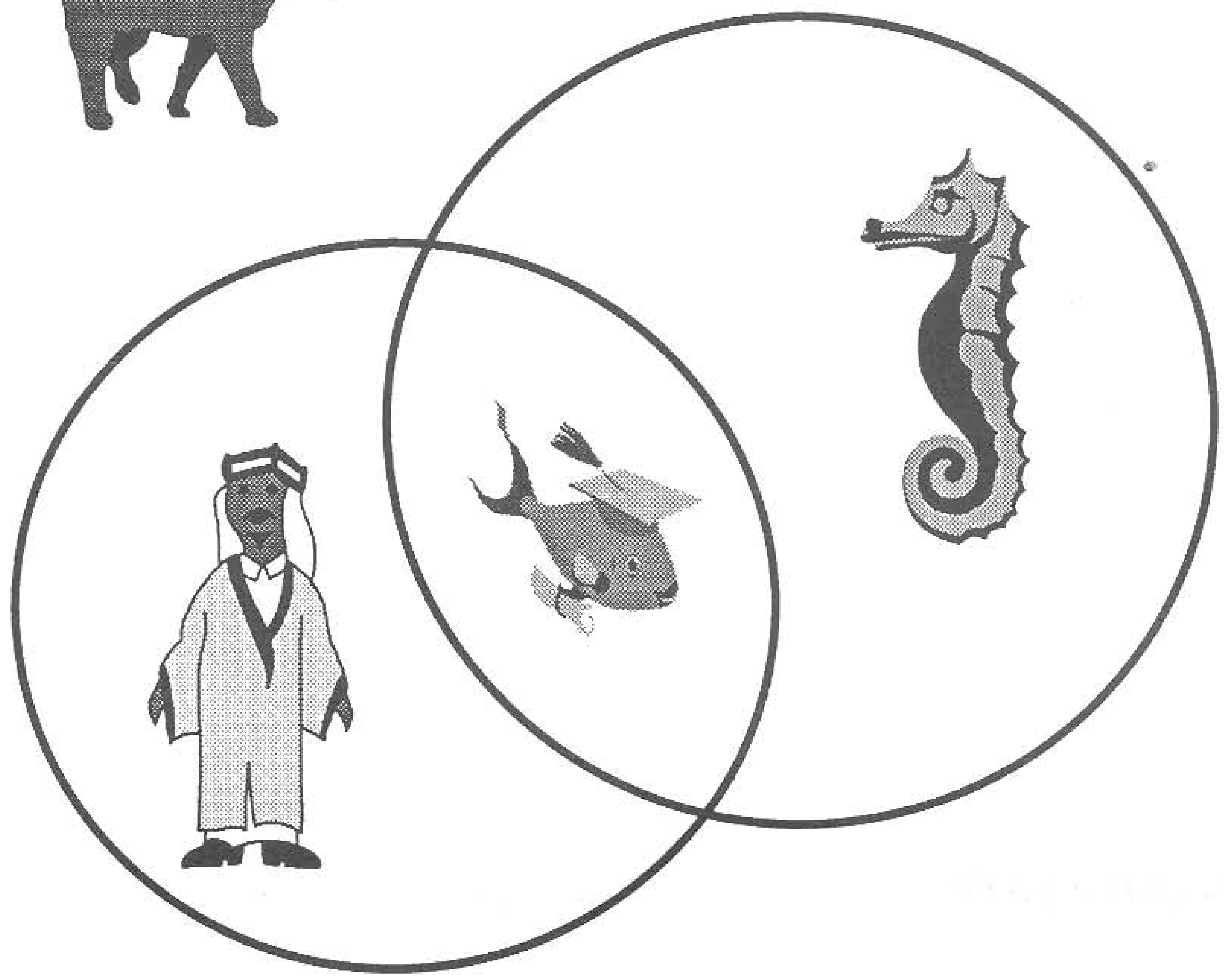
Leadership Program in Discrete Mathematics
P. O. Box 10867
New Brunswick, New Jersey 08906



Tangram Puzzle
(Article on p. 3, solution on p. 6.)

Can you guess a rule for this Venn Diagram?
(Article on p. 5; solution on p. 6)





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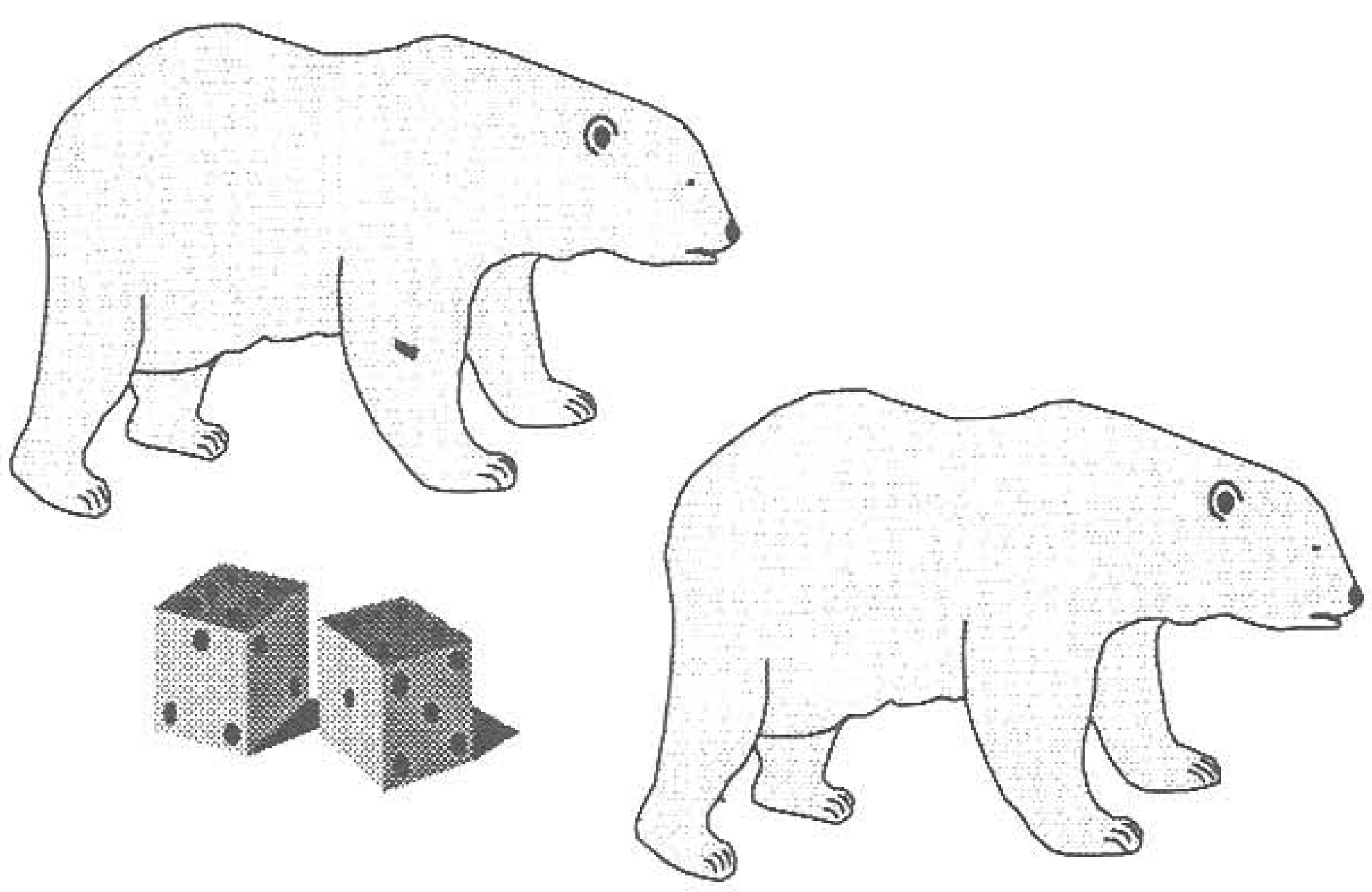

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