IN DISCRETE MATHEMATICS

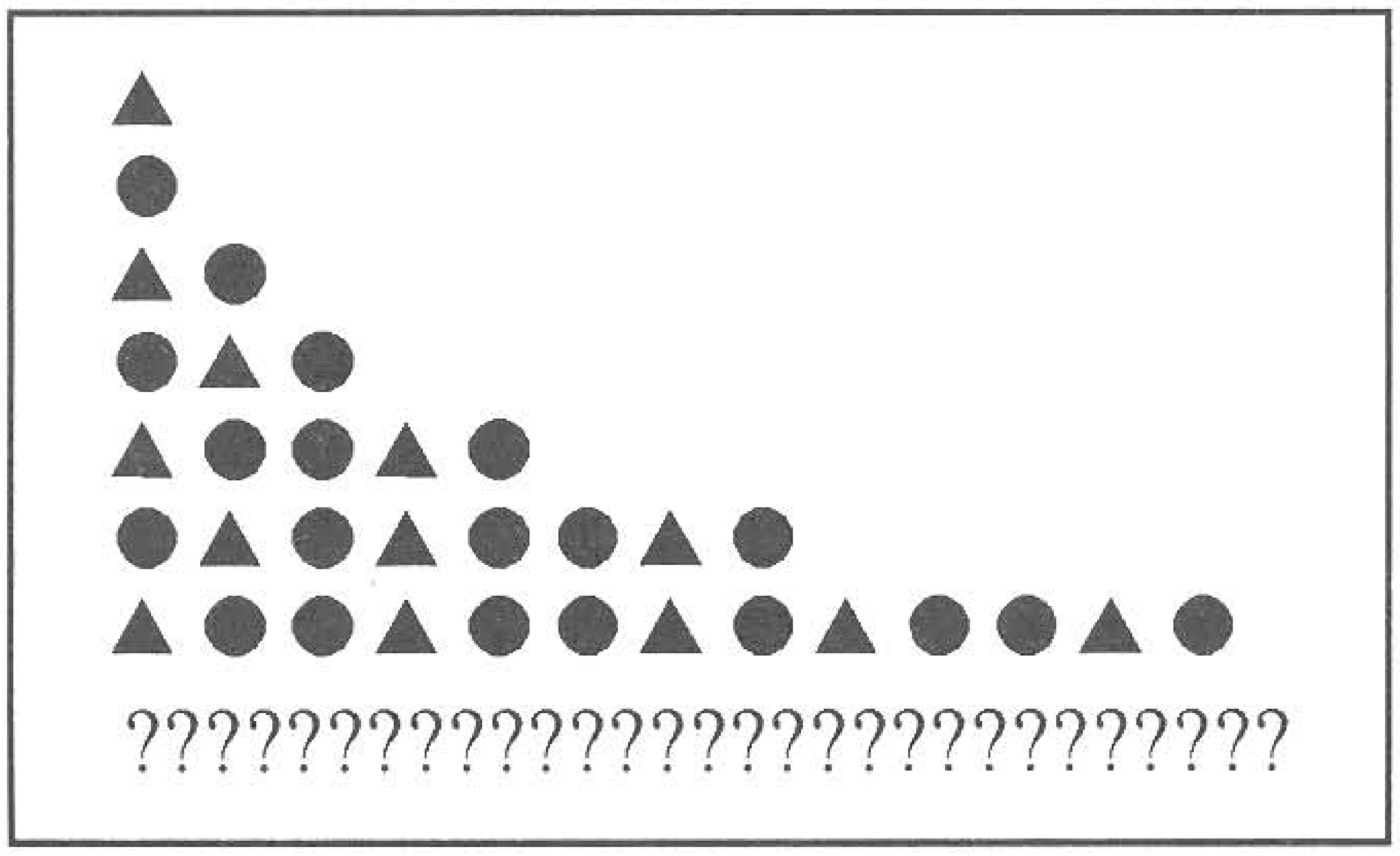
Using Discrete Mathematics in the Classroom

Issue #8 *Fall/Winter 1996*

Speaking Discretely...

Robert Hochberg

One of the themes of this issue is "voting." The lead article by Joe Malkevitch and T.C. Wu discusses ranking the students in a classroom and reveals some surprising consequences of dropping the lowest grade. Paul Dreyer's article on page 9 outlines an interesting method for polling a group of students without any student having to reveal his or her actual opinions. Thus a pollster would be able to poll people on sensitive issues without individuals needing to reveal personal information. Another theme of this issue is geometric patterns. On page 5, Kerry Simmons relates her success with tessellations in the kindergarten, and on page 10, Suzanne Foley and Deborah Franzblau give hints for using quilts as a vehicle for introducing discrete math into the classroom.



Judy Ann Brown begins a new regular feature on Internet resources, and Janice Kowalczyk reviews books which can be used in conjunction with teaching DM. We have also included humorous (and true) classroom anecdotes as space provided. Please send yours for future issues!

Please share the announcements on pages 4, 6, 11 and 12 with your colleagues. There are excellent programs to help teachers in your district become more acquainted with DM, especially the "Workshops in your District" program.

In This Issue

Is Dropping the Lowest Grade Fair?

Joseph Malkevitch and T.C. Wu

It is common for students, say, high school students in what follows, to request instructors to determine their final grades by dropping the lowest test grade that each student got. Presumably this suggestion is based on the idea—similar to the notion of a crime without victims—that no one is "hurt" by doing this. The purpose of this note is to examine the procedure "drop the lowest grade" from a mathematical modeling point of view so as to determine whether or not it is indeed true that no one is hurt by this practice, and to consider the pros and cons of this procedure.

Historically, mathematics has been taught from the point of view of developing different mathematical techniques, often without relation to contexts. This approach sometimes results in students' not thinking as much as they might about what they are learning. As will be seen from the discussion below, various mathematical ideas emerge out of the situation we have posed concerning dropping a student's lowest grade.

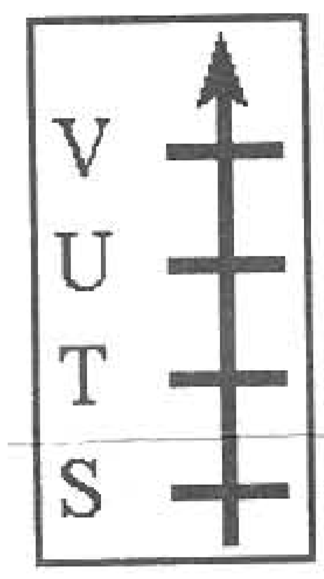
In examining the question of whether or not to drop a student's lowest grade, it is reasonable to ask about what the context involved. If a teacher uses the system of converting numerical averages to letter grades using fixed ranges for each letter (i.e. 60–69 = D, 70–79 = C, etc.), then the procedure of dropping a student's lowest grade will result in all students getting at least as high a grade as they would have otherwise. (Have your students show this is true.) The students may think that this is nice, but it does result in grade inflation for the school as a whole if all the instructors adopt this policy. The consequence of this may be that colleges who admit students from this high school are less likely to think highly of its applicants. Rather than no one at the high school being hurt, all students who apply to college may be hurt. It is "system" thinking of this kind that mathematical modeling exercises can encourage. Alternatively, if a teacher curves his grades, the grade a student gets is arrived at by assigning A's to a certain percentage of students, B's to a certain percentage, etc. (One way to grade on the curve might be to assign 10% A's, 20% B's, 40% C's, 20% D's and 10% F's.) Teachers who grade on the curve must arrange their students in a rank ordering from highest to lowest so that they can decide which students get which letter grades. Another reason a ranking may be necessary is that the examination grades may serve as a way to rank students for receiving a departmental prize or scholarship.

It may be helpful at this point to introduce a specific example (Figure 1) to focus our thinking. Here we have students S, T, U and V who have taken three examinations.

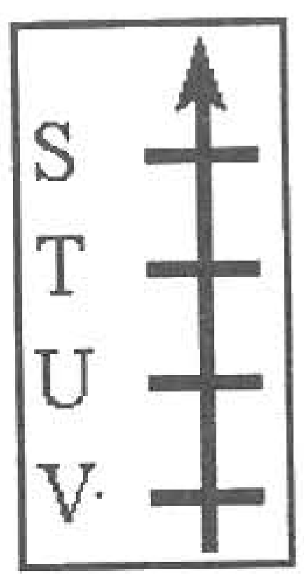
	S	T	U	V
Test 1	96	55	70	77
Test 2	80	90	83	82
Test 3	11	84	81	80
Total (all tests)	187	229	234	239
Total (lowest grade dropped)	176	174	164	162

Figure 1

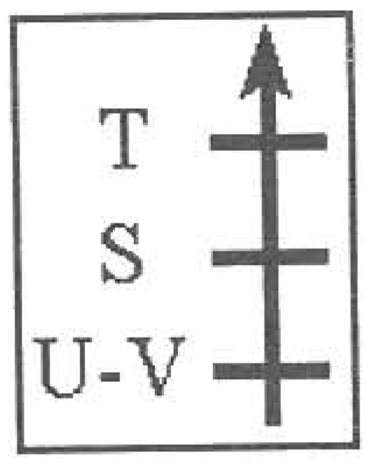
How can we arrive at a ranking of the students? One method of ranking the students is to consider their averages on the three tests. The higher the student's average, the higher rank the student gets. Note that instead of using the highest average, we can instead merely look at the sum of the grades on the three tests. The ranking obtained based on this sum will be exactly the same as the ranking obtained by using the average but will save a fair amount of computation. (This is a nice fact for students to prove.) Based on the data above, the test score sums are: 239, 234, 229, and 187 for V, U, T and S, respectively. This would result in the ranking shown to the right.



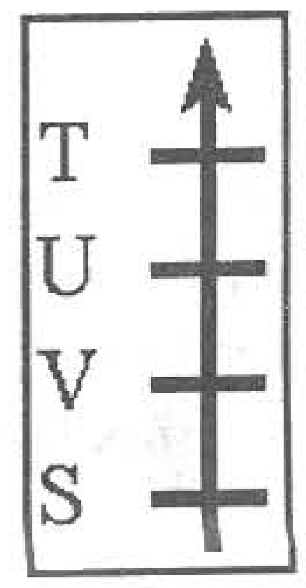
If only the highest ranked student gets the scholarship, the scholarship would go to V. What happens if we drop the lowest grade and rank the students in terms of the sum that they now obtain? The new data is given in Figure 1 and the ranking which results is shown to the right.



Perhaps, unintuitively, not only does V not win the scholarship, but the ranking of the students is now completely reversed! In this ranking S would win the scholarship. (In the context of grading on a curve (with 25% of the students getting A, B, C and D grades), this example shows that dropping the lowest grade might result in a student's getting a D grade instead of an A grade!) These are not the only methods that could be used for giving out the scholarship. For example, we could count the number of exams on which each student got the highest grade and rank the students accordingly. If this is done, S came in highest on one test, T on two, and U and V were never highest on any tests. The ranking which results is shown to the right. Notice how we incorporate the fact that U and V are at the same level. This yields a ranking in which T comes in the highest.



This is reminiscent of situations where individuals rank alternatives and the individual rankings must be combined into a group ranking. On important example is the problem of deciding the unique winner or ranking the candidates who participated in an election. There are a large number of methods (see [1]) that one can use, and the disconcerting fact is that different but seemingly reasonable methods result in different winners (rankings). One popular method for elections is the Borda Count, where alternatives are given credit for how high up on a preference schedule they appear. Thus, 4 points are assigned for a first place, 3 points for a second place, etc. This idea can be adapted to the current context. For each examination, one can see what place the student came in on that examination. Four points are assigned for a first place, 3 points for a second, etc. For example, since S's grades of 96, 80, and 11 were the highest, lowest, and lowest on the examinations, respectively, S would get 4 + 1 + 1 = 6 points. Similarly, T would get 9 points, U would get 8 points, and V would get 7 points. The ranking obtained by this method is shown to the right. How would you change this to allow for dropping the lowest grade?



In this example we have used 4 different methods to produce a ranking and have found 4 different rankings, though the number of different individual winners is only 3. (U is not a winner using any of these methods.) You may wish to have your students construct an example similar to the one produced here where all 4 methods yield different winners. You may also wish to see what other methods your students might develop to rank the students.

We have seen that the issue of whether or not it is fair for an instructor to drop every student's lowest grade raises some interesting questions. When a mathematical algorithm is employed, one expects the algorithm to output a unique answer. Thus, $3 \times 5 = 15$, $2x - 3 = 11$ has only 7 as its solution, and $1/2 + 2/3 = 7/6$. Emphasis only on mathematical algorithms gives students the impression that mathematics is a dry, relatively static subject. By adopting a modeling environment, it becomes apparent that mathematics is a more complex subject than students might have otherwise realized. Furthermore, students can see how new mathematics develops and how old mathematics finds new applications.

Acknowledgement: The work of the first author had partial support from the National Science Foundation (Grant Number: DUE 9555401) to the Long Island Consortium for Interconnected Learning (administered by SUNY at Stony Brook).

[1]. Malkevitch, J. and G. Froelich, *The Mathematical Theory of Elections*, Consortium for Mathematics and Its Applications, Lexington, 1985.

Cyber Space Learning

Judy Ann Brown

Future historians, looking back at the developments of the late 20th century, will refer to our era as the "Information Age." In order to prepare our students for the developments yet to come in the 21st century, we must seize every opportunity to employ new techniques for the exchange of knowledge. The World Wide Web provides precisely this opportunity. Teachers must become evangelists spreading the word of the rich global community blossoming on the Internet.

Yes, there are those who speak of the "evils" of the Web. Only the naive would deny that offensive material does exist on the Internet. In the past there have also been those who have led crusades to ban great works of literature, and to remove televisions and videos from the path of our children. Yet somehow, we have managed to use these devices to the benefit of education because dedicated teachers have shown the way. With support from the education community we can establish a safe learning environment on the Internet for our students.

Teachers must understand that the Internet is a powerful research tool and like other educational tools students should be taught its proper use. They need to learn to discriminate between "good" and "bad" Internet resources. If not informed, students may assume that everything they read is correct. Every site should be evaluated using the following questions:

- Is the information accurate and up-to-date?
- Are the links to other sites maintained?
- Is the information biased?
- Was the site created by a person knowledgeable about the subject matter?
- Is the site affiliated with an organization?
- Can you confirm the truth of the information given?

In a recent class discussion the question was asked, "When does the 21st century begin." Students debated the issue, eventually narrowing the field to January 1, 2000 or January 1, 2001. Two students journeyed to the library to find the answer. They returned with a book titled *Major Events of the 20th Century*. The book began with the year 1900, their conclusion was that the 20th century began in 1900 therefore the 21st century must begin in 2000. A more directed search led to the World Book Encyclopedia and the correct information that the turn of the century will be January 1, 2001. An internet search gave similar results. At one site,

Judy Ann Brown's Top-12 List of Hot Web Sites

1. For those impossible-to-answer questions that students often pose there is Ask Dr. Math. Students/teachers can submit questions for the doctor. Please read the FAQ list before submitting your question.
`<http://forum.swarthmore.edu/dr.math/drmath.html>`

2. AIMS Puzzle of the Month Hotlist is a site you will want to visit the first week of each month. There you will find ready-to-use worksheets with a new puzzle each month.
`<http://204.161.33.100/Puzzle/PuzzleList.html>`

3. Chaos in the Classroom: Robert L. Devaney, Department of Mathematics Boston University.
`<http://math.bu.edu/DYSYS/chaos-game/chaos-game.html>`

4. Eisenhower National Clearinghouse for Math and Science Teachers at Ohio State University. The ENC Catalog of Curriculum Resources records materials which are designed to assist teachers in selecting appropriate and high quality items for classroom use. `<http://www.enc.org>`

5. To add a touch of the History of Mathematics to your daily lessons visit Mathematical MacTutor. There you can find the name of the mathematician of the day or do a search for a specific topic in mathematics.
`<http://www-groups.dcs.st-and.ac.uk:80/~history/Mathematical_MacTutor.html>`

6. PCTM Puzzle of the Week Page:
`<http://dimacs.rutgers.edu:80/~judyann/POW/puzofwk.html>`

7. MetaCrawler Search Engine will help you to find almost anything on the web:
`<http://metacrawler.cs.washington.edu>`

8. Learn the rules of the Net:
`<http://www.fau.edu/rinaldi/netiquette.html>`

9. Learn the rules of Netiquette
`<http://www.dse.vic.gov.au/loti/netiquet.htm>`

10. NCTM Standards: The National Council of Teachers of Mathematics standards for math education.
`<http://www.enc.org/online/NCTM/280dtoc1.html>`

11. The Classroom Connect's home page features links to educational resources:
`<http://www.wentworth.com/classroom>`

12. Web page full of software for Mac and IBM
`<http://www.netaxs.com/people/dmorgen/stoney.html>`

Good luck and Happy Surfing!

<http://www.mediabridge.com/nyc/bids/tsbid/quotes.html>, you can find information for a turn-of-the-millennium celebration to be held in New York City on December 31, 1999 at midnight. On the other hand, at <http://ecuvax.cis.ecu.edu/~pymccart/2000.html> you will find that the first day of the 21st century is January 1, 2001.

Getting Started, What do you need?

Starting with basics, you will need a computer with a modem, an Internet Service Provider (ISP), and a browser. All come in a variety of sizes and quality. A Mac or PC with limited RAM may limit your ability to access applications. A faster modem may increase your enjoyment of Web surfing.

The ISP you select is crucial. The requirements for an acceptable provider include: A local phone number for dial-up access, low rates (about $19.95 per month) for unlimited access time, and technical support during evening and weekend hours. You can find an ISP by visiting the site <http://thelist.com/> and searching by area code; at <http://www.cnet.com/Content/Reviews/Compare/ISP/> you will find ratings for over 500 ISP's. At last check AOL <http://www.aol.com/> was ranked 502 out of the 518 ISP's rated, while EPIX <http://www.epix.net/>, my provider, was listed at 449. The number one ISP as this time is NTR.NET Corporation <http://www.ntr.net/>. The ratings at this site are based on responses by visitors to the site; the numbers provide only benchmarks—the user must provide the interpretation.

Most services provide software that is compatible with their system; this software should include telnet, gopher, and browser programs along with programs to read mail and news groups. Netscape is one of the top browsers on the market and if your service does not provide it, you can download a copy (free to educators) from <http://home.netscape.com/>. Once correctly configured (here is where you may need the assistance of the technical support staff), Netscape can be used to read your mail and newsgroups, browse the Web, and download software.

Where Do I go from Here?

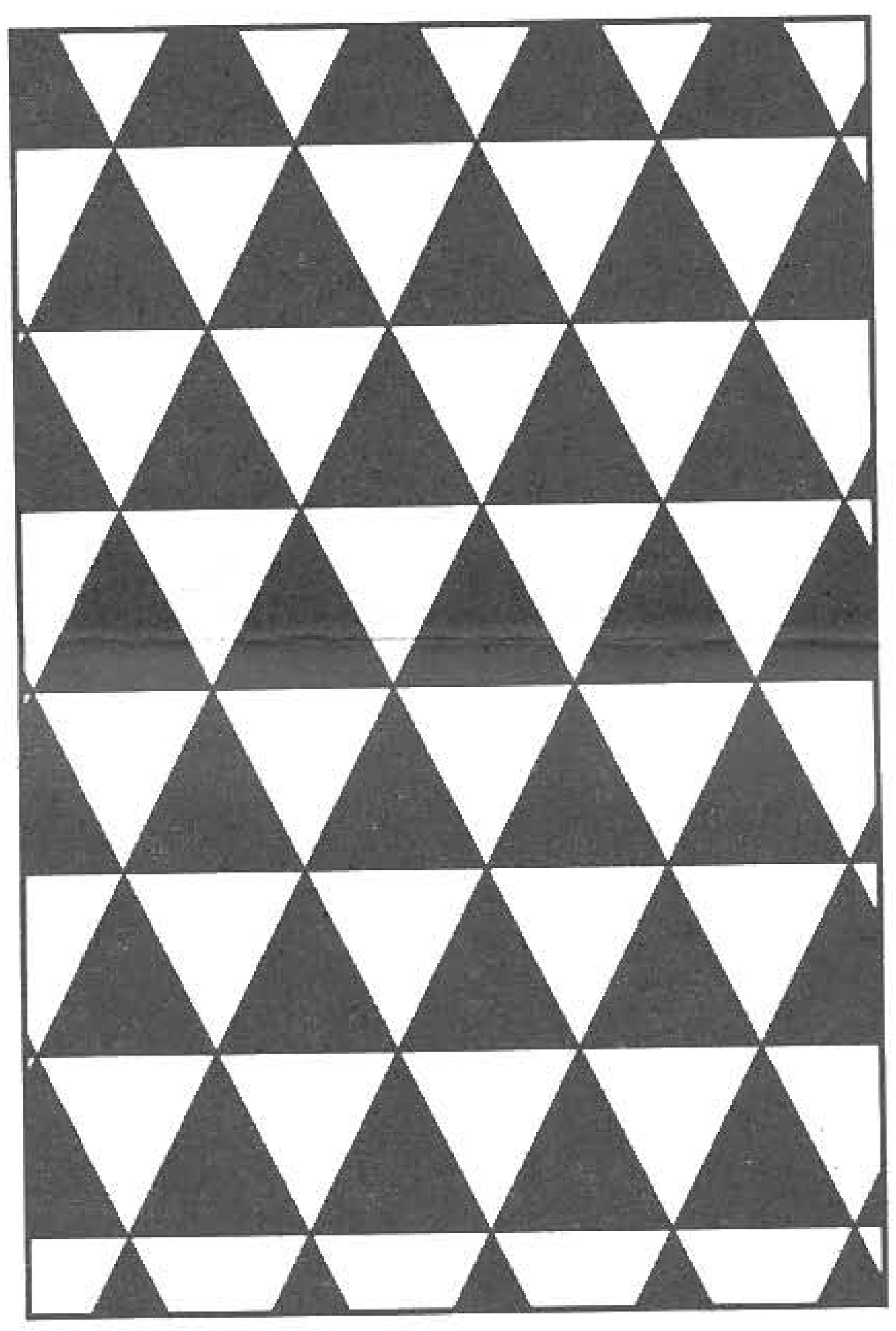
Once you are on-line you will need a few URL's to visit. The shaded box on the previous page is my short list of favorite sites. For the complete list you will need to visit my home page at <http://dimacs.rutgers.edu/~judyann/> where you will be able to click on highlighted text or graphics to access the sites.

Tessellations in the Lower Grades

Kerry Simmons

Tessellations seem to be very popular among intermediate and junior high school students. It might then be assumed that this concept would be too advanced or too abstract for students in the primary grades. This, however, was not the case with my Kindergarten class. Tessellations have in fact become one of their favorite topics. It appears that when approached at its very basic level, tessellations can be easily recognized and understood by primary students.

My class is comprised of 34 Kindergarten students in a self-contained full day setting. It is neither the size nor structure of the class which sets the tone for this experience, but inevitably the individual interest of each student. What was accomplished with this group could, I believe, be accomplished with a number of groups of various sizes and settings.



While reviewing basic shapes, my Kindergarten class began to experiment to see if certain shapes could be placed next to each other in a manner that would neither cause them to overlap nor leave spaces. When such a shape was discovered, it was then added to a list which we were compiling. As the list expanded, we discussed what these shapes all had in common. The responses included the fact that each shape was made up of sides and corners. Others pointed out that the shapes were straight and did not curve as did the circle which was not included on this list.

After all the basic shapes which did not overlap or leave spaces were identified, the word "tessellate" was introduced to the class. The word—because of its size and sound—intrigued the children. The class was separated into groups of four or five and given pattern blocks of the same shape to tessellate. These blocks also became useful as tracers. The children were able to pick their own shape and make an illustration using the block as a pattern to trace.

One of the things which the students found most exciting was the search for tessellations in their own home and school environments. They discovered that the tiled ceilings and floors of the classrooms were tessellations, as were the brick and cinder block walls. They also explored their own homes and reported that such things as kitchen floors, checkerboards, tablecloths and fences were tessellations as well.

These discoveries at the class level allowed us to discuss how most tessellations exhibited a pattern involving either color or shape. The checkerboard, for example, was a wonderful representation of a tessellation organized in a color pattern. The students were then able to take their own tessellations of squares and color them in a pattern of their own. The class was also able to recognize at a very simple level, that a tessellation of triangles involved the rotation or change of position of every other triangle. We identified this as a pattern and used it in the creation of Native American headbands for our Thanksgiving feast.

Motor skills at the Kindergarten level vary quite distinctly from student to student so that creating and tracing their own individual tessellations would be too frustrating for children at this developmental level. It is very rewarding, however, to see these children begin to understand the mechanics of a tessellation and recognize basic shapes in a tessellated form. Using pre-cut shapes, the class was also able to work together to form one large class tessellation. it was quite a demonstration of cooperation. In addition, the challenge in creating this one large tessellation was that the completed tessellation should follow a color pattern.

Paper folding is an additional method we used to illustrate the tessellation of a basic shape such as a square or rectangle. The use of paper folding was more in tune with a Kindergarten student's motor ability and the results were more than satisfactory.

In concluding my unit on tessellations, I invited our eighth grade class to visit the Kindergarten and exhibit their own tessellations which they had been working on. The eighth graders were given the opportunity to explain their work, and the Kindergartners were given the opportunity to question the eighth graders about their tessellations. It was a very worthwhile experience for both classes, and an excellent way to culminate the unit.

The students were so fascinated with this topic that even after a month had passed, I was still receiving pictures of tessellations. One student even brought in a tessellation of Lego blocks to use as his show-and-tell for the day. he explained quite clearly how he used the same shape block over and over again—making sure not to overlap them or leave any spaces.

This unit was very rewarding. It allowed students at the primary level to identify and demonstrate concepts which have in the past been connected with upper grade levels. These students have been given the basic concrete foundation as well as the descriptive vocabulary necessary to continue their study of this topic.

Credits...

This Newsletter is a project of the Leadership Program in Discrete Mathematics (LP). The LP is funded by the National Science Foundation and is co-sponsored by the Rutgers University Center for Mathematics, Science and Computer Education (CMSCE) and the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS). **Joseph G. Rosenstein** is Director of the LP and Founding Editor of this Newsletter. **Valerie A. DeBellis** is Associate Director of the LP.

This issue was typeset using QuarkXPress, with graphics generated on Corel Presentations.

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Editors...

Judy Ann Brown is our World Wide Web resources column editor.
Janice Kowalczyk is the Discrete Reviewer, editing the column on Discrete Mathematics resources.
Robert Hochberg is the new general editor.

***IN DISCRETE MATHEMATICS:* Using Discrete Mathematics in the Classroom**

Published by the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS) at Rutgers University

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Discrete Mathematics Summer Institutes for K-8 teachers

The Leadership Program in Discrete Mathematics will conduct its ninth annual summer programs in 1997 for K-8 teachers, featuring two-week residential institutes at Rutgers University, New Brunswick (NJ), during July 14-25, and two-week commuter institutes at Rutgers (June 25-July 11), Scotsdale AZ (June 2-17) and Warwick, RI (Aug. 4-20).

Participants will be expected to attend follow-up sessions during the school year and a follow-up institute during the summer of 1998. Graduate credit will be available. Teams of teachers in schools or districts are welcome to apply. Funding by the National Science Foundation provides for all costs of the institutes and a stipend of $600 for the two-week program. Participants will be expected to assume leadership roles in bringing discrete mathematics to their classrooms and schools, and in introducing their colleagues to these topics. For information, call Bonnie Katz, 908-445-4065, e-mail her at bonnie@dimacs.rutgers.edu, download the materials from `http://dimacs.rutgers.edu/Education/k8/k8_97.html`, or write to: Leadership Program, P.O. Box 10867, New Brunswick NJ 08906.

I was walking with one of my kindergarten age children many years ago. As we crossed Hancock street in our town, I mentioned that it was named after an early governor of Massachusetts. My child replied: "He must be a very tall man." When I asked why, there came the reply "Because he has a very tall building named after him." —Erica Voolich (LP '94)

The Discrete Reviewer

Janice Kowalczyk

The Discrete Reviewer continues as a column with teachers' recommendations and suggestions for discrete mathematics resources for the classroom. The material in this issue comes mostly from the article being assembled by Debbie Franzblau and myself in "Discrete Mathematics in the Classroom: Making an Impact" to be published in 1997. Since an increasing number of K-8 teachers are bringing discrete mathematics topics into their classrooms, I have decided to dedicate this column to resources that make the literature/discrete mathematics connection.

Teachers, especially those at the primary level, have found that one of the ways to interest and engage students in mathematical thinking is through literature. In the early grades, weaving literature in with mathematics may be a necessity, since so much classroom time at this level is already dedicated to developing literacy. This column will highlight a short list of some of the books that teachers have found connect well with discrete mathematics topics. This is not meant to be a comprehensive list and while there are a number of publications that provide more complete lists of literature–mathematics connections we are not aware of any that highlight topics in discrete mathematics.

As always, I would appreciate your comments and recommendations on this column in order to make future columns more useful to you. Your feedback on the resources that you try as a result of this column is also encouraged, so that we can develop a better appreciation of the usefulness of these materials to classroom teachers and continue to develop a discrete mathematics resource directory to share with others. Feedback and recommendations can be sent to:

Resources— Janice Kowalczyk
P.O. Box 10867
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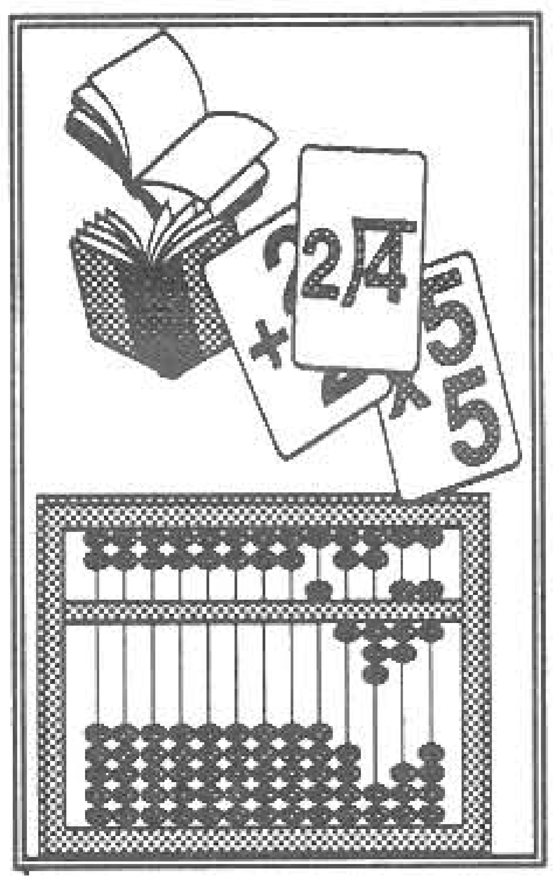
or e-mail to: kowalcjn@ride.ri.net
or: JCKRISF@aol.com

Literature and Discrete Mathematics



Katy and the Big Snow
by Virginia Lee Burton
Houghton Mifflin Co., Boston, 1943
Cost: $4.95
Grade level: K-3
This is a classic children's book about a red tractor named Katy who as a snowplow saves the city of Geoppolis following a huge snowstorm. It is because of Katy that the fire, police, mail, water department, and other city services are restored. Teachers have found this book a natural introduction to the topic of Euler paths and circuits. The book even includes a map of Katy's route that could be used or modified for such activities.

Sam Johnson and The Blue Ribbon Quilt
by Lisa Ernst
Wm. Morrow & Co., 1983
Cost: $5.00
Grade level: K-3
This beautifully illustrated book raises both mathematical and social questions. While mending the awning over the pig pen, Sam discovers that he enjoys sewing the various patches together but meets with scorn and ridicule when he asks his wife if he can join her quilting club. The borders of the book feature various American quilt patterns which can be used to connect to mathematical concepts such as symmetry, tessellations, and transformational geometry.

A Cloak for a Dreamer
by A. Friedman
Scholastic, 1995
Cost: $15
Grade level: K-3
A delightful book about the three sons of a tailor who are asked to sew a colorful cloak that will keep out wind and rain. The first two sons sew watertight cloaks using rectangles, squares and triangles, but the third son, the "dreamer", makes a cloak using circles, that is full of holes. This book is a nice introduction to tessellations and contains (for parents and teachers) a section on the underlying mathematical concepts in it.

Dr. Seuss Books
by Dr. Seuss
Random House Inc.
Cost: $8 - $15
Grade level: pre-K - 4
A number of the Dr. Seuss books contain the seeds for thinking about iteration and recursion. *The Cat in the Hat* and *Green Eggs and Ham* are just two that come to mind. In these stories, events or activities are repeated over and over but in each repetition a new event is added. Another Dr. Seuss book, *The Lorax*, is a variation on this with an environmental theme. As events happen in the Lorax other events are triggered. This can lead to a rich lesson on systems thinking. Events in *The Lorax* could be examined by students recursively to determine the critical events that cause the environment to go out of balance. Note: I still enjoy these books and especially the Lorax which has a message for us all.

Grandfather Tang's Story
by Ann Tompert
Crown Inc., 1990
Cost: $15
Grade level: K - 3
This is a Chinese folk tale told with tangrams. Using tangrams, Grandfather tells a story about the shape-changing fox fairies to his granddaughter. Fox fairies are an integral part of Chinese folklore and are believed to be endowed with supernatural powers. In the story, two fox fairies try to best each other until a hunter brings danger to both of them. Using tangrams students can be encouraged to investigate geometrical concepts and to retell or invent their own stories. Another good book for motivating students to use their visual imaginations and explore geometry with tangrams is:

The Tangram Magician
by Lisa Campbell Ernst
Harry N. Abrams, NY, 1990
Cost: $20
This story involves a magician who can change shape, and is illustrated with tangrams. The reader is asked to supply the end of the story by creating the next shape that the magician will take on.

A Three Hat Day
by Laura Geringer
Harper Collins, 1987
Cost: In paperback, $5
Grade level: K-3
This amusing tale about a hat collector and his search for a perfect wife has provided an opportunity for teachers to ask "How many different ways are there to...?", and introduce the concept of systematic counting in combinatorics.

Two of Everything
by Lilly Toy Hong
Albert Whitman & Co., 1993
Cost: $15
Grade level: 1-4
A Chinese folk tale about a couple who finds a magic pot that doubles everything they put into it. The story is retold and illustrated by the author. This is a good starting point for thinking about iteration and exponential growth.

One Hundred Hungry Ants
by Elinor Pinczes
Houghton-Mifflin, 1993
Cost: $15
Grades: 1-3
This is a simple story told in verse of one hundred hungry ants heading towards a picnic. Different formations of 100 ants are tried in order to speed their way to the food, and in the process the author delightfully introduces the factors of 100 and the problem of counting factors. A fun way to inspire mathematical thinking.

Jurassic Park
by Michael Crichton
Random House Inc., New York, 1990
Cost: In paperback $5.99
Grade level: 6-12
Jurassic Park, which connects mathematics, biotechnology and prehistoric legend, has proved to be a "student magnet" in a number of mathematics classrooms and served as a stepping stone to the introduction of fractals. In the beginning of each chapter, the dragon curve fractal is constructed to a few more levels, as a mathematical foreshadowing of the events to come. One eighth grade teacher told us she observed some of the best learning of her teaching career take place, with lessons centered around this novel, as the learning was sparked by real student interest and a real need to know.

Anno's Mysterious Multiplying Jar
By Masaichiro and Mitsumasa Anno
Putnam Publishing, 1983
Cost: $17
Grades 1-6
One of my very favorites and a delight for both the mind and the eye. Simple text and beautiful illustrations tell a tale about a porcelain jar with a sea inside. In the sea is one island, and on the island are two countries, and in each country are three mountains and so on up to 10. This book provides a rich introduction to the the concept of factorials (e.g. 5-factorial, or 5!, is the product $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$).

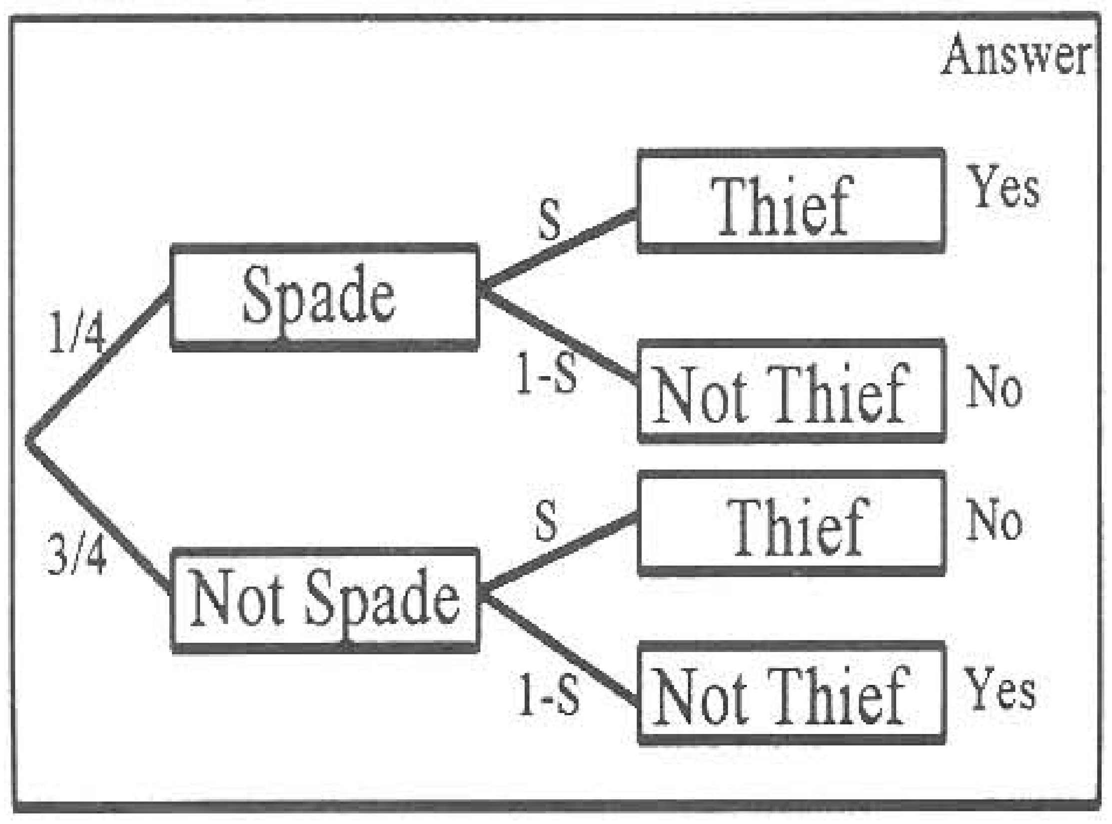
I was buying nails and washers at a hardware store to make a classroom set of the Towers of Hanoi. I picked out six washers, each smaller than the one before, and brought them to the counter. The hardware man was concerned that I would have trouble nailing up the largest washer, so told him that I wouldn't be nailing them up anywhere. After a moment's confusion, he exclaimed "Oh, I know what you're doing. You're making that puzzle with the three pegs!" Even the hardware man knows about Discrete Math! — Judy Nesbit (LP 94)

You want to know WHAT?

Paul A. Dreyer, Jr.

When pollsters ask people who they are going to vote for in an upcoming election, people usually have few qualms about telling the pollster who they support. But imagine that a pollster asked the question, "Have you ever shoplifted anything from a store?" If a person has stolen something, chances are that person will still say "no" because it is embarrassing, and no one has any reason to expose the skeletons in their closet to a pollster. Nevertheless, pollsters could still ask questions on sensitive topics and get pretty accurate results. How they could do this provides an entertaining excursion into probability.

Imagine a pollster walks up to you and says, "I'd like you pick a card from this deck. If it is a spade, I would like you to answer the question, 'Have you ever shoplifted anything from a store?' truthfully. If it is any other card, I want you to lie." Whatever you say, the pollster has no idea whether you told the truth or not, so no information about you is imparted to the pollster. Surprisingly, when many people are polled this way, it is possible to approximate what percentage of the people have actually shoplifted.



Y = (1/4)S + (3/4)(1-S)

Let us consider this from the pollster's point of view. When the pollster meets a person whom he wishes to poll, there are three random events that take place:

1) The person either draws a spade or does not draw a spade,
2) The person either is a shoplifter or is not a shoplifter, and
3) The person answers either "yes" or "no." The combination of these events, together with all possible outcomes, are shown in the tree diagram in the figure, where S represents the probability that a person is a shoplifter, and "Thief" is the title unflatteringly assigned to those who shoplift. For example, if a shoplifter approaches the pollster and draws a heart, he will respond "no" to the question, as indicated by the bottom "no" in the figure.

Let us use Y to denote the probability that a person who is polled answers "yes" to the question. Then we have three probabilities to consider:

- 1/4, the probability that a spade is drawn,
- S, the probability that a person is a shoplifter, and
- Y, the probability that a person says yes.

(Notice that this implies that the probability that a person draws a spade is $1-1/4$, or $3/4$, the probability that a person is not a shoplifter is $1-S$, and the probability that a person answers "no" to the question is $1-Y$.)

But now we make a crucial observation: A person's response *depends* on whether or not they shoplift and on the card that they draw, so we are able to write down an equation describing this relationship. First notice that there are two cases in which a person answers "yes" to the question:

- the person draws a spade and is a shoplifter, and
- the person draws a non-spade and is not a shoplifter.

Since the card a person draws is independent of their shoplifting habits, we can use the product rule to determine that "the probability that a person draws a spade and is a shoplifter" equals $(1/4)\cdot S$. Similarly, "the probability that a person draws a non-spade and is not a shoplifter" equals $(3/4)\cdot(1-S)$. Since these two cases are mutually exclusive, and since these are the only ways that we can get "yes" as an answer, we use the addition rule for probability to see that $Y = (1/4)\cdot S + (3/4)\cdot(1-S)$, as shown beneath the figure.

But we are interested in discovering S based on the number of people who answer "yes." So we solve that equation for S and obtain $S = (3-4\cdot Y)/2$.

Let us try an example: In a classroom with 30 children, suppose we ask the question "Are you in love" (a particularly prickly question!) If 10 of the students answer "yes", that that gives Y a value of 10/30 or 1/3. Thus $S = (3-4\cdot(1/3))/2$, or $S = 5/6$. So we conclude that about 5/6 of the students, that is, 25 students are in love. That answer is only approximate, of course, since this is all based on probability. Nevertheless, the outcome is striking—although only 10 out of 30 said "yes", about 25 are actually in love.

This provides a fun project for students to work on. In particular, for younger students, it is always amusing to find out what questions they find sensitive. Also, it is possible to set up more interesting randomizing systems. For example, a pollster can say, "If you pick a spade, just say yes." A warning, however. Consider the system "Flip a coin. If it is a head, lie; if a tail, tell the truth." Returning to the equation above, we get $Y = (1/2)S + (1/2)(1 - S) = 1/2$, *always*. This means that Y gives no information about S. Students can figure out what are valid and invalid randomization systems. I hope you and your students have fun with this, I certainly have!

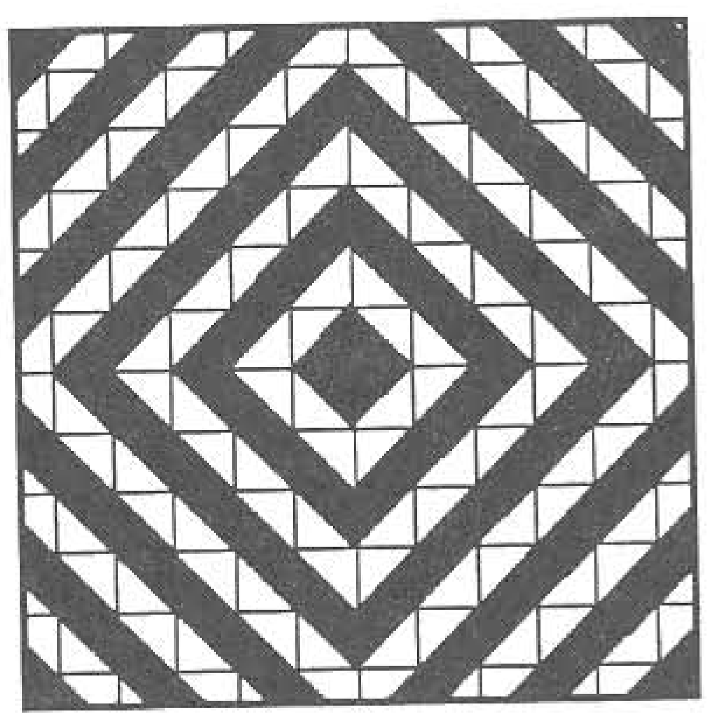
I worked at an elementary school while going to college. I missed a month of work because I donated a kidney. When I returned, one of my kindergarten students tugged on my pants and asked, "Ms. Price, how did they get a kitty out of you?!" — Carol Price (LP '94)

Quilting: More Than Meets the Eye

Suzanne Foley and Deborah Franzblau

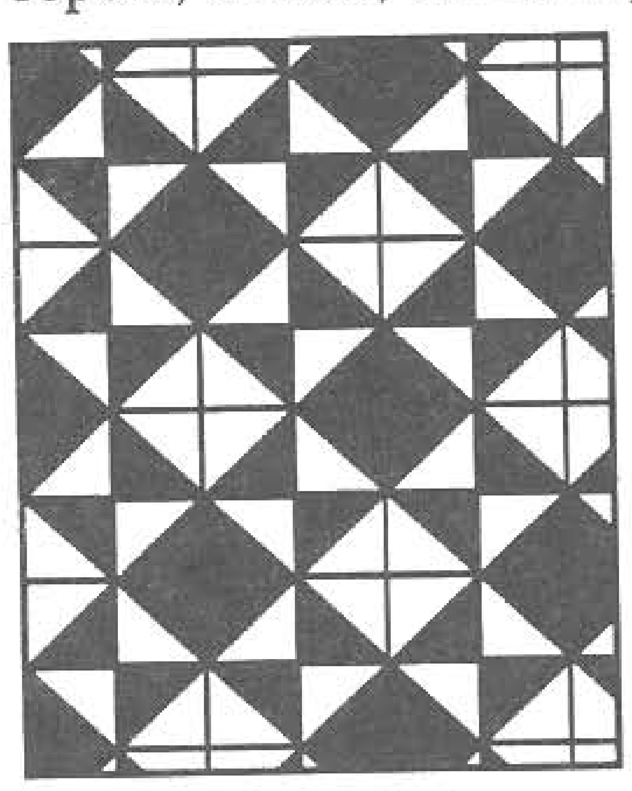
At first glance a quilt may appear to be just pieces of fabric sewn together. Upon closer inspection you may notice its colors, stitching, and patterns.

"From the earliest grades, the curriculum should give students opportunities to focus on regularities in events, shapes, designs and sets of numbers. Children should begin to see that regularity is the essence of mathematics." (NCTM Standards, 1989A p. 60)



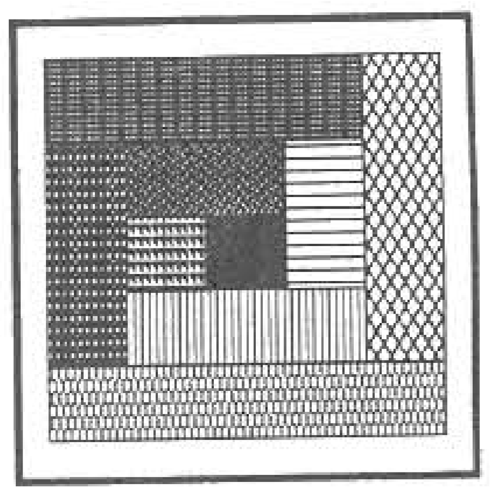
In the lower grades (K-3), students usually get their first experiences with patterns through auditory, physical, and linear patterns. Auditory patterns could be iterations, for example, "Clap, Tap, Stomp", a song that builds like "I Know An Old Lady Who Swallowed a Fly," or a story that repeats such as *Brown Bear, Brown Bear* by Eric Carle. An example of a physical pattern is an arrangement of students in a Sit, Sit, Stand configuration. Linear patterns can be created by stringing macaroni or beads onto a string or constructing a paper chain. Experiences with a Hundreds Chart, or with arranging pattern blocks, beans or buttons, can also provide worthwhile experiences with patterns.

Quilts provide an excellent way to introduce two-dimensional patterns. Quilting is more than material and batting. Quilt construction uses counting techniques, tessellations, and measurement. Some pattern concepts include repeat, reflect, slide, flip, turn, skip, rotate, as well as symmetry, parallel, and perpendicular, all of which are good vocabulary words for young students to grasp. People of all ages and backgrounds are fascinated by quilts.



Most quilts that you see are comprised of one basic block that is reproduced and either translated, reversed, or rotated to achieve different effects. For example, a simple block could be a square divided diagonally into two triangles, one light and one dark. These blocks can be combined in many different ways to construct a quilt, as shown in the the two quilts above. Also, if more than one basic block is used, you can make interesting variations (see bottom of page 11).

The first author has used quilting and quilt patterns with students at levels K-4 to broaden their knowledge of patterns and as a vehicle to link quilting to other areas of the curriculum. For example, the log cabin pattern shown to the right was created during the Lincoln presidential campaign. Its name is in honor of Abraham Lincoln's humble beginnings. The pattern is created by sewing rectangular material strips around a center square, as shown here. The first log cabin quilts were made of fabric scraps from various articles of clothing and other recycled material. Different quilts are created by manipulating the basic block, which is simply a variation on the square divided diagonally. When made using log cabin blocks, the top quilt pattern in the left column is called "Barn-Raising."

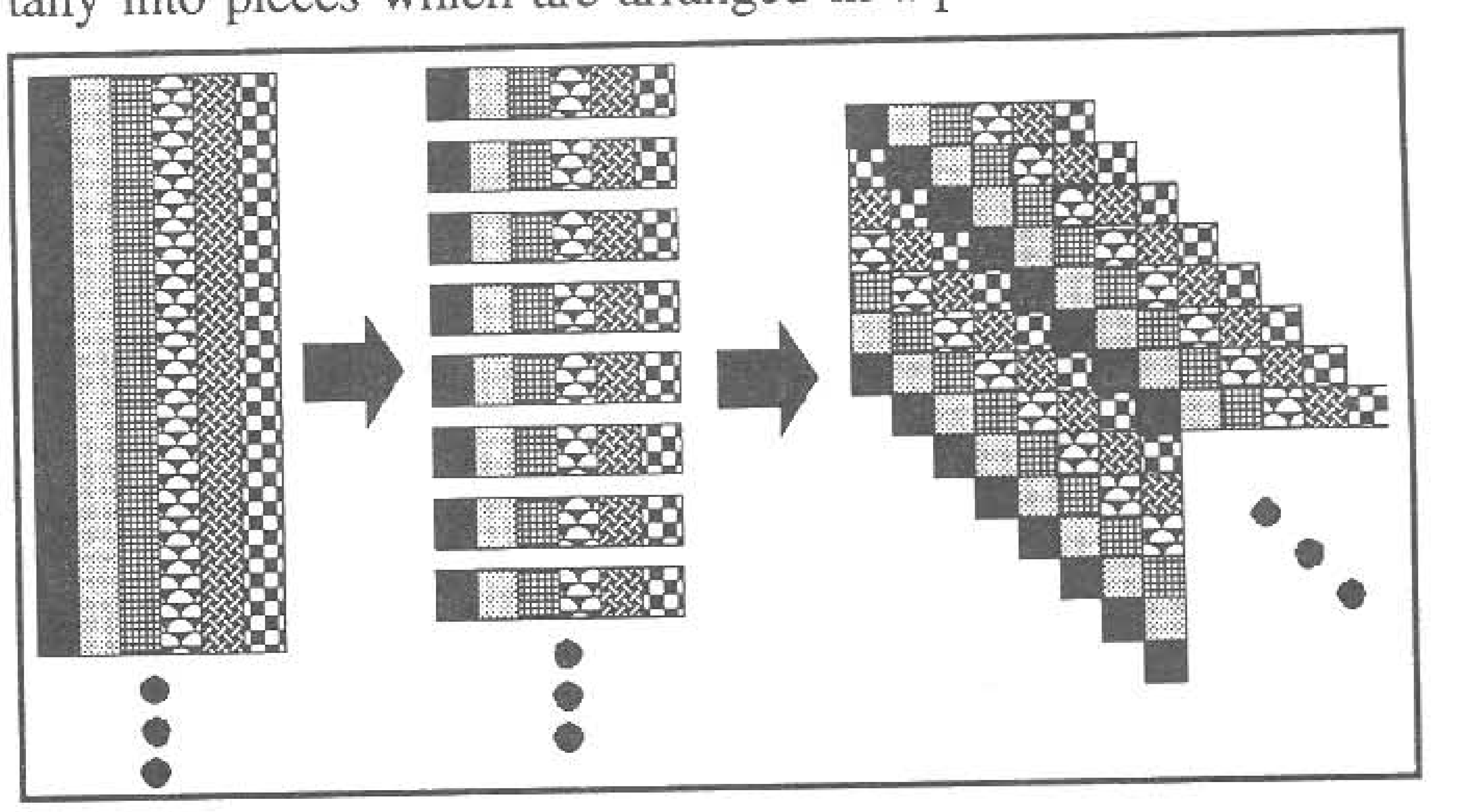


Wallpaper scraps provide good material for making log-cabin quilts. Students can sort the strips into dark and light and measure and cut strips of the desired lengths. After gluing down a center square, students place strips around the center in a coiling fashion to create the blocks. Students can work in groups of four to see how many variations can be produced by combining four blocks.

Quiltmaking raises mathematical questions as well. If a piece of material must be 1/4" larger on each side to provide a seam allowance, how much material of each color must be purchased? How much material is "wasted" for scraps and or seam allowances versus the actual quilt top? How many different geometric shapes can be found within a given quilt?

Traditionally, each piece of material was cut out individually and sewn together to form the top. Today, modern quilters use various devices and methods to increase their accuracy and to save time. These include using a special plastic guide mat with a grid, a ruler and a rotary cutter. The rotary blade looks like a very sharp pizza cutter. Quilters can cut up to six thicknesses of fabric quickly and accurately. Given a piece of fabric, what is the best method to use to produce a given number of shapes using the fewest cuts?

After the material is cut, quilters often use a speed-piecing or strip-quilting technique. In a split rail quilt several pieces of fabric are sewn together, ironed and then cut to a given length. The long rectangle in the picture is cut horizontally into pieces which are arranged in a pattern of horizontal,



vertical, horizontal... and sewn together to form a quilt top. How many straight cuts and straight seams are required both for this method and the traditional method (cutting and sewing individual squares)?

The next time you look at a quilt, take a step back and consider how this quilt was constructed. Perhaps you can identify the basic block of the quilt, other ways that block could be arranged, or the most efficient way to construct the quilt.
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- *America's Heritage Quilts.* Better Homes and Gardens. Edited by Patricia Wilens. Meredith Corporation, Des Moines, IA. 1991
- *The Complete Book of Quiltmaking.* Michele Walker. Alfred A. Knopf, New York, NY. 1990.
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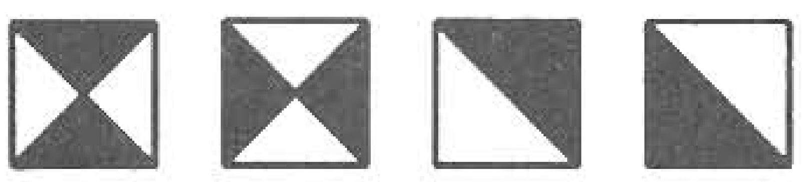
"Mod 4 Quilts"



0 1 2 3

When one uses a mathematical rule to aid in the design of a quilt, there frequently arise some very pleasant patterns and symmetries. For example, consider the multiplication tables below: The first one is familiar,

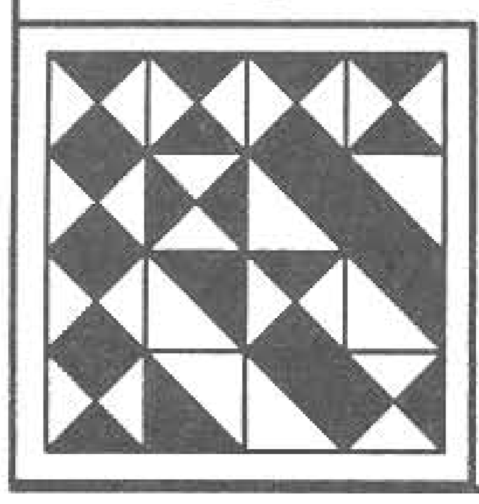
×	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	4	6
3	0	3	6	9

but the second one is created by replacing each entry with its remainder when divided by 4. For example, when 6 is divided by 4, the remainder is 2, so the sixes become twos in the new table. (This is called *multiplication mod 4*, by the way.)

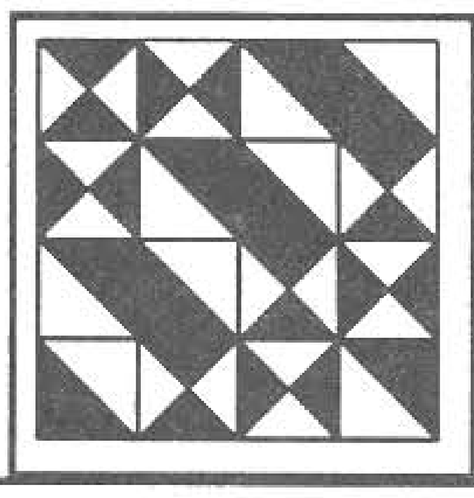
×	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Now we refer to the basic quilt squares shown up top. Substituting them into the multiplication table shown to the right we obtain the quilt block labeled "A" below. You may wish to do the same thing with addition instead of multiplication. Why not take a minute and try it now. You should get the pattern labeled "B". In the next section, we will take advantage of the symmetry in these patterns.

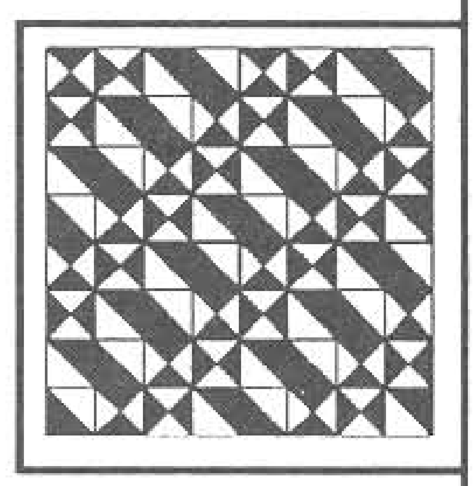
A



B

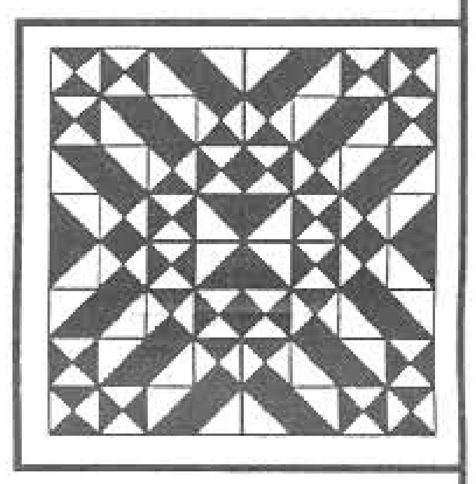


Build the quilt

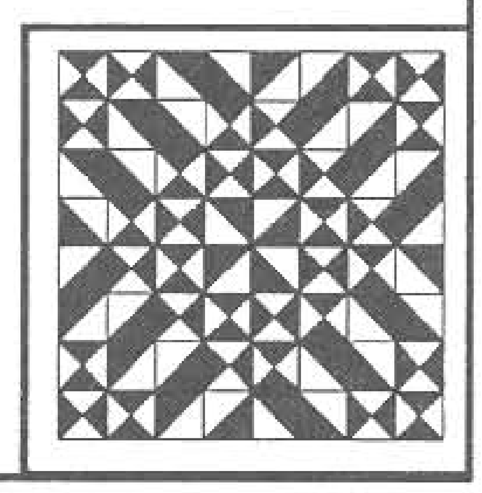


We can now sew together many copies of pattern B to generate a quilt. Can you tell which of the quilts on the right were generated by rotations, reflections and mere translations of pattern B?

Some questions



There are many questions which can be asked about the various stages of the quilt-building process. For example, young children can be asked to design symmetric tiles like 0, 1, 2 and 3 above. Or, given pictures of the tiles, they can be asked to find them in the finished quilt, and to tell whether they are in their original position, or have been rotated or reflected. Older children can be asked to find the number of ways to arrange the given tiles into a grid of squares. For this exercise, you might give a set of 2 tiles (instead of 4) and a 2×2 grid (instead of 4×4), and have them find them by systematic listing. Or, more advanced students can use the multiplication rule on these examples, or using 3, 5, or 6 tiles. There are many good discrete questions; the possible directions are limited by the teacher's imagination alone.



Leadership Program in Discrete Mathematics
Crash Course for High School Teachers

The Rutgers Leadership Program in Discrete Mathematics will be offering a 2-day "crash course" for high school teachers at Rutgers University on June 23-24, 1997. The content will include paths and circuits in graphs, sequences, voting methods, codes, and fractals. Meals will be provided, as well as lodging on the night of the 23rd. Persons interested in this program should contact Bonnie Katz at 908-445-4065, e-mail her at `bonnie@dimacs.rutgers.edu`, or write to Leadership Program, P.O. Box 10867, New Brunswick NJ 08906. This program is sponsored by the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS) with funding by the National Science Foundation (NSF).