

IN DISCRETE MATHEMATICS

Using Discrete Mathematics in the Classroom

Issue #9

Fall/Winter 1998

Speaking Discretely...

Robert Hochberg

Welcome to the Fall/Winter 1998 edition of *In Discrete Mathematics*. This issue is especially diverse across grade levels and offers something for everyone.

Ann Lawrence has "uncovered" some discrete mathematics in the children's classic *From the Mixed-up Files of Mrs. Basil E. Frankweiler*, and has created a series of activities

which can be adopted or modified for any classroom old enough to read the book. In her article, Erica Dakin Voolich introduces us to a toy which can provide food for mathematical thought in any primary or secondary grade. Judy Gugel and Judy Grogan show us how successfully coloring can be introduced into Kindergarten and first grade, respectively,

and Jill Dunlap presents a coloring exercise, *The Farmer's Daughter*, which challenges her gifted sixth graders! Finally, Janice Kowalczyk reviews some discrete book resources. Enjoy!

M3 (Midnight Mischief at the Met)

Ann Lawrence

Connecting Children's Literature With Discrete Mathematics Topics

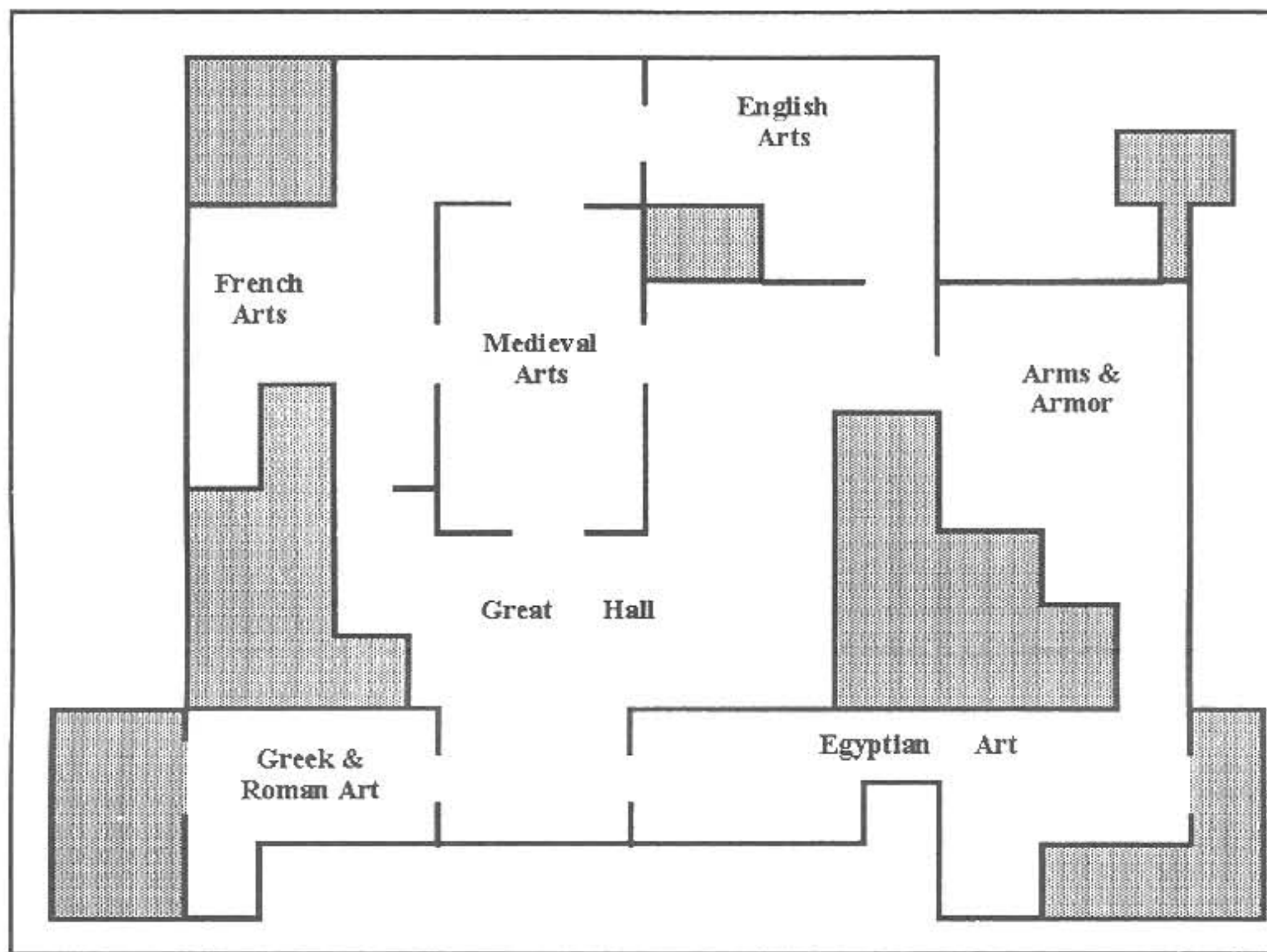
As teachers, we search for ways to get the most out of every classroom minute. Many of us have found that children's literature offers a rich source for explorations in mathematics. Numerous stories provide authentic, thought-provoking examples of human experiences which also offer vivid, engaging contexts for mathematical investigations.

Many discrete math topics can be explored in such contexts. For example, *From the Mixed-up Files of Mrs. Basil E. Frankweiler* by E.L. Konigsburg (New York: Yearling Books, 1967) provides a setting for introducing Euler paths and circuits in middle school. To introduce the lesson, read a passage from the book or have a student

who has read it summarize the plot. In the story, Claudia ran away from home and hid in the Metropolitan Museum of Art to make her parents appreciate her. She took her brother too, and the book describes their adventures there, including solving a mystery about a beautiful statue. She also wants to feel different after her adventure, and this goal adds a wrinkle to each experience.

You can pose the following problem to your students: *For entertainment one night, Claudia took several coins from those she and Jamie found in the fountain and placed one in each major doorway on the main floor of the museum. She challenged "Sir James" to pick up all the coins one by one. According to Claudia's rules, in order for Jamie to succeed he must gather all the coins by going **only once** through each doorway connecting major galleries to each other or to the Great Hall. (Ignore doorways to the shaded areas.) Can you find a path Jamie could follow? Your solution may start and end in any of the major galleries, but remember that Jamie may pass through each doorway only once!*

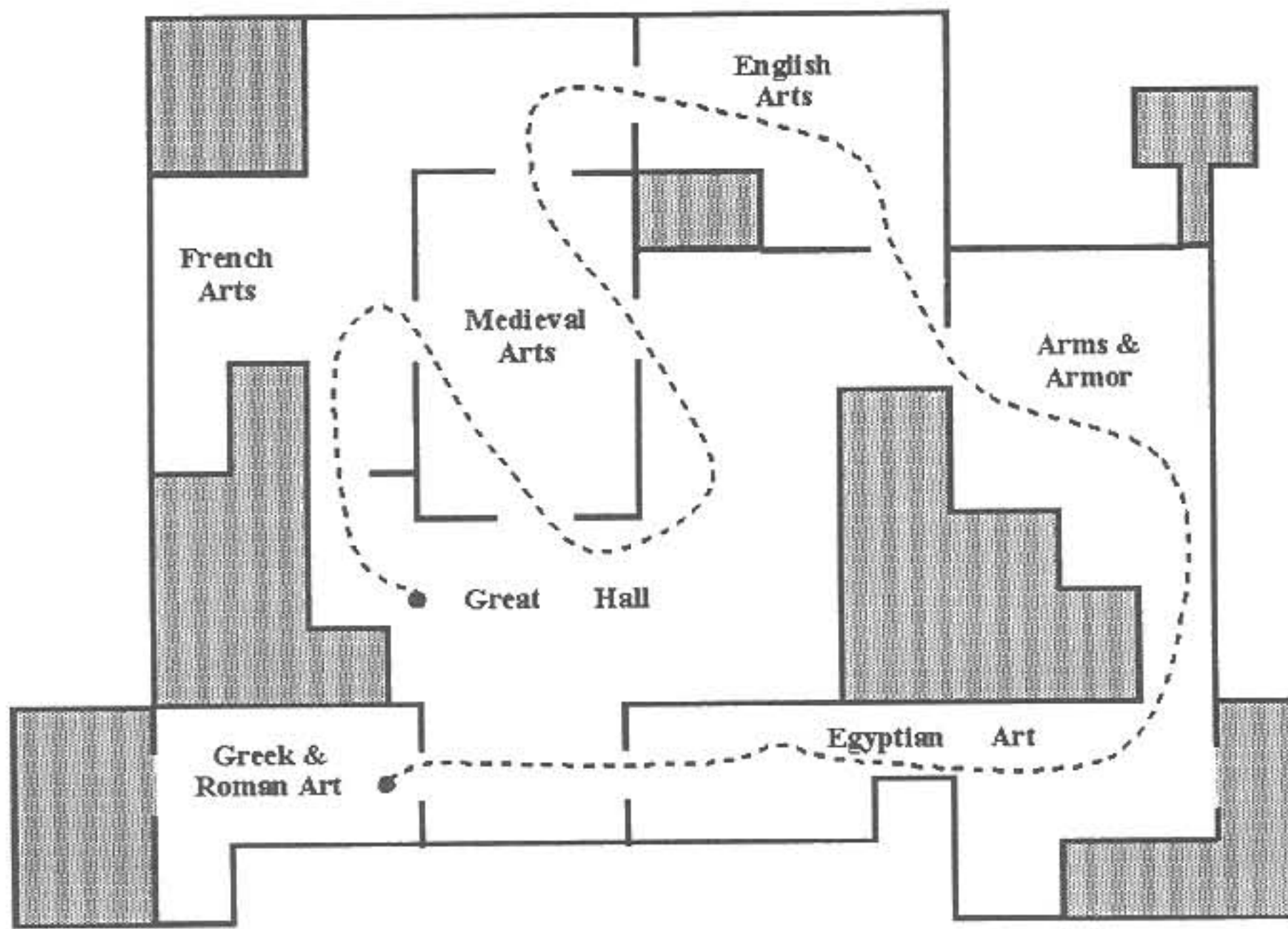
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In This Issue

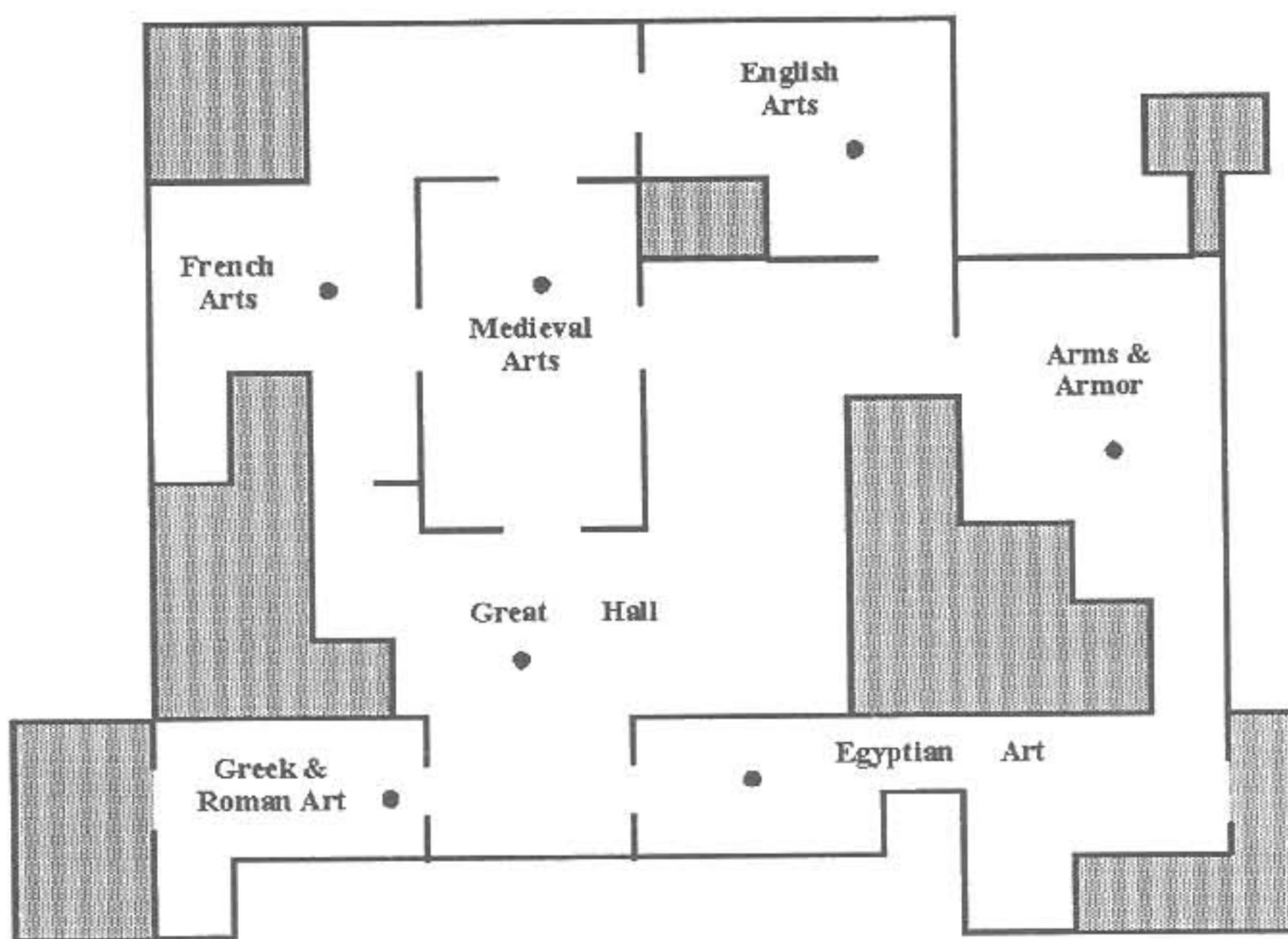
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Pass out the map of the Met's first floor (Page 1) to the students. Have them work in pairs. While they work, circulate among the students to facilitate construction of an accurate path by each pair. Have students share their graphs to ensure that all are correct. Elicit that all correct paths begin in either the Greek and Roman Art gallery or the Great Hall and end in the other. One correct path is shown below.

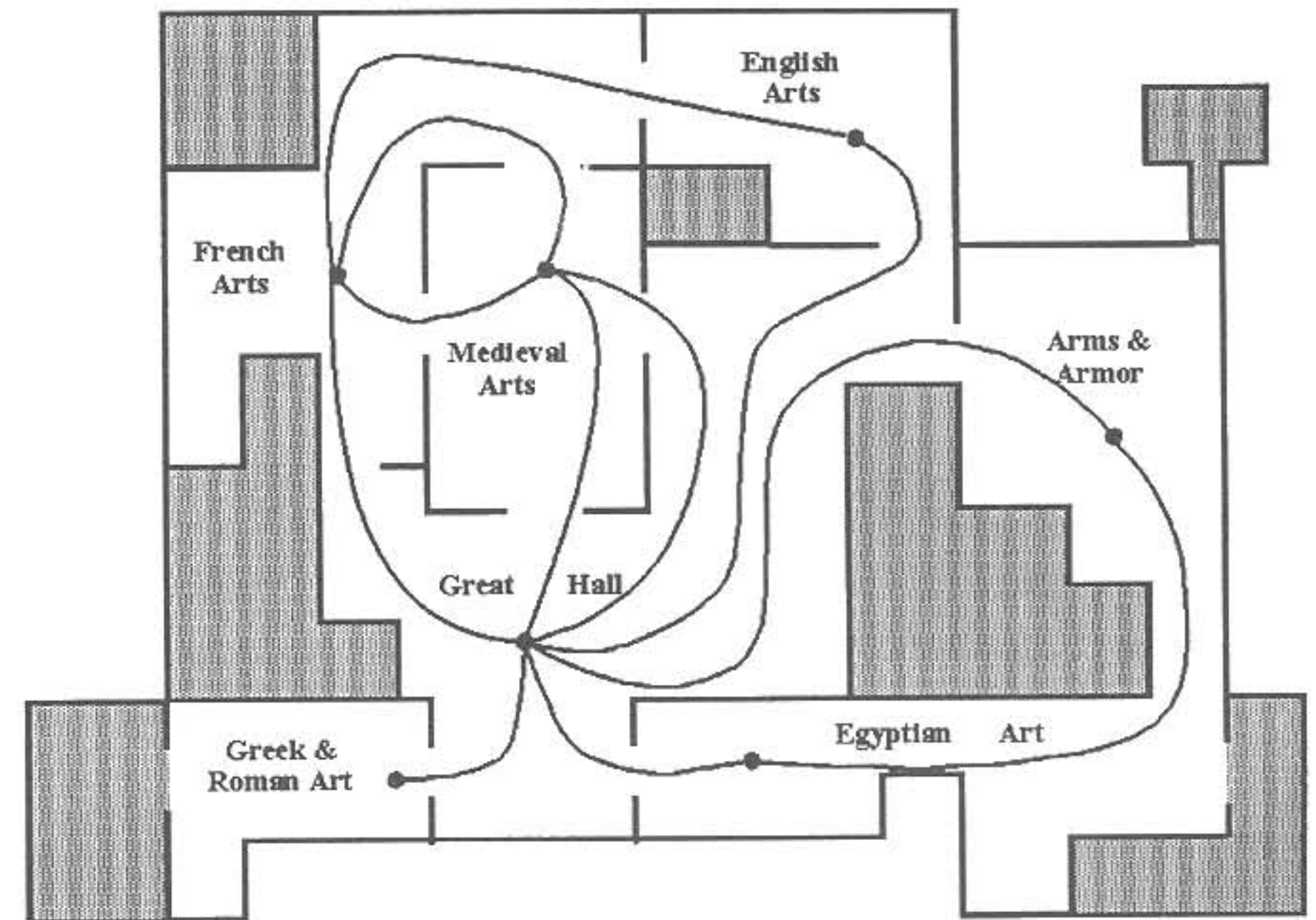


Next, model how to make a graph to represent the main floor of the Met, using transparencies. To do this, use a clear transparency laid over a transparency of the Met's first floor. As you construct the graph, emphasize the appropriate vocabulary.

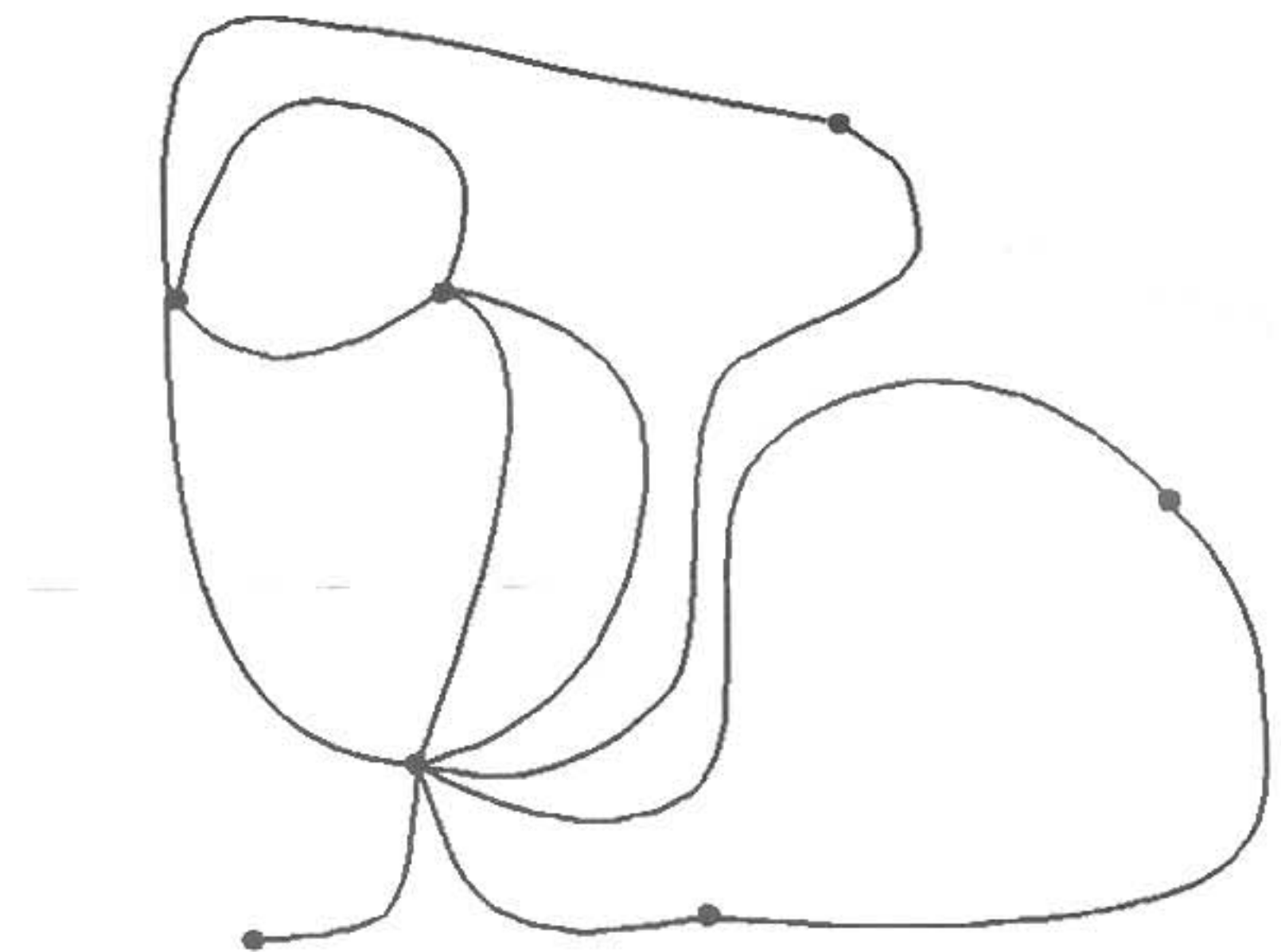
Steps: (1) Construct a vertex in each relevant room:



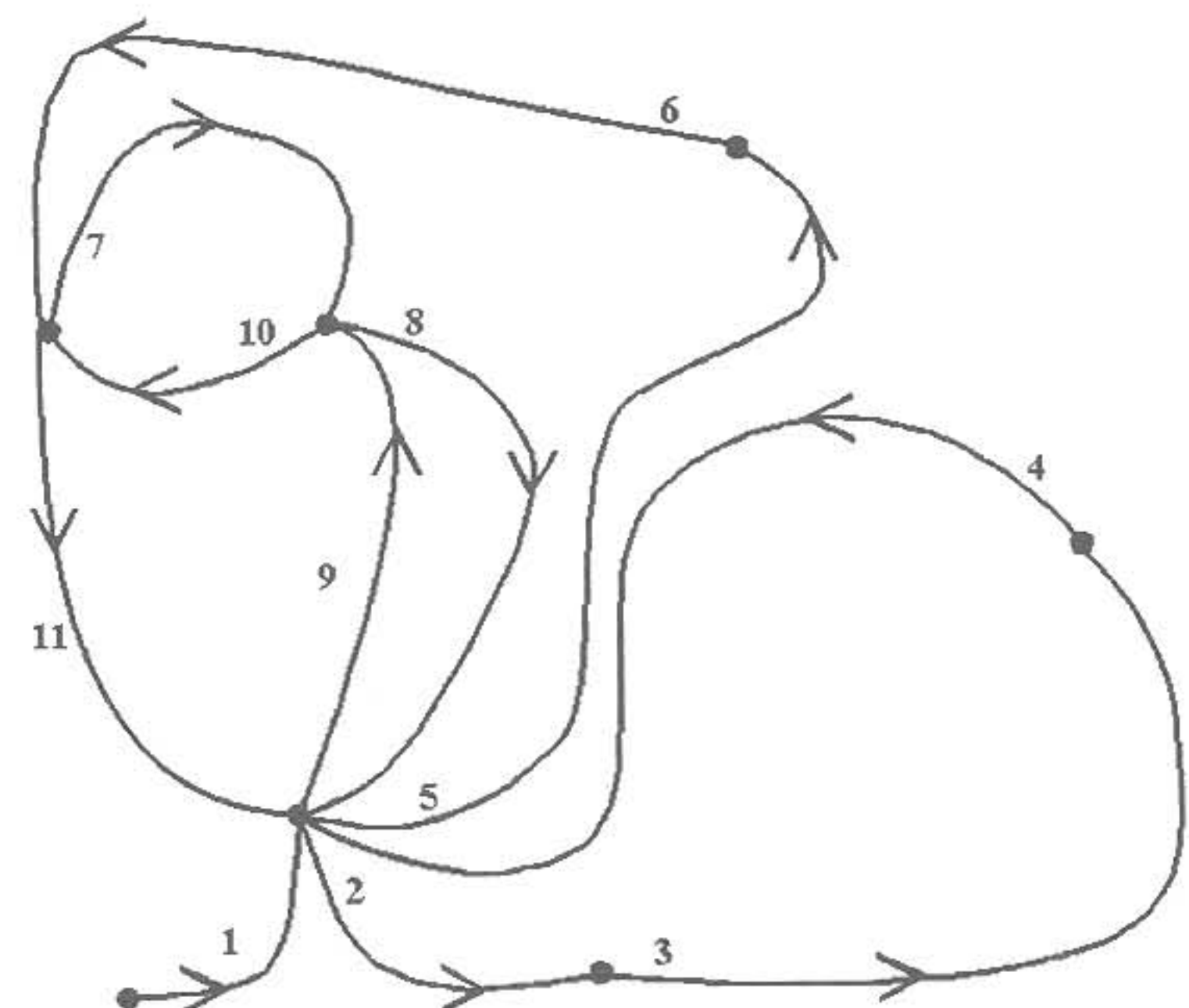
(2) Use edges to connect vertices between rooms which have doorways between them: *(at top of next column)*



(3) Remove the map of the Met from the overhead to reveal the graph, as shown below.



(4) Now ask students to trace their path on the graph. Elicit that a successful path should travel each edge exactly once. Circulate among pairs to ensure successful completion of this activity. Then show (or have a student show) how a successful path for Jamie can be shown on the graph, as illustrated below. You might draw their attention to the fact that the



Euler path in the graph is the same as the dotted-line path on page 2.

(5) Have students share other successful paths. Then review the terminology (*graph*, *vertex*, *edge*), if needed. Introduce *vertex of odd degree* and *vertex of even degree*. Elicit that this graph contains only two vertices of odd degree and that every successful path for this problem began at one of these vertices of odd degree and ended at the other.

Define *Euler Path* in reference to this activity; e.g., “What you have created here is known as an Euler path. Mathematicians define an Euler path as a path that includes every edge of the graph exactly once.” Pose the question, “Can you have an Euler path with more than two vertices of odd degree?” Students, of course, will need to explore with other graphs to answer this question. Summarize the activity and answer questions from students.

According to the level, knowledge, and experience of your students, drawing the map on a large tarp and constructing the graph using tape and large dots (small paper plates work well as vertices) is an effective way to carry out this activity. Having learners actually walk along the edges to show their paths is important. As an edge is traveled, place a marker of some kind on that edge, so that when the path is finished, everyone can easily observe that each edge has been traveled exactly once.

Many other books could serve as the springboard for a similar activity. Among appropriate titles are the following:

- Ahlberg, Alan and Janet. *The Jolly Postman*. New York: Wilteinemann Little, 1987.
- Burton, Virginia Lee. *Katy And The Big Snow*. Boston: Houghton Mifflin, 1943.
- Dubois, William. *The Twenty-One Balloons*. New York: Puffin Books, 1989.
- Fitzhugh, Louise. *Harriett the Spy*. New York: Dell, 1964.
- Juster, Norman. *The Phantom Tollbooth*. New York: Knopf, 1961.
- Lewis, C.S. *The Lion, the Witch and the Wardrobe*. New York: MacMillan, 1950.
- Lobel, Arnold. *Frog and Toad Are Friends*. New York: Harper Trophy, 1979.
- Lowry, Lois. *Number the Stars*. New York: Dell, 1989.
- Taylor, Mildred. *Roll of Thunder, Hear My Cry*. New York: Penguin Books, 1976.

The Discrete Reviewer

Janice C. Kowalczyk

Above-Average Discrete Math Titles

This column highlights four (somewhat) recently published books for discrete math educators and “wannabees”. The books I have chosen range from serious to the very playful. In this sample of four, there should be at least one match for anyone has an interest in discrete mathematics

Discrete Mathematics in the Schools

Joseph G. Rosenstein, Deborah S. Franzblau and Fred S. Roberts, eds.

American Mathematical Society & National Council of Teachers of Mathematics, 1997

ISBN: 0821804480

Cost : \$30

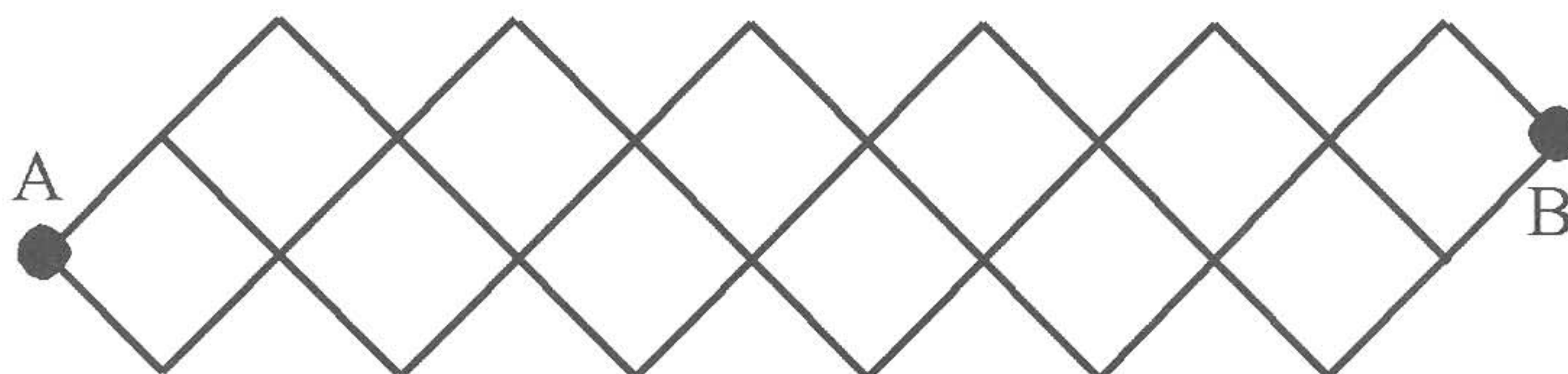
Grades: Anyone with an interest in mathematics education and mathematics reform

From the preface: “Discrete mathematics can and should be taught in the K-12 classrooms. This volume, a collection of articles by experienced educators, explains why and how, including evidence for “why” and practical guidelines for “how.” It also discusses how discrete mathematics can be used as a vehicle for achieving the broader goals of the major effort now underway to improve mathematics education.

“This volume is intended for several different audiences. Teachers at all grade levels will find here a great deal of valuable material that will help them introduce discrete mathematics in their classrooms, as well as examples of innovative teaching techniques. School and district curriculum leaders will find articles that address their questions of whether and how discrete mathematics can be introduced into their curricula.”

The complete preface, an introduction entitled *Discrete Mathematics in the Schools: An Opportunity to Revitalize School Mathematics*, a description of the organization of this volume and an annotated summary of the articles can be found at the Website:

<http://dimacs.rutgers.edu/Volumes/Vol36.html>



Interesting Answer

How many paths are there from A to B in the graph to the left, moving only in an eastward direction?

In addition, two practical articles from the book can be read in their entirety from the website: Joseph Rosenstein's article *A Comprehensive View of Discrete Mathematics: Chapter 14 of the New Jersey Mathematics Curriculum Framework* contains a comprehensive discussion of topics of discrete mathematics appropriate for each of the K-12 grade levels accompanied by numerous suggested classroom activities. An article by Deborah Franzblau and myself *Recommended Resources for Teaching Discrete Mathematics* identifies outstanding discrete mathematics resources for the K-12 classroom, including books, modules, periodicals, literature, Internet sites, software, and videos.

Discrete Mathematics in the Schools is of interest to anyone who wants to listen to voices of experience to understand how and why discrete mathematics can be used to improve mathematics education. This volume makes the case that discrete mathematics should be included in K-12 classrooms and curricula, and provides practical assistance and guidance on how to implement this goal.

Investigating with Power Solids

by Erica Voolich
Cuisenaire, 1997
ISBN: 1574520296
Cost: \$9.95
Grades: 6-8

This practical book contains sixteen classroom tested activities designed to be used with the Power Solid manipulatives available through the Dale Seymour or Cuisenaire catalogs. These activities help students discover relationships between shapes; surface area and volume; and three dimensional shapes and their corresponding two dimensional nets. Each activity is accompanied by comprehensive teacher's notes, discussion prompts and an explanation of the mathematics behind each task.

Discrete mathematics connections are nicely made. In this book you will find activities or extensions on Euler and Hamilton paths and circuits, eulerizing, instant insanity (vertex coloring), sorting activities, Euler/Descartes vertices formula, pentominoes & hexominoes, patterns in nets of solids, and number pattern activities. Erica, the author and a 1994 LP participant, managed to include some mathematics history, which is one of her favorite topics.

The Book of Numbers

by John Horton Conway and Richard K. Guy
Springer-Verlag, 1997
ISBN: 038797993X
Cost: \$29.00
Grades: HS, College, Teacher resource

The new book in the Copernicus series by Springer-Verlag takes a comprehensive look at different aspects and kinds of numbers. It contains many non-standard ideas, examples, and things to think about. Chuck Biehl (LP in DM '90), a teacher from The Charter School of Wilmington, reviews it for us:

"*The Book of Numbers* is an extremely pithy and engaging look at all kinds of numbers, from naturals to irrationals to transcendentals and beyond. Since many types of numbers exist only in the arena of pure mathematics, applications for the novice are a bit strained, but this does not prevent this wonderful book from being an excellent professional resource as well as an inspiration for teaching interested students at a deeper level about number patterns and properties using a broad level of sophistication. Its intense nature makes it very educational for student and teacher alike."

Cool Math: Math Tricks, Awesome Activities, Amazing Factoids and More

by Christy Maganzini
Price Stern Sloan Publishing Inc., 1997
ISBN: 0843178574
List: \$6.95
Grades: 3-8

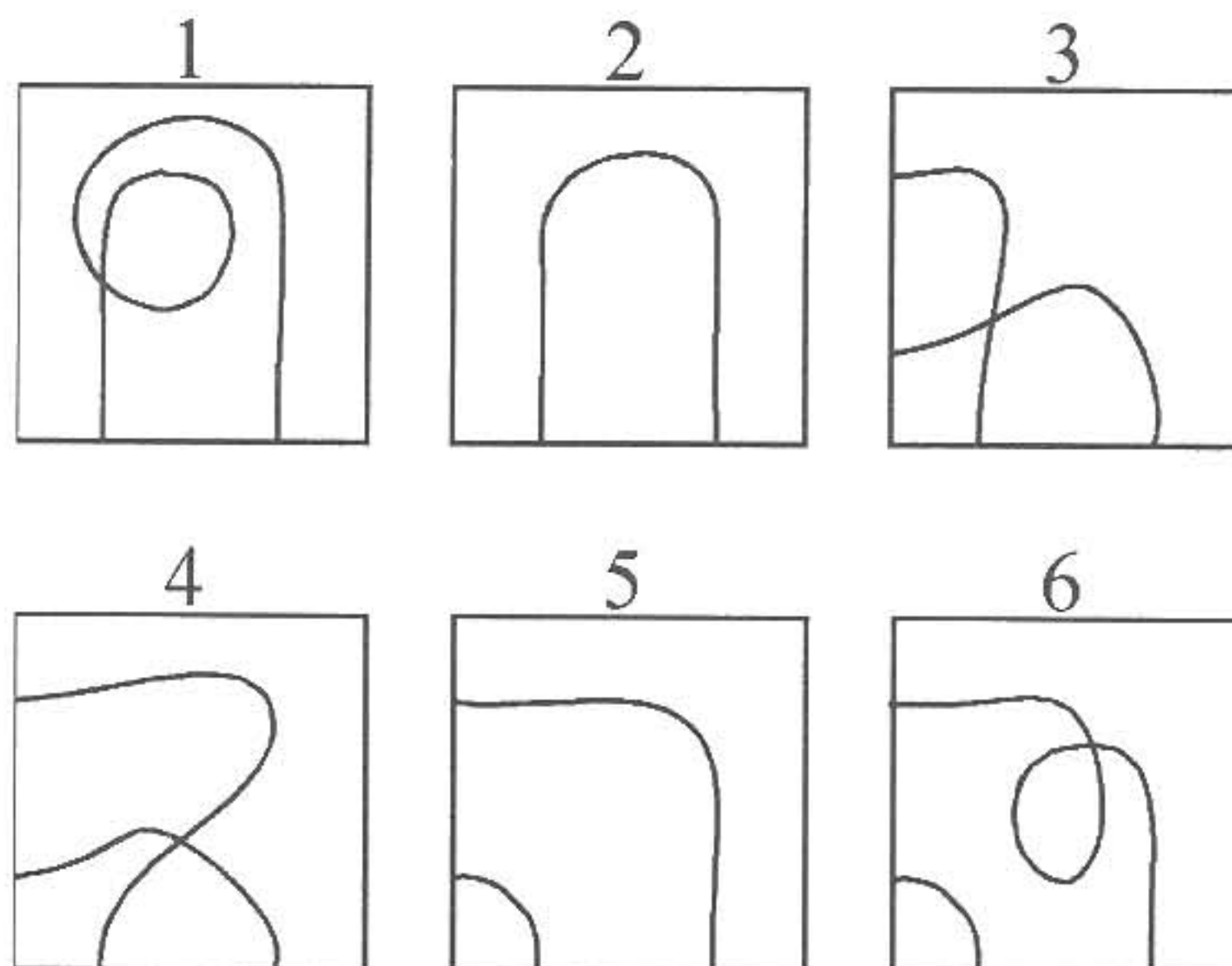
This interactive, playful introduction to the wonders of mathematics uses stories, history, a host of games, quizzes, hands-on activities, trivia, and more to explore some of the mysteries of mathematics and numbers. The book has many discrete connections including palindromes, counting systems, codes and ciphers, the Fibonacci Numbers, topology, Euler, the Konigsberg bridge problem, map coloring, probability, triangular numbers, square numbers and other number patterns.

I walked into the room where I was giving my discrete math presentation, and saw that the advertisement for my presentation had me talking about "discreet" math. So I quickly made a slide and put it up there..."You don't need to be discreet about discrete math."
—Linda Boland (LP '93)

Combinatorics, Euler, and Toy Tracks

Erica Dakin Voolich

There is a cute toy made by PLAS-TOY called Puzzle Vehicle Set. It consists of six interlocking track pieces, a wind-up vehicle to run on the tracks, and three traffic signs. The box begs a mathematical question: "Six inter-changeable puzzle blocks for over fifty ... different layouts." Of course, in order to answer the question of "how many ways" you need to define what counts as different ways. Note that the pieces are so shaped that you can connect any two pieces along any edge where both have tracks.

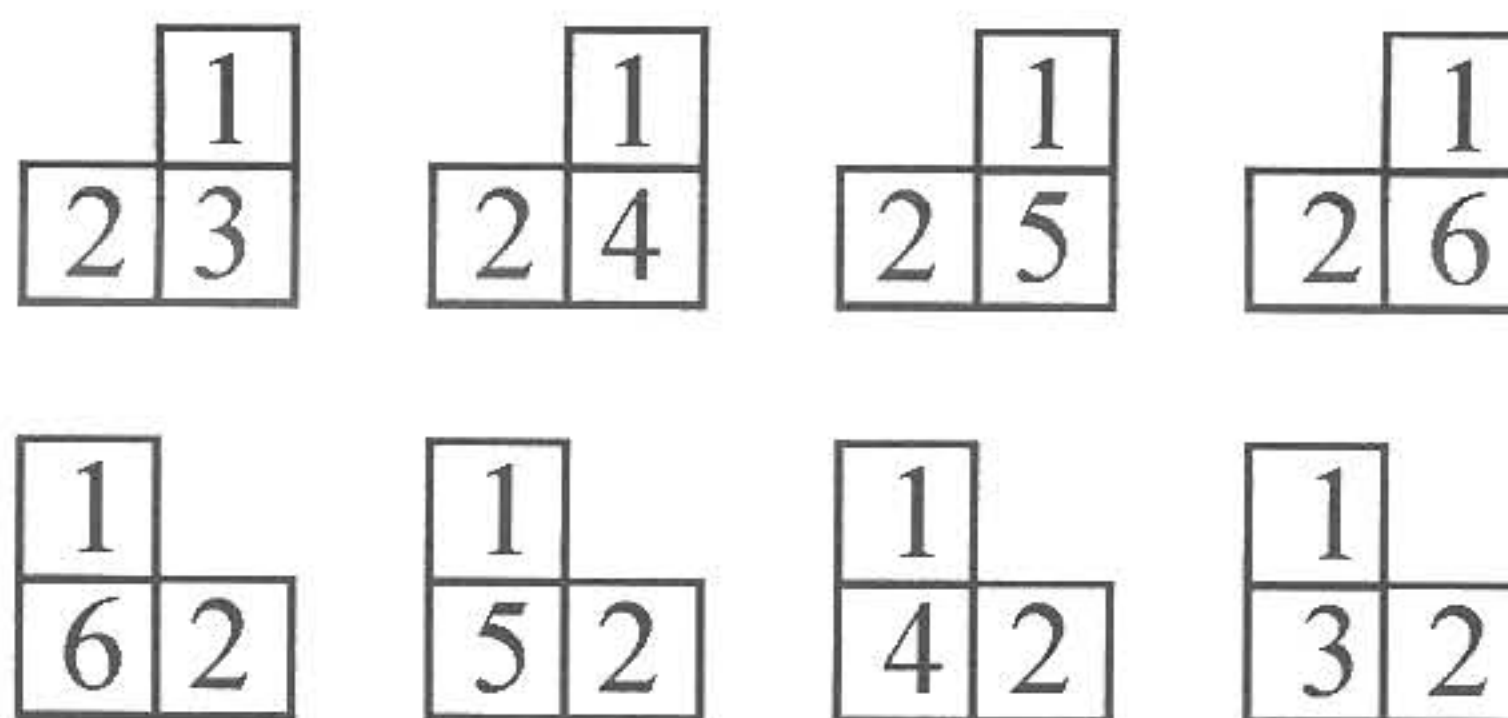


The six interlocking track pieces

copies of the numbered track pieces to cut out and move around.

At first glance, the solutions resemble various polyominoes. However, some polyomino arrangements are impossible to build with the tracks while maintaining a circuit. For example, it is possible to put two pieces together (shapes #1 and #2) $\square\square$ in a straight line and still have a circuit, but not three, because pieces #3, #4, #5, and #6 involve a 90 degree turn when $\square\square\square$ connecting.

Number the pieces (as shown to the right) and pass the toys out to groups of students with instructions to decide if the statement on the box is correct. Are there really over fifty different ways to put the track together? Initially, instead of defining what constitutes putting the track together legally, let the students explore and record what they find. Some groups may start out trying to draw sketches of track assemblies, others might work more systematically to try to come up with all the ways to put three pieces together. Some examples are shown to the right.

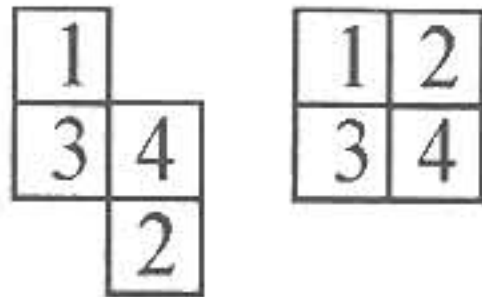


Eight ways to put three pieces together

Finding all of the layouts which include pieces #1 and #2 involves first finding the number of possible track-design layouts starting with #1 and ending at #2 (see Figure A, soon to be discussed), and then counting the number of ways each of those layouts can be obtained using shapes #3, #4, #5 and #6.

For example, if you have a three-piece track which uses #1 and #2, you can have two basic designs, as shown below. The question becomes "How many ways can you put one piece from #3, #4, #5, and #6 between pieces #1 and #2. If you wish to place one piece between #1 and #2 (as in the figure below) then you have 4 choices (#3, #4, #5 or #6).

After some initial exploration, my students agreed that:

- To count as a way to put the track together, the train has to be able to travel onto each piece without running off the track. These are similar to Euler circuits because it is possible to travel over the whole track ending up where the train started without going over any part of the track more than once.
- There are different ways to put the same pieces together, for example, at the right are two ways to put pieces #1, #2, #3, and 4 together. 
- Rotating a completed track layout is not a new way.
- If you use either piece #1 or #2, then you have to use the other one also.
- The answer is not 6! (many students' original guess) because you can put together fewer than six pieces and still get a track that the train will stay on.

Because the track pieces can only be put together sharing a whole side, and all of the shapes are numbered, students are able to record their solutions using graph paper. Students were encouraged to discuss how they could systematically find all of the arrangements. They each had

Refer now to Figure A, which depicts all the ways to lay out a connected track using the pieces #1 and #2. Notice how a tree diagram is used to systematically list all the ways, starting at the top with just the pieces #1 and #2. The arc in each square (except #1 and #2) indicates the two edges at which the tracks end in that square. Each row is obtained from the previous row by adding a new square in all possible ways. Then each row contains all track layouts which use a given number of pieces, including #1 and #2. For example, the bottom row shows 10 layouts using all six pieces.

There are 4 possible ways of having a four-piece track — see the third row of Figure A. In each, you have to place two pieces between #1 and #2. How many ways are there to do this? You have 4 choices for which piece comes first (meaning nearer to the end-piece #1) and then 3

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Credits...

This Newsletter is a project of the Leadership Program in Discrete Mathematics (LP). The LP is funded by the National Science Foundation and is co-sponsored by the Rutgers University Center for Mathematics, Science and Computer Education (CMSCE) and the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS).

Joseph G. Rosenstein is Director of the LP and Founding Editor of this Newsletter.

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Just send us your name, address, phone number and school.

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**IN DISCRETE MATHEMATICS:
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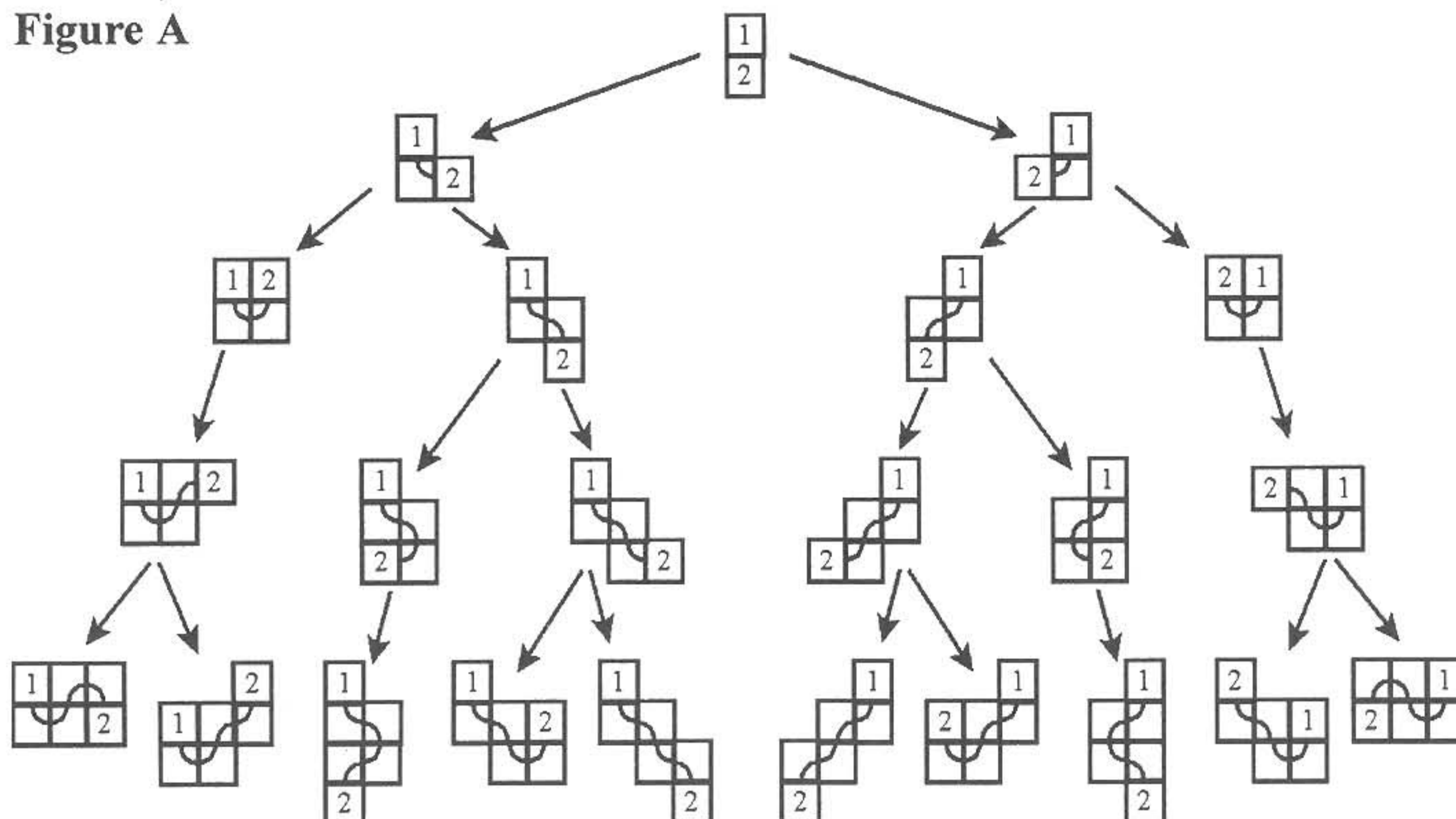
Please inform your K-8 colleagues about our

Summer Institutes in Discrete Mathematics for K-8 teachers

The Leadership Program in Discrete Mathematics will feature two two-week residential programs at Rutgers University, New Brunswick NJ, during June 28 to July 9 and during July 12 to July 23, and three two-week commuter programs at Scottsdale AZ (June 3-18), at Rutgers (June 28 to July 14), and in Rhode Island (August 2-17).

Participants will be expected to attend follow-up sessions during the school year and a one-week follow-up institute during the summer of 2000. Graduate credit will be available. Teams of teachers in schools or districts are welcome to apply. Funding by the National Science Foundation provides for all costs of the institutes and a stipend of \$600 for the two-week program. Participants will be expected to assume leadership roles in bringing discrete mathematics to their classrooms and schools, and in introducing their colleagues to these topics. For information, call Bonnie Katz, 732-445-4065, e-mail her at bonnie@dimacs.rutgers.edu, download the materials from <http://dimacs.rutgers.edu/lp/institutes/>, or write to: Leadership Program, P.O. Box 10867, New Brunswick NJ 08906.

Figure A



(continued from page 5)

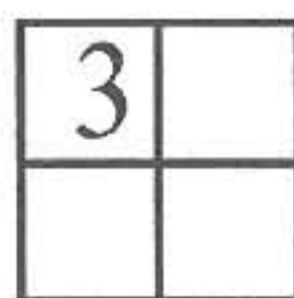
choices for which piece comes second. This yields a total of $4 \times 3 = 12$ ways to select the pieces. For three intermediate pieces you have $4 \times 3 \times 2 = 24$ ways, and for four intermediate pieces, there are $4 \times 3 \times 2 \times 1 = 24$ ways. Figure B combines this information with the number of track layouts to get the total number.

Number of Pieces	Number of Track Layouts	Middle Piece Arrangements	Total
2	1	1	1
3	2	4	8
4	4	12	48
5	6	24	144
6	10	24	240

Figure B

Thus there are $1+8+48+144+240=441$ different track layouts using pieces #1 and #2.

If you do not use pieces #1 and #2, you can put the other four pieces together to form a square. Consider choosing piece #3 for the upper-left corner (and note that there is only one orientation for this piece which will yield a continuous track): There are $3 \times 2 \times 1 = 6$ ways to put the other three pieces into the square. For each of these, there is only one rotation of each piece which will yield a "legal" track, so this square shape yields 6 new arrangements. Thus the total number of tracks, in which the train can travel over each square in the arrangement, is $441+6=447$.



It is quite possible that the only layouts that the puzzle makers would consider acceptable are those in which the six pieces form a rectangle. In that case, there are two lay-

outs — at the extremes of the bottom row of Figure A — and each can be realized in 24 ways, for a total of 48 layouts — not quite the "over 50" that they advertised.

If we change the rules to our game just slightly, the answer will be larger than the 447 found above. We required that the train must be able to reach each square in the layout. What if we drop that condition? The track pieces would be locked together but the train would not be able to cross each piece. An example is shown below.

In addition to the 447 ways already found, we have those arrangements, as in the example here, which are formed by placing a #1-#2 unit along a side of one of the 6 square patterns.

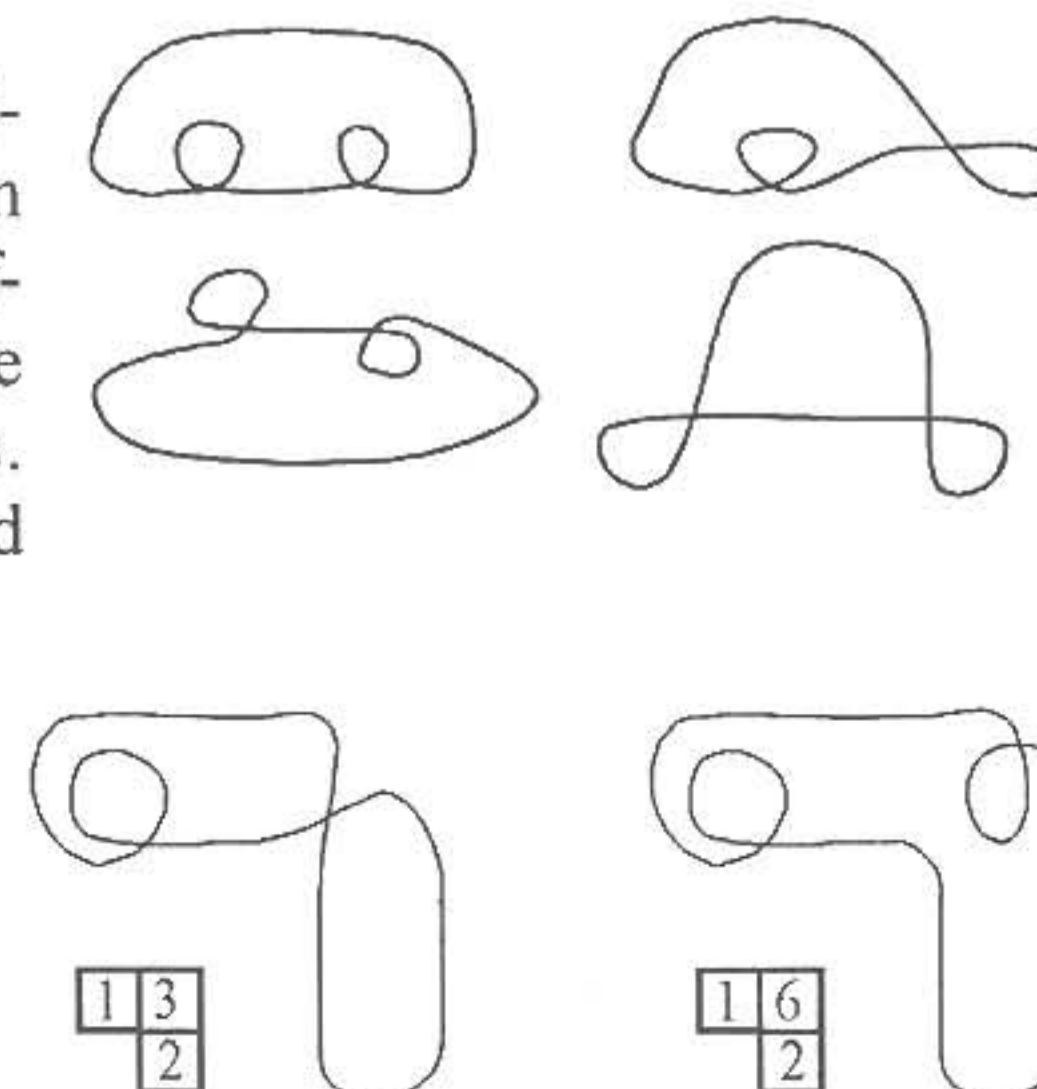


There are 4 sides to the square, and the #1-#2 unit can be placed two different ways on each side of the square, so there are $6 \times 4 \times 2 = 48$ new such ways. So with these relaxed rules we have $447+48=495$ ways to assemble the track pieces.

On the other hand, if you want to define "different layouts" to mean "essentially different graphs which are circuits," then it becomes a totally different question. For example, we can define *vertices* to be places where two tracks cross and *loops* as track sections that begin and end at the same intersection without going through another intersection. We can then say that in order for two layouts to be

different, they cannot have the same number of vertices which are connected in the same way and which have the same number of loops at each vertex. For example, these four graphs are essentially the same:

With the definitions above, one can count the essentially different circuits that can be built using the tracks. For example, if you build a circuit using pieces #1-#3-#2 and another using #1-#6-#2, they are each made up of



(continued on page 11)

Coloring in Kindergarten?

Judith Gugel

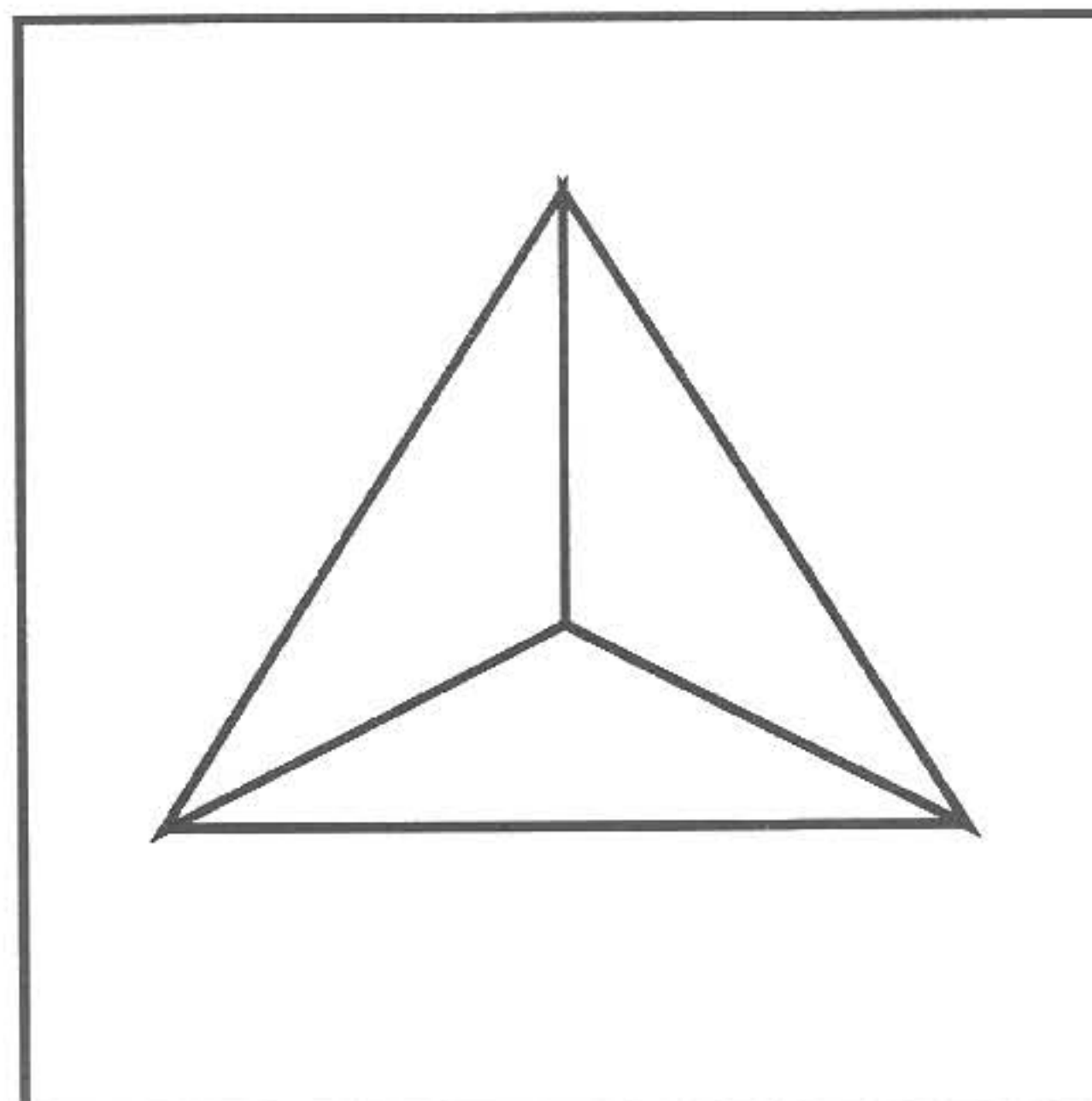
I am so excited with what is happening in the kindergarten classes regarding map coloring/conflict resolution skills. When I first introduced this concept I did it with the intent of creating an AB pattern. Another focus I had was the concept of same and different (without one of these concepts you can't successfully be shown how to color two regions). After some work with AB patterns I decided to introduce the coloring of a picture. One of these pictures goes home with the children for homework once a week.

Prior to sending the picture home, I do the coloring with the children on the overhead. I ask the children to try to color the picture with only two colors if possible. As a class we decide what to do with each region of the picture and we try to limit the number of colors. Coloring the pictures with the children ahead of time helps the children to succeed with their homework (unless a parent convinces them otherwise!).

Up until yesterday we had completed approximately 4 of these homeworks. Yesterday, as usual, I placed a picture on the overhead. This time I asked the children to take a few moments to look at the picture to predict if they could color it with only two colors.

After a few moments of silence I asked the class if they indeed felt we could color the picture with only two colors. As I expected, most said "yes," but some said "no." I asked one child why he said "no." His response was, "You need three colors." I thought that maybe this was a lucky guess so, as I always do, I asked him why he thought that we needed three colors. He answered, "Well, there are three things that touch."

Now I was becoming excited so I had him go up to the screen with a pointer to show me what he meant. Sure enough he showed me three regions that were touching, and we discussed why that meant we needed at least three colors. We used that as our starting point for coloring the picture and found that indeed, we needed three colors. I then asked the children "what would happen if I put a square around this picture" and Tammy said we would need a fourth color and was able to go to the screen to



justify her answer. All of this was really exciting, and some of the children are really getting it. While many are being successful when *doing* the coloring I can now see that some are developing *strategies* in order to succeed.

DIMACS RESEARCH & EDUCATION INSTITUTE

Rutgers University - New Brunswick, New Jersey

July 19 - August 6, 1999

An invitation to Middle Atlantic high school teachers of mathematics and science to participate in a three-week program on...

Graph Theory and its Applications to Problems of Society

The intent of the NSF-funded DIMACS Research and Education Institute (DREI '99) is to integrate education and research in the mathematical and computer sciences. The institute will invite many of the world's foremost researchers in graph theory and combinatorics to participate. In addition to giving technical talks in their areas of expertise, invited researchers will also give lectures designed for a more general audience. Participating teachers will have special classes in graph theory and will be introduced to related software. Formal and informal interaction among all participants in the Institute will be encouraged. Applicants who are selected will be fully supported and will receive a stipend; they will also receive three graduate credits in mathematics education if they complete all requirements. Details available at <http://dimacs.rutgers.edu/drei/1999/> along with the application form. Or you can call or email Elaine Foley at 732-445-4631 or elaine@dimacs.rutgers.edu.

Coloring in Grade 1

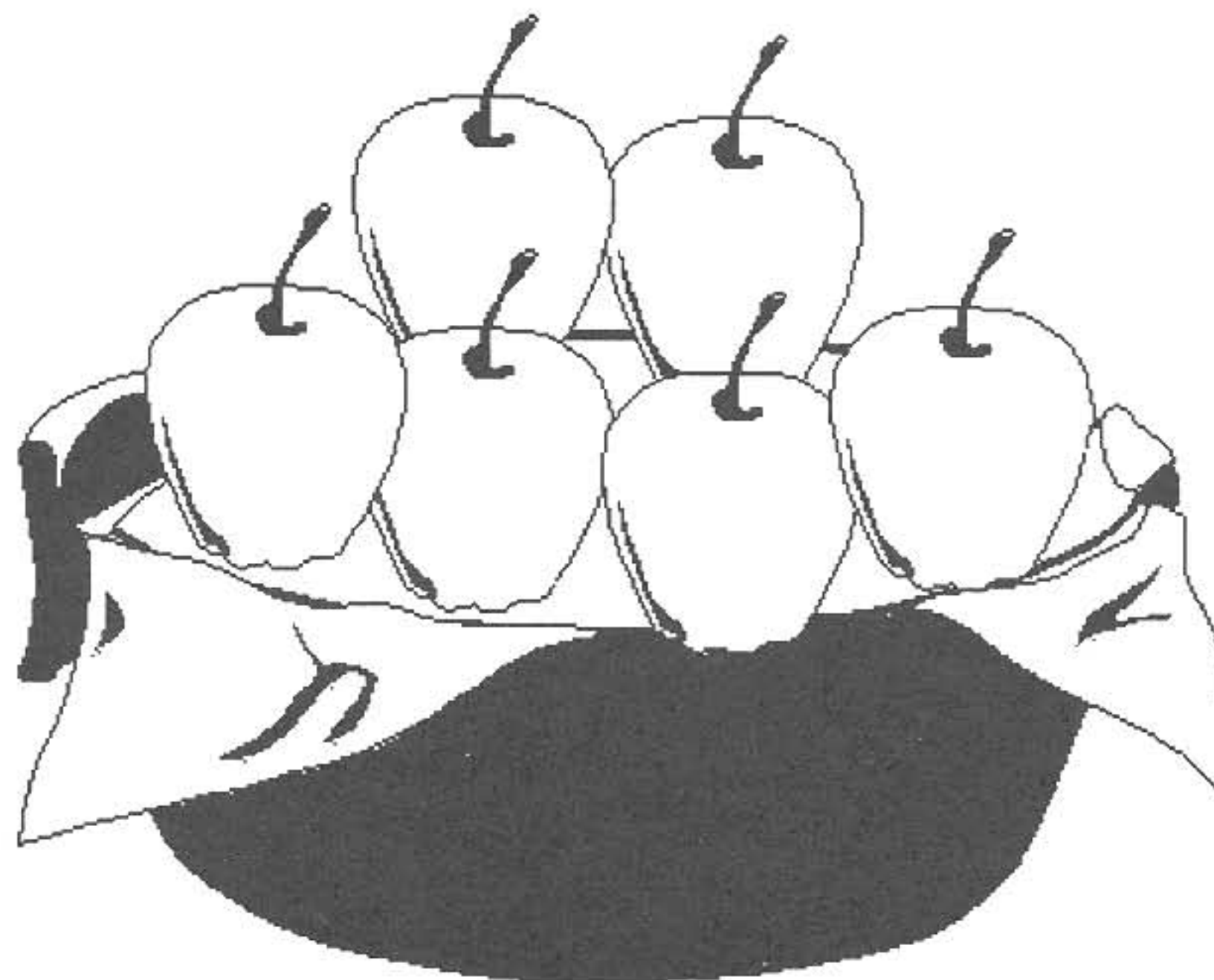
Judy Grogan

Teaching first grade is always a challenge, and I am continually looking for new ways to motivate my students. When a colleague suggested I take a Discrete Math Course with her, I jumped at the chance. I learned a great deal and found the course packed full of information, but I'm not sure how much of it can be simplified enough for first grade.

I work in a suburban school district and have 24 children with a range of abilities. I also have two students who come to me for math that are mainstreamed from a perceptually impaired class.

My first area of choice was graph coloring. Most children come to first grade with a love of coloring. After spending the first few weeks on color recognition we were ready for our first exercise: coloring apples and placing them in baskets so that two apples of the same color didn't touch.

It's been my experience that telling a story is a good way to motivate children. I made up a story about Farmer Brown who grew and sold apples. He grew three kinds: red delicious, granny smith, and golden delicious. (At this point I drew one each of a red, green and yellow apple on graph paper so demonstrate what they would look like.) He needed to find a way to show off his apples so that people would buy them. One way would be to put them in a basket so that each apple would stand out, and not touch one that was the same color. I told the children that they would be his helpers, and that we would figure out a way to solve his problem.



Each child was given a worksheet with six apples on it and a piece of paper with a basket drawn on it. They were directed to color the basket, and to color two of the apples green, two red, and two yellow. Then they could cut them out and arrange them. They could work either in pairs or alone. This part of the lesson was very successful, although some of the children had a hard time figuring out how to arrange their apples. Once several of the more advanced students figured out a good way, then everyone wanted to do theirs the same way. (The children were not provided the picture on this page.)

My next step was to call the children together to discuss how we had solved the problem. Children were invited to share their apple baskets. Finally we drew all of the different arrangements that the

children had made on chart paper, using markers.

This lesson was followed by a similar one using balloons, and finally one on coloring a partial map of the United States. All of these worked quite well.

I think that primary teachers have to remember that they are introducing the foundation for the upper grade teachers to build on. If we keep it simple then it will be fun and the children will develop a love for problem-solving using the tools that we give them. It is my hope that they will consider math one of their favorite subjects throughout their school years.

Leadership Program in Discrete Mathematics Crash Course for High School Teachers

The Leadership Program in Discrete Mathematics will offer a two-day crash course in discrete mathematics for high school teachers at Rutgers University on August 11-12, 1999. The content will include paths and circuits in graphs, patterns in numbers and geometry (fractals), voting methods, and codes. The anticipated cost of the program is \$150 (including dinner and lodging on August 11); the cost may be reduced if grants are obtained. For information, call Bonnie Katz, 732-445-4065, email her at bonnie@dimacs.rutgers.edu, download the materials from <http://dimacs.rutgers.edu/lp/crash-course/>, or write to: Leadership Program, P.O. Box 10867, New Brunswick, NJ 08906.

The Farmer's Daughter

Jill Dunlap

Once upon a time there was a farmer who had two strong sons and a young daughter. His daughter was known throughout the land as a deep thinker and an excellent problem-solver. Well, one day the farmer decided to take his small farm and divide it into eighteen different fields for each of which he would purchase some livestock. So, while the farmer and his two big strong sons were busy building fences, he gave his daughter the following problem to solve so that when the fencing was completed he would know exactly **how many** of each different kind of livestock he needed to buy, and **where** he should place them.

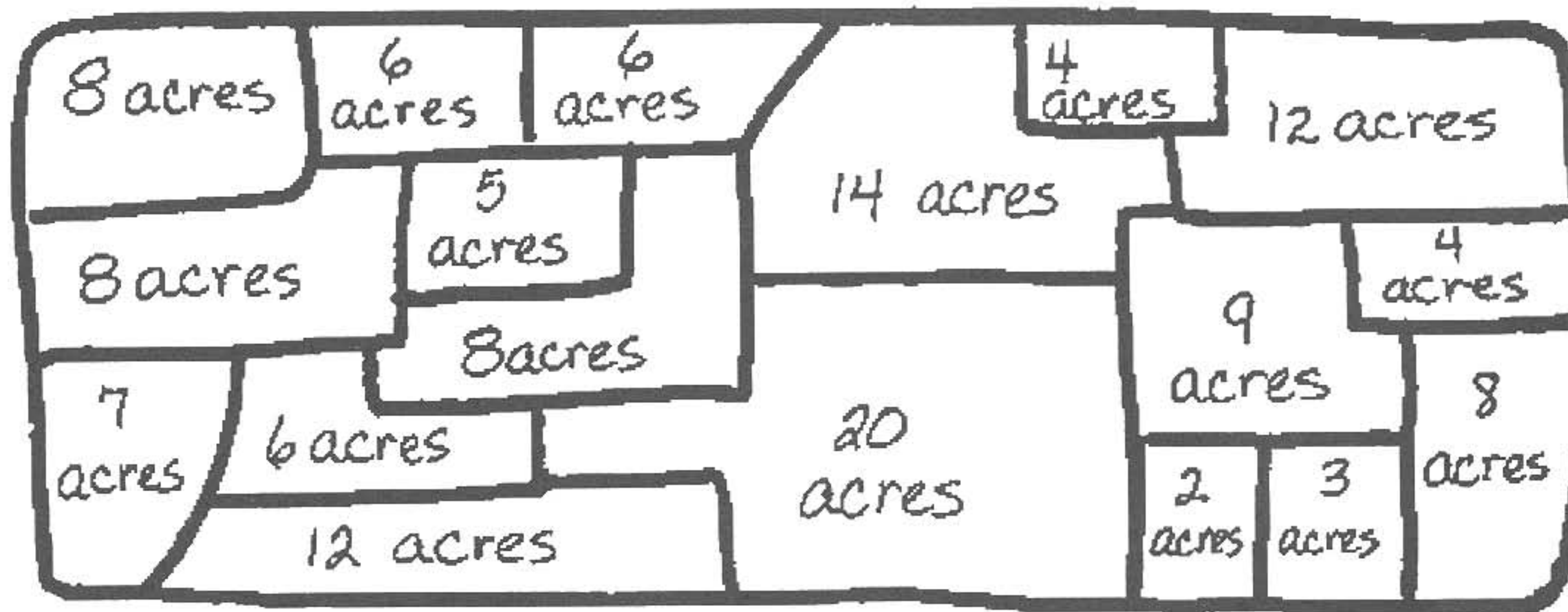
While the farmer enjoys many animals, he wants to limit his farm to as few different animals as possible. Now, everyone knows that a good farmer does not put like animals in pastures that share a common fenceline, for they will surely try to tear down the fence in order to join their friends on the other side. He also had a priority listing of preferred livestock, so that he wanted:

1. As many acres of cows as possible
2. Then as many acres of sheep as possible
3. Then as many acres of horses as possible
4. Then as many acres of goats as possible
5. Finally, as many acres of pigs as possible.

Note that he could actually end up with more sheep than cows, that is okay...as long as cows occupied the most acreage.

The farmer decided that each acre could hold either 8 cows, 20 sheep, 6 horses, 12 goats or 18 pigs. With these rules established, he provided his daughter with a diagram of the farm (see figure).

The daughter was given two weeks to work on this problem. At the end of this two-week period, she was to



present to her father:

1. A plan for which type of animal would be assigned to each of the 18 fields, and how many of each type of animal he should buy.
2. A written explanation as to why her plan works and is the best solution to the problem.

Now, **YOU** are given two weeks to work on this problem!

(This project was given to my 6th grade talented class following a unit on map-coloring. I have never seen students attack a project as they did this one. One thing out of the ordinary that I did was to encourage parent and family involvement. The response from the parents was great! One mother told me that she and her husband worked on this problem for over an hour one evening while their son was not home, and were sure that they had arrived at the best-possible solution. When the son returned, Dad told him that he had better get busy on this problem because it was not as easy as it appeared. The young man sat down and promptly produced an answer that bettered Mom's and Dad's in just five minutes. Mom said it was a "humbling experience!"

The students' tasks were to 1. decide on the smallest number of different animals that are needed to meet all of the requirements, 2. decide how to arrange the animals in the fields so that the preferred animals got more acreage, 3. design a clear way of displaying which animal was to be located in each field, and 4. determine the number of livestock of each type that the farmer should buy.)

A Discrete Challenge

The editors of *In Discrete Mathematics* invite readers and their students to come up with their best solutions to *The Farmer's Daughter* problem. The solution with the most acres of cows will be the winner, with ties broken by the most acres of sheep, then horses, goats, and pigs. The best solution received by April 1, 1999 will appear in the next issue of *In Discrete Mathematics*!

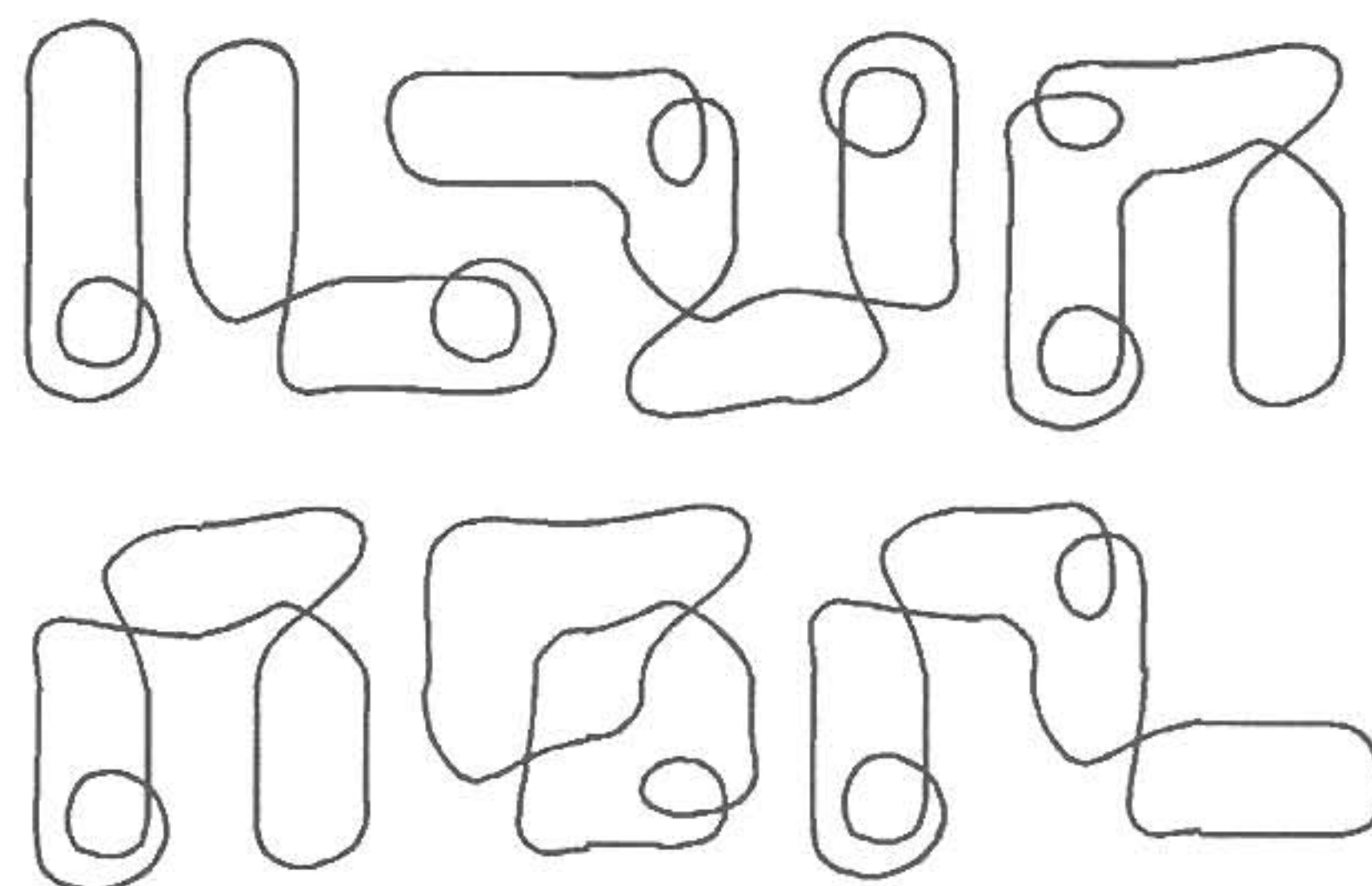
-eds.

(continued from page 7)

two vertices with two loops, and the vertices are connected to each other by two edges. They appear different but are essentially the same, as shown on page 7.

With some experimentation, your students can discover that there are altogether seven essentially different graphs, as shown at the right.

So the answer to our original question can be either 447, 489, 48, or 7, or even something else, depending on your rules as to what constitutes an "allowed" arrangement.



One of my students presented me with a gift last June...a chalk-holder..."so you won't get chalk all over you when you write at the board..." (I am notorious, especially when wearing black!) I noticed it was well worn with a used piece of chalk, and graciously said "thank you." She said, "It was my father's...and I wanted you to have it..." He was a college professor who died last fall...My eyes welled with tears

— Mary Ann Harasymowycz (LP '97)

Please tell your colleagues about...

Workshops in Your District

Discrete Mathematics for K-4, 5-8, and 9-12 Teachers

EXPLORING DISCRETE MATHEMATICS: Would you like the teachers in your school or district to become familiar with topics in contemporary mathematics and use them in your classrooms? If your answer is "yes", consider hosting a workshop on discrete mathematics in your district.

WORKSHOPS IN YOUR DISTRICT! A trained workshop leader will come to your school or district to lead hands-on interactive workshops on contemporary topics in discrete mathematics. Full-day or half-day workshops are available; workshops deal with a variety of topics in discrete mathematics and are addressed to a variety of grade levels of teachers.

WHEN? Workshops will be scheduled on an individual basis at your request. They may be scheduled during the school year or during the summer.

BY WHOM? These workshops are presented by experienced workshop leaders who have participated in the Rutgers Leadership Program in Discrete Mathematics and have used the workshop materials in their own classrooms.

FOR FURTHER INFORMATION, INCLUDING FEES: Contact Program Coordinator Debby Toti by email at toti@dimacs.rutgers.edu or by phone at 732/445-4065, or check the web at <http://dimacs.rutgers.edu/lp/workshops/>.

If you know a high school student who would enjoy an enjoyable and challenging month attending a mathematics summer camp, suggest that he or she apply to the

Rutgers Young Scholars Program in Discrete Mathematics

For whom: This residential program is for talented students currently in grades 9-11.

When and Where: July 12 to August 6, 1999 at Rutgers University, New Brunswick, NJ. The program goes from Monday to Friday. Most students will go home each weekend, although activities will be arranged for those who wish to stay.

Program: Students will participate in workshops in discrete mathematics with college faculty, and in research projects, field trips, computer activities, and workshops on careers.

Cost: \$1900, including tuition, materials, housing, and food (weekends extra). Scholarships are available. The program is supported by DIMACS and the AT&T Foundation.

For information: Call or email Lisa Estler at 732/445-4065 or estler@dimacs.rutgers.edu, or check the web site <http://dimacs.rutgers.edu/ysp/>. For early admission, apply by March 26; the Application Kit includes solutions to a set of problems.

DIMACS—DM NEWSLETTER

In Discrete Mathematics ...

... Using Discrete Mathematics in the Classroom

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