IN DISCRETE MATHEMATICS

Using Discrete Mathematics in the Classroom

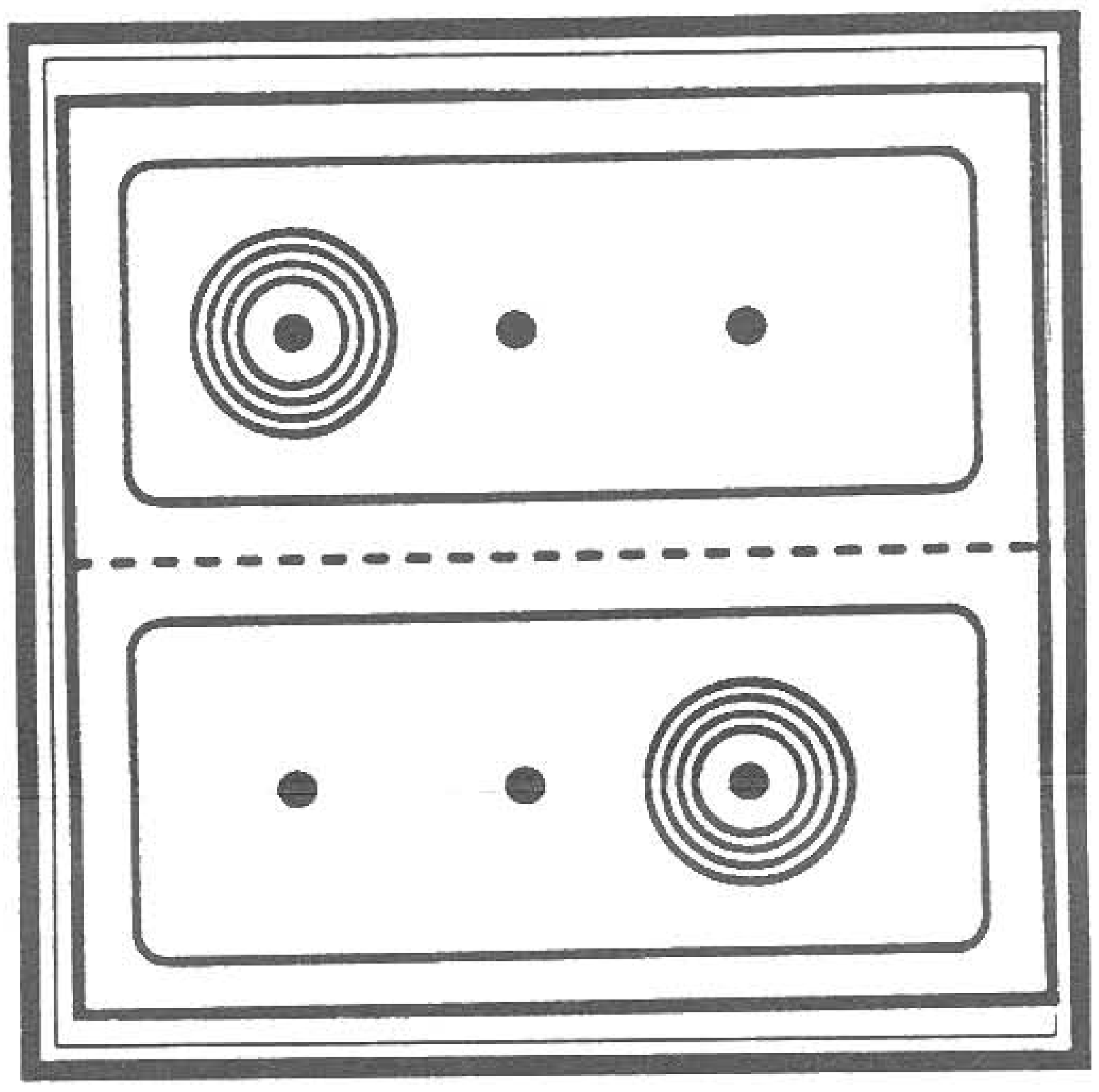
Issue #4 *June 1994*

Speaking discretely...

by Deborah S. Franzblau

This is the fourth issue of the Newsletter, and my first as your new editor. I hope you will shower me with your contributions: articles on classroom activities, outlines for new courses, book reviews, reports on software, cartoons, bibliographies...

In this issue, you will find several excellent teaching ideas from participants in the Rutgers Leadership Program in Discrete Mathematics. All of these address the question "How can I use Discrete Mathematics Topics in Algebra or Basic Mathematics?", and require only simple materials like maps (p. 4), junk mail (p. 5), or old boards (p.7), along with a bit of imagination.



Also featured are two articles by participants who have played important roles in developing new discrete mathematics curricula: Lina Bowyer in Tennessee, and Susan Picker in New York City (p.2). Joseph Malkevitch takes a look at the history of mathematics courses for liberal arts students, and the new role of discrete mathematics in such courses. This issue also features a special section on news from DIMACS, with an article on a breakthrough on the Traveling Salesperson Problem by researchers here last summer (p.3).

We are again including information on the program "Workshops in Your District," one-day workshops given by Leadership Program participants (p. 11). Feel free to copy this page to use as a flyer to give to colleagues and distribute at conferences.

In This Issue

Discrete Mathematics for Liberal Arts: A Historical View

by Joseph Malkevitch

At the college level, not only has Discrete Mathematics emerged as an important area in the training of computer scientists and mathematicians [3, 7], but it is evolving to take on a crucial new role in the teaching of mathematics to liberal arts students. The appearance of the text "For All Practical Purposes" [2], first published in 1988, has, I believe, played a key role. In order to understand this new development it is useful to take a brief look at the history of the liberal arts mathematics course since World War II.

Just after the War, if there was any mathematics requirement, it was a course called Mathematical Analysis, a mixture of algebra and elementary differential calculus, taught without much emphasis on theory or applications. The course did not serve liberal arts students well, however, and two new types of courses were developed. One course, pioneered by John Kemeny and J. Laurie Snell of Dartmouth, was called Finite Mathematics [5, 6]. The other course, harder to label and characterize, I will call the Topics Course [1, 8].

Finite Mathematics broke new ground with its content, which included sets, logic, matrices, probability, and graphs. However, just as important was its emphasis on the application of this material to problems in business, life sciences, and social sciences. Finite Mathematics caught on slowly, but, by the late 1960's, Finite Mathematics books, all variations on the Kemeny/Snell text, dominated the market.

The Topics Course, on the other hand, aimed to show students the beauty and elegance of mathematics, but with little attention to applications. One of the best of the texts for this course was Sherman Stein's *Mathematics, The*

(Continued on page 8)

New Discrete Mathematics Courses in Tennessee...

by Lina Bowyer

Two years ago, the Principal and the Math Department Chair asked me to develop a one-semester course in Discrete Mathematics to complement our existing semester course in Statistics. My background at the time did not include any courses in Discrete Math. Luckily, I found out about the Leadership Program in Discrete Mathematics offered at Rutgers University, which has proven to be an invaluable resource to me in developing and implementing this course. I piloted a course, which I have taught for two years, using early versions of COMAP's new text, *Discrete Mathematics Through Applications* [1]. Now, the state of Tennessee is in the process of adopting Discrete Mathematics as a one-semester course at the high school level; through a combination of lucky circumstances, I am responsible for developing the curriculum guidelines and piloting the program for the whole state.

I teach at a public magnet school for academically talented students, where we require all students to take Math and Science for four years. Discrete Math, Statistics, and Calculus are all offered to those students who have completed or are taking Advanced Math/Trigonometry. The enrollment for the Discrete Math course has been 40-45 students each year, taught in two sections.

The first time I taught the course, after surveying several Discrete Math texts, I chose COMAP's text because it included most of the topics I wanted to cover, and I liked its emphasis on student discovery, projects and computer explorations. The curriculum I designed for the course centers on four main themes: Graph Theory, Social Decision Making, Matrix Modeling, and Recursion. My first syllabus included three 6-week sessions on the first three topics, with only a brief look at recursive processes. One of the highlights was a unit on voting theory which corresponded nicely with the presidential election of November 4, 1992. To complement this unit, the Government teacher and I coordinated a survey which was written, conducted, and analyzed by both of our classes.

The second time I taught the course, after having taken Terry Perciante's unit on fractals during the summer, I included much more on fractals and chaos, using the workbook, *Fractals for the Classroom* [2]. (See the sidebar for the current syllabus.) This unit was great fun to teach and a favorite among students:one of the highlights was constructing a 4-foot-high, stage-5 Sierpinski tetrahedron!

In teaching these topics I have found that I need different methods for evaluating student work than those I had used in other math courses. I let the learning process center more on student discovery and less on lectures. The students do a lot of work in cooperative groups. Part of the grade is based on participation in class discussions and problem-solving activities. I give writing assignments on homework and tests, which has been a challenge both for me and the students! I also require each student to complete a six-week project which further explores a topic we have covered in class.

As their final semester project, I ask students to put together

(Continued on page 5)

One-Semester Discrete Math Course

I. Graph Theory (6 weeks)
Euler and Hamilton paths and circuits, critical path analysis, graph coloring, scheduling, minimum spanning trees, Steiner points, traveling salesperson problem, and shortest paths.

II. Matrix Models (4 weeks)
Leslie model, Leontief input-output models, Markov chains, game theory, and cryptography.

III. Fractals and Chaos (4 weeks)
Chaos game, basic fractal curves, fractal dimension, iterated function systems, cobweb diagrams, Mandelbrot and Julia sets.

IV. Tesselations (1 week)

V. Social Decision Making (3 weeks)
Election theory, voting power, fair division, and apportionment.

...and New York City.

by Susan Picker

All students in New York City must now complete 3 years of mathematics to meet new diploma requirements. However, there are many students who have completed two years of Algebra, yet are ill-prepared for or have not passed the first term of the third year in the Algebra sequence. The Manhattan Superintendent, Patricia Black, has now given the go-ahead to Manhattan to create a completely new course in Discrete Mathematics to fill the need for another third-year course. Manhattan will be the first borough to implement such a course.

The curriculum committee of the Office of the Superintendent is currently preparing a Discrete Mathematics curriculum for implementation in September 1994. The committee includes six alumnae of the Leadership Program in Discrete Mathematics at Rutgers University. This two-semester course is intended as an academic college preparatory course. The syllabus will include such topics as voting, apportionment and fair division, graph theory and coloring, coding theory, combinatorics, matrices, algorithms, number sequences and recursion, introduction to fractals and chaos, and tesselations.

The proposed new Leadership Program for 1995 will include a special section for teachers implementing curricula like that of Manhattan.

DIMACS Researchers Win a Gold Medal in TSP Competition

by Fred Rispoli

The traveling salesperson problem (or TSP for short) is to start from a given city, visit all the cities on a particular list, and return to the initial city by traveling the shortest possible total distance. For over a decade the TSP has received an enormous amount of attention in the Discrete Mathematics and Operations Research literature. Recently many computer codes for solving the TSP have been tested on a standard set of problems or "benchmarks" known as the TSPLIB. This practice is reminiscent of a chess tournament in which computer algorithms compete against a select set of expert players, in order to rank different algorithms. Some of the TSPLIB problems were obtained from recreational sources, such as the problem of finding knight tours on a chess board. Others were obtained by considering sets of cities together with the actual distances. Still others were obtained from applications such as drilling holes on a circuit board and from setting x-ray crystallography equipment.

In the spring of 1992, a team consisting of DIMACS members David Applegate (AT&T Bell Labs), Vasek Chvatal (Rutgers) and Bill Cook (Bellcore) along with Bob Bixby (of the Center for Research on Parallel Computation, Rice University) claimed a gold medal by solving fifteen previously unsolved problems from the TSPLIB with sizes ranging from 417 to 3,038 cities. The largest previously solved TSPLIB problem had 2,392 cities. In June 1993, the team broke its own record by solving a TSPLIB problem with 4,461 cities using a parallel implementation. The computations took 27 days on a network consisting of 75 processors working in parallel (the estimated running time on a single workstation comes to about eighteen months). The main ideas used were inspired by a landmark 1954 paper of Dantzig, Fulkerson and Johnson [1] who used "cutting planes" to convert fractional solutions from the associated linear programming problem to integer solutions, and were the first to solve a non-trivial TSP with 48 cities (see below for an explanation of cutting planes). To make this approach work on very large -scale problems the DIMACS team developed a fast technique to identify the cutting planes, and a "shrinking strategy" for dividing the problem into smaller subproblems.

Somewhat surprisingly, real-life applications of the TSP have not been numerous. So why all the attention? Perhaps the best explanation is that the TSP is a problem that is natural and easy to state, but so hard to solve that it has given new meaning to the phrase "hard problem". Just ten years ago researchers used to only dream about solving some of the larger problems in the TSPLIB that can now be solved in hours, making the problem an irresistible challenge.

References

1. Dantzig, G., Fulkerson, R., Johnson, S., "Solution of a large-scale traveling salesman problem," *Operations Research,* Vol. 2 (1954), 393-410.

2. Kolata, Gina, *Math Problem, Long Baffling, Slowly Yields,* New York Times, March 21, 1991.

3. Michaels, J., and Rosen, K., eds., "The Traveling Salesman Problem" in *Applications of Discrete Mathematics,* McGraw-Hill, 1991.

Cutting Plane Method for Solving the TSP

A full description of cutting-plane methods is quite difficult, however the intuition behind the method is just "successive approximation." The first step is to convert the TSP into a vector problem. Assume that there are n vertices (cities), and for each edge {i, j} between vertices i and j there is a cost c_{ij} (distance). A traveling salesperson tour, or TS tour, is a cycle that visits each vertex exactly once (a Hamiltonian cycle). The *best* TS tour is the one with the minimum total cost (sum of costs of the edges). Now, think of {i,j}and {j,i}as different edges; for each edge {i, j} introduce a variable x_{ij}. Let $x_{ij} = 1$ mean that edge {i, j} is used in the traveling salesperson tour and $x_{ij} = 0$ mean that {i, j} is not used. Create a vector of 0's and 1's ordered lexicographically where, for example, x_{12} precedes x_{14} and x_{24} precedes x_{31}. Then, each tour can be represented as a 0-1 vector of length n(n-1).

A small example showing how to represent a tour as a vector is shown on page 6. Assume that n = 4 for the moment.

Certainly, not all 0-1 vectors represent tours. To represent the problem correctly, we must add a number of geometric constraints. For example, $x_{12} + x_{13} + x_{14} = 1$, since exactly one edge must leave vertex 1. Similarly, $x_{12} + x_{32} + x_{42} = 1$, and so on. Geometrically, each of these equations is a "hyperplane": all the vectors representing tours are restricted to the intersection of these planes.

Finding the minimum cost TS tour in this case (n=4) is equivalent to finding a 0-1 vector which (1) satistisfies all the relevant geometric constraints; and (2) minimizes the sum $c_{12}x_{12} + c_{13}x_{13} + \dots + c_{43}x_{43}$.

(Continued on page 10)

Using the Ohio Highway Map to Teach Mathematics

by Marialice Kollar

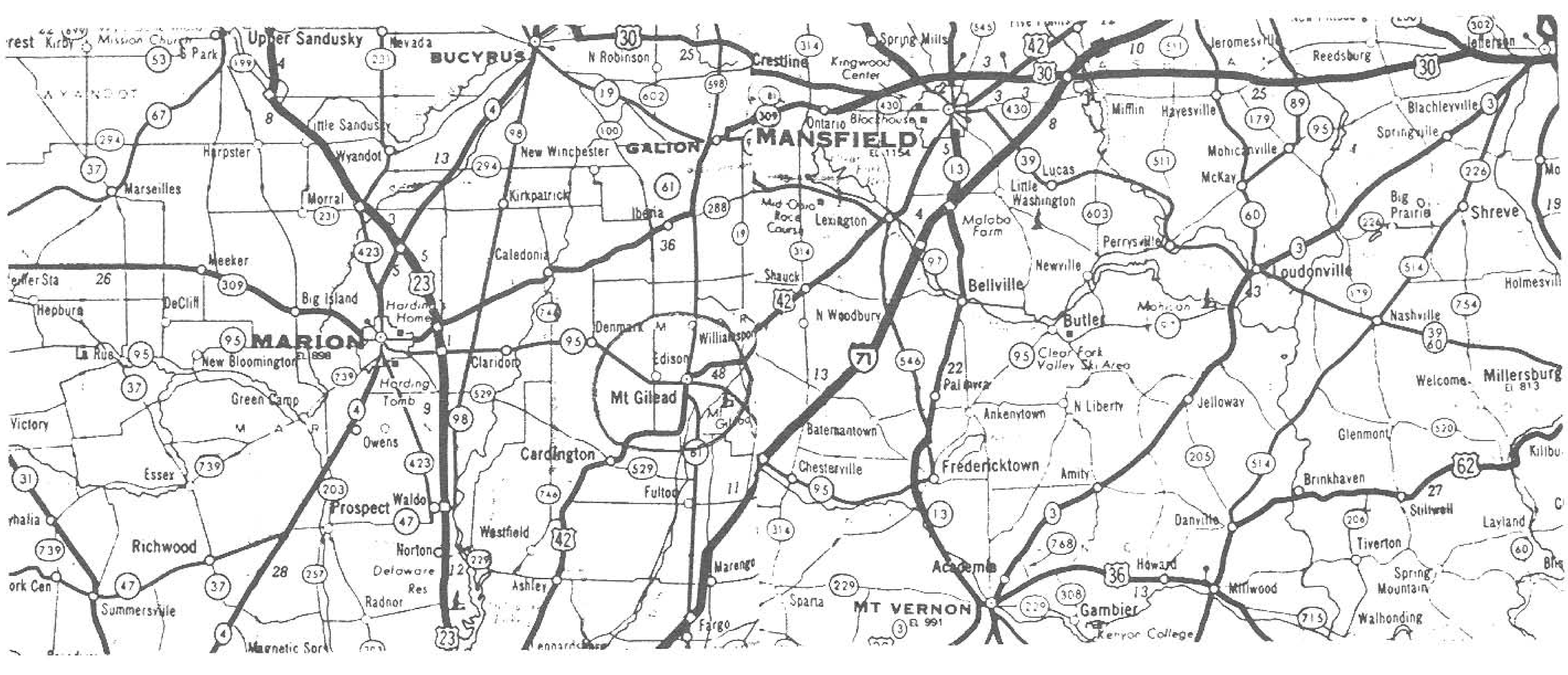
Mt. Gilead is a small, rural community about an hour north of Columbus, Ohio, with two exits off the interstate. We have 400 students in the high school with 50 students currently enrolled in Pre-Algebra. These students have been unhappily going through the repetitions of general math topics and the Ohio Map unit described below actually excited a few of them! Here is what I did. First, in August, I requested 50 "Official Ohio Highway Maps" from the Department of Transportation. In September, we started Pre-Algebra with our maps. The first day we found Mt. Gilead on the map using the grid of letters across the top and numbers down the side. We made the connection between the map grid and the x and y coordinate axes, and talked about how easy it was to find a community on the map thanks to the grid labels. We talked about the best places to visit in Ohio, like Sea World, Kings Island, the Pro Football Hall of Fame and the State Capitol, and used the maps to figure out how far these places are from Mt. Gilead. We used the legend to find the mileage scale and used rulers to measure the distance, then converted to the real distance using ratios. More math! Another fun part of the class was at the end when we had to fold the maps! Many of the students commented that they had never used a map before and some students admitted that even their parents did not ever use a map!

The second day, we again brought out the maps. This time, we learned about mileage markers and exit numbers and what they mean. In Ohio, the mileage markers and exit numbers correspond to the distance either north or east of the border depending on the direction of the interstate. We also discussed odd/even rules for interstates and state routes. Most students had never heard of the reasons for the numbers and were surprised by them. Some students actually went home and looked at the whole United States to see that those same rules worked nationwide. We talked about patterns of numbers and looked up zip codes to see how they were also patterned. More math! Students actually like math!

We then looked at the counties. This turned out to be a perfect introduction to graph theory and the four color theorem! We traced the counties and created a graph with vertices for counties and edges between counties which share a border. I let the students work together and help each other find a coloring with four colors. There are 88 counties in Ohio, so this took quite a bit of time!

We figured distances between the 5 major cities of Ohio and tried to minimize driving time from Mt. Gilead to these cities. We discussed the UPS and Federal Express delivery systems and related them to our maps. This was a good introduction to Euler and his famous "Konigsberg Bridge" problem, as well as Hamiltonian circuits and the "Traveling Salesman Problem". The students began to immediately try to connect Dayton, Toledo, Cincinnati, Columbus, and Akron/Canton. It was great! Many students were impressed to find that the interstates were constructed to help in commuting between these cities. Students wondered if bus routes were constructed using similar strategies. "Is this really math?", they wondered. We continued to use the maps for two more weeks; by using the maps, my students and I had fun learning math and were excited about doing something different!

To order Ohio maps, write to
the Ohio Department of Transportation
25 South Front Street
Room 712 Columbus, Ohio 43215,
or call (614) 466-7170.
It takes about three weeks for them to arrive so write early!



A Discrete Look at Discretion?

by Anne Carroll

Codes, if not the heart of communication today, are at least the feet! Without the bar codes on our mail and packages, written communication would slow to a crawl. However, codes are entered by human hands or machine printers, and mistakes will always occur. Thanks to a presentation by lead teacher Br. Pat Carney (LP '91) and a little independent research, I now feel a bit more secure that if I request 20 copies of *The Joy of Mathematics*, a small mistake (human or otherwise) will not result in an avalanche of manuscripts of "Hamlet".

My students learned about codes after I decided to begin the New Year with something a bit different. I teach the second part of a two-year Algebra I course, Consumer Mathematics, Essentials of Calculus (non-AP), and two sections of Advanced Mathematics. My students range in ability and desire from traditional college-bound seniors to hard-to-motivate, non-college-bound sophomores. We also have a heavy concentration of non-English speaking students. I tried a version of this lesson in each of my classes and found that the topic of codes was accessible to all and held some degree of interest for each group.

Using the bar-coded zip code on reply mail torn from magazines as my tease, I asked the students to discuss its purpose and to explain how it works. Someone in each of my five classes was able to identify the purpose, but no one knew the way in which the bar code is analyzed beyond "a scanner does it." The door was open---the teacher knew something that the students didn't, and it was something that affected their daily lives. There is nothing like a good secret! And that, after all, is what codes are all about.

I explained the algorithm for reading the bar code for zip codes (see box).

It was interesting that the students in the calculus and advanced mathematics classes found it easier to use the expansion algorithm to decode, while the others found the matching list faster.

The last block of 5 bars (before the end frame bar) is the correction digit, or check digit. In the zip code, the rule is that the correction digit must create a sum that is divisible by 10. This brought us to a discussion of modular arithmetic, i.e., that the sum is equal to 0 (mod 10). I tackled this topic with the calculus and advanced mathematics classes. With

(Continued on page 10)

The first and last bars form "frames." The rest of the bar code is broken into blocks of 5 strokes or bars. Each bar represents a digit (short bar = 0, long bar = 1) and each block of 5 bars represents a number:

11000 = 0	00110 = 3	01100 = 6	10100 = 9
00011 = 1	01001 = 4	10001 = 7	
00101 = 2	01010 = 5	10010 = 8	

These numbers can also be obtained by the "expansion algorithm" by multiplying the value of the bars in sequence by the numbers 7,4,2,1,0 respectively, and adding (mod 11, so that 11000 = 0).

For example, 10010 is equivalent to (1x7) + (0x4) + (0x2) + (1x1) + (0x0) = 8.

Block 1 | 2 | Check Digit | 1

Bar Code for 20077-7330

Tennessee...

(Continued from page 2)

a portfolio of their work, including an essay on Discrete Mathematics. These essays have been very positive! The last day of the semester was the 2nd Annual "Discrete Math Appreciation Day", in which students in my class let other students in the school know what Discrete Math is all about by presenting results of their projects in other classes. The students have greatly enjoyed learning about these new approaches to math, and have especially enjoyed applying mathematics to real-world problems.

References:

1. Crisler, N., Fisher, P. and G. Froelich, *Discrete Mathematics through Applications,* W. H. Freeman, New York, 1994.

2. Peitgen, H., et al., *Fractals for the Classroom: Strategic Activities* (Vol 1), Springer-Verlag, New York, 1991.

Note: If you are interested in reviewing the text [1] for a future issue of the Newsletter, please contact the editors.

Credits...

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Encouraging words...

Read any good discrete mathematics textbooks lately? Do you have a good story or problem involving discrete mathematics? Have your students worked on interesting projects? Don't keep it to yourself! Share your ideas and experiences with other readers of *In Discrete Mathematics.* Even if you don't have a "finished article," send a brief outline or description to Deborah Franzblau (telephone: 908-932-4573) or franzbla@dimacs.rutgers.edu to discuss your idea.

Subscriptions...

Please send us the name, address, phone number, and school of any teacher who should receive a copy of this Newsletter, and we will include him/her on our mailing list. (See back page for form.)

Solutions...

<u>Pizza Cutting</u> (p. 8)

If p(c) is the number of pieces with c cuts, $p(c) = 1 + 1 + 2 + ... + c$
$= 1 + (c+1)(c)/2$.

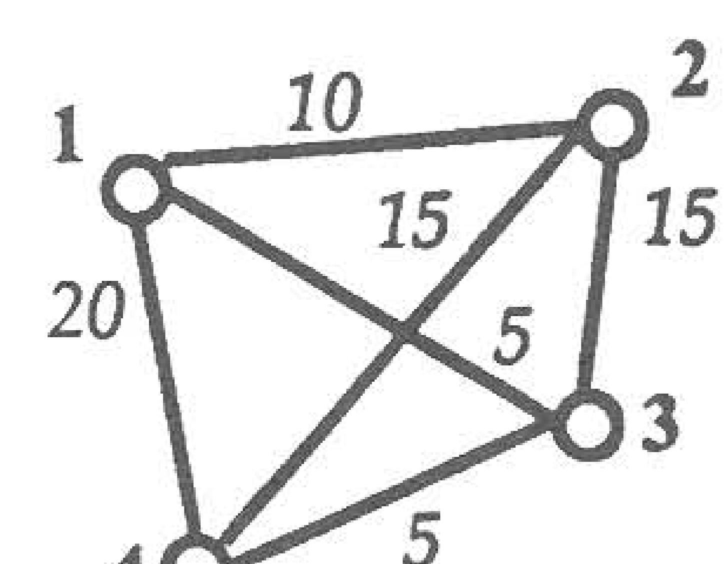
<u>Zip Codes</u> (p. 10)

The correct zip code is 14207-9967 (the error is in block 5).

You can only correct 2 errors if they are in the same block <u>and</u> you know which block. For example, suppose the true code is 00000-333 (check digit 1), but you read 60000-333. The true code might just as well be 60400-333 if all you know is that 2 errors are made.

TSP example (p. 3)

Representing a Tour as a Vector (n = 4)



Cost Vector
(10, 5, 20, 10, 15, 15, 5, 15, 5, 20, 15, 5)

Tour: {1,2} {2,4} {4,3} {3,1}

Tour Vector
(1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1)

Cost = 10 + 15 + 5 + 5 = 35

The Towers of Hanoi in Algebra

by Louis Lo Bosco

During the three weeks I spent in the Leadership Program, I was awakened to many interesting topics in Discrete Math. My school already offers a one-semester elective in Discrete Mathematics, taught by Lou D'Angelo (LP '91), so my goal has been to implement the topics throughout the curriculum.

In my Honors Algebra II course, a sophomore-level course (with a few freshman), I decided to introduce the concept of recursion. I remembered how impressed I was with Lead Teacher Phil Reynolds' (LP '89) presentation on the Towers of Hanoi, and decided to try it. First I needed some working models. I found some old 1-by-2-foot boards lying around, cut the boards into 8-inch lengths and hammered three nails with small heads into each board. I used six metal washers of decreasing size for a set of discs.

I put the students into groups of four and told them the classic story: "According to an ancient Hindu legend, Brahma piled 64 gold disks one on top of the other. The disks, each a different size, were stacked in order with the largest at the bottom and the smallest at the top. Priests were told to transfer the disks one at a time from one pile to another, using a third pile if necessary, so that at no stage would a larger disk be placed on top of a smaller disk. When the work was complete, according to the legend, the world would end."

I gave students a worksheet (see sidebar), gave them time to investigate, find the results, then record and describe their solutions.

The students really seemed to enjoy this hands-on work. All the groups eventually figured out a recursion formula for the number of moves,and everyone realized that the time needed to move all 64 disks was extremely large*. The students enjoyed the problem enough that I decided to venture one step further. All of my students use the TI-81 calculator, so I asked them to write a program to compute the number of moves needed using their recursive formula. Most of them had never written a program before, but as a group, we established a correct algorithm; I then helped them to enter the program into the calculator. The students were impressed with how simple and short the actual program was.

This project took about three days to complete, and was well worth it. I was most impressed with those groups that said that even though we could write down a mathematical procedure to move all 64 disks, we could not actually perform this task in the real world. They had understood the difference between a theoretical construction and a practical one.

* 585 billion years approximately!

Towers of Hanoi Student Worksheet

Objective: move all disks from Tower 1 to Tower 3, using Tower 2 as needed. Rules:
a. Move only one disk at a time.
b. Never put a larger disk on top of a smaller one.
c. Try to use the minimum number of moves.

1. Record your moves using first 3 disks, then 4, then 5, using a table like the one below, which is an example for 2 disks. A is the larger disk: AB in the column labeled T1 means that B is on top of A on the first tower.

Move #	T1	T2	T3
0	AB	-	-
1	A	B	-
2	-	B	A
3	-	-	AB

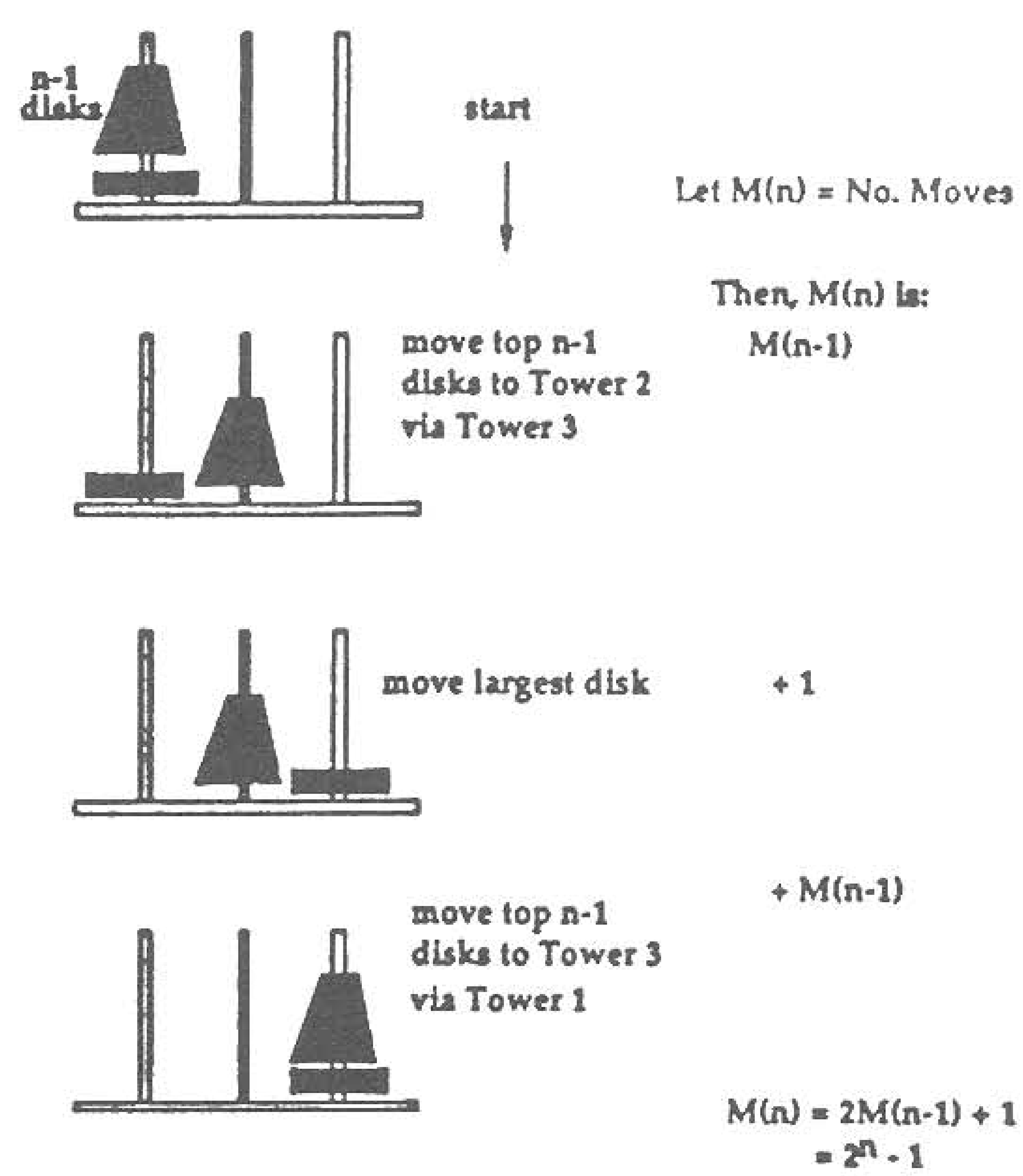
2. Look for a pattern, and try to find a general solution that will work for any number of disks.

3. Explain how you arrived at this solution.

4. Assume that you can move one disk a second. Determine the number of years it would take to move the 64 disks (the number of years it would take for the world to end according to the legend).

What conclusions can your group draw from this exercise? In particular, what did this exercise help you to understand? What conclusions can you draw from part 4?

Towers of Hanoi: a solution



Historical View...

(Continued from page 1)

Man-Made Universe [8], which treated a range of subjects from number theory, combinatorics, and modern algebra.

By 1970, departments such as business or accounting were requiring Finite Mathematics for their majors, changing the emphasis in the course to make it less valuable for general liberal arts students. Moreover, a higher proportion of students entering college were weak in algebra (and symbol manipulation in general), yet both the Finite Mathematics and Topics courses assumed a strong background in algebra. In fact, in many schools the liberal arts mathematics course degenerated into a review of basic skills, unsatisfactory for both students and faculty. At the same time, there had been vibrant growth in Discrete Mathematics, stimulated by the development of the digital computer; much of the mathematics is based on relatively elementary logical and geometric ideas rather than algebraic manipulation.

These developments together spawned the text *For All Practical Purposes,* which represents a truly new approach for the liberal arts mathematics course. The content of this innovative book is best described as Applied Discrete Mathematics. The topics include game theory, voting systems, fair division, graph theory, algorithms, and statistics, but the point of view is quite different from that of the new generation of Finite Mathematics texts. The new approach underlines the importance for liberal arts students of analyzing and understanding real-world situations and building mathematical models (see [4]) rather than gaining facility with solving exercises based on the models.

The number of liberal arts mathematics courses adopting this point of view is increasing, and new texts with similar content are appearing (e.g., [9]). My hope is that such courses will ultimately foster a climate in which the public perceives that mathematics is responsible for dramatic improvements in technology, and has direct benefits to society.

References:

1. Beck, A., M. Bleicher., and D. Crowe, *Excursions Into Mathematics,* Worth Publishers, New York, 1969.

2. COMAP, *For All Practical Purposes* (3rd edition), W.H. Freeman, New York, 1994. (First Edition, 1988.) This text is supplemented by 26 half-hour video tapes focusing on applications and on those who develop and use Discrete Mathematics.

3. Franzblau, D., "New Models for Courses in Discrete Mathematics," SIAM Activity Group on Discrete Mathematics, *Newsletter,* Volume 4, #2, 1993-1994.

4. Malkevitch, J., and W. Meyer, *Graphs, Models and Finite Mathematics,* Prentice-Hall, Englewood Cliffs, 1974.

5. Kemeny, J., and J.L. Snell, *Introduction to Finite Mathematics,* Prentice-Hall, Englewood Cliffs, 1962.

6. Kemeny. J., J.L. Snell, and G. Thompson, *Introduction to Finite Mathematics* (3rd. edition), Prentice-Hall, Englewood Cliffs, 1974.

7. Ralston, A. (ed.), *Discrete Mathematics in the First Two Years,* MAA, Washington, 1989.

8. Stein, S., *Mathematics, the Man-Made Universe,* W.H. Freeman, 1963.

9. Tannenbaum, P., and R. Arnold, *Excursions in Modern Mathematics*, Prentice-Hall, Englewood Cliffs, 1992.

Editors' note: In a previous Newsletter (#2, p. 10), we printed an enthusiastic review by Anthony Piccolino of *For All Practical Purposes.* The article above was adapted from a longer article by the author which will appear in the Newsletter of the SIAM Activity Group in Discrete Mathematics.

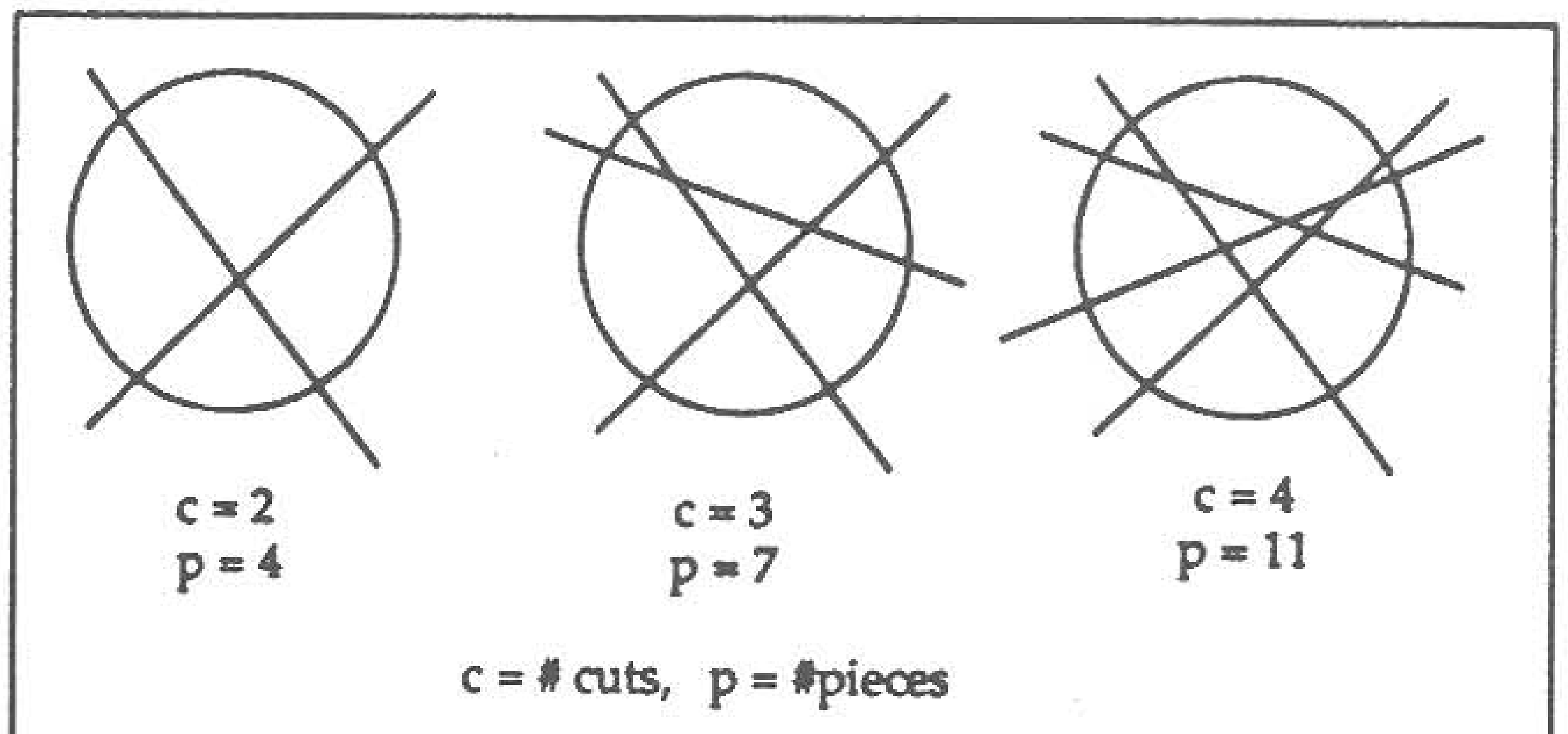
A Pizza Cutting Problem

by Constance Cunningham

This is a lesson I implemented in a Basic Math I class, of primarily "at risk" students in grades nine and ten who had no formal algebra training. The lesson was based on ideas from an article by Mary Kim Prichard [1]. I began by presenting a pizza cutting problem:

How many cuts would you need to make in a giant pizza so that each student in our school could have one piece (not necessarily the same size)?

(Continued on page 9)



Pizza...

(Continued from page 8)

I then gave out a worksheet asking students to make one, two, three, etc. cuts to the pizza and determine the maximum number of pieces produced for each number of cuts.

It took a while to convince the students that the cuts did not have to go through the center and the pieces did not have to be of equal size! The students soon found sketching the cuts to be a clumsy method of counting, so we decided to look for a pattern. I suggested that the students organize their data in a table and then examine it. The students were quick to see a pattern, but it took some discussion to get the students to arrive at a generalized recurrence formula for determining the number of cuts.

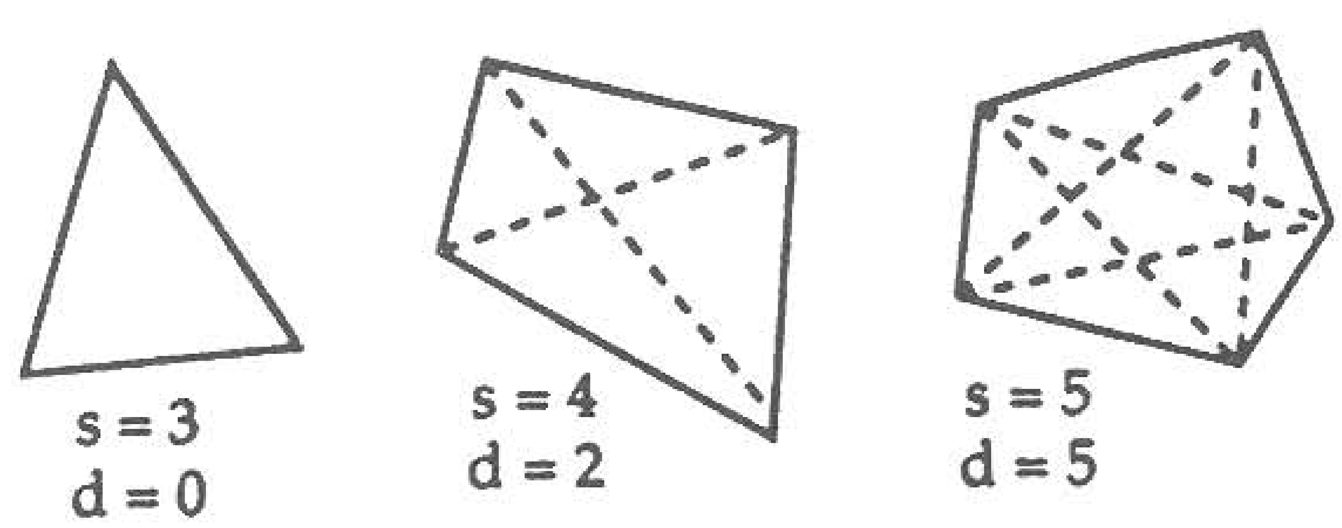
#cuts	1	2	3	4	5
# pieces	2	4	7	11	16

General Pattern: If c is the number of cuts and p(c) is the number of pieces, then $p(c) = p(c-1) + c$.

More advanced students may be able to find a formula for p(c) without programming. (See solutions, page 6.)

In her article, Prichard suggests having the students write a BASIC program based on the formula to determine the number of pieces given the number of cuts. However, since my students had no programming experience, I adapted Prichard's program to Macintosh TrueBasic (see sidebar) but left out steps of the program for the students to fill in. In class I explained the program commands, and the class discussed what would be needed to fill in the program blanks. The students found it difficult at first to understand the programming concepts, but eventually completed their programs; they then went to the computer lab to determine how many cuts would be needed to provide a piece of pizza for all 620 students in our school.

Next, I asked the students to determine a pattern for the number of diagonals in a polygon (see figure). This time they were asked to determine the number of diagonals in a 100-sided polygon.



s = # sides, d = # diagonals

The class as a whole wrote a program from scratch to determine this (the program is similar to that for pizza cutting), and again went to the computer lab. Although writing the program for the number of pieces of pizza had seemed very taxing, this time the students had caught on, and wrote the program with ease. I found these activities both worthwhile and rewarding. The students loved it and have frequently asked when they can do more programming (now that they think they're experts!).

Prichard's suggestions were extremely helpful; although I did not have sufficient time to try other similar problems, the article offers suggestions and programs for other good iteration problems such as triangular numbers and Fibonacci numbers.

1. Prichard, Mary Kim, "Mathematical Iteration Through Computer Programming," *Mathematics Teacher,* February, 1993.

TrueBasic Program (Adapted from Reference [1])

```
10 Rem ***********************************************
20 Rem Pizza Problem
30 Rem ***********************************************
40 Print " Cuts " , " Pieces "
50 Print "____ " " "______": Rem set up table headings
60 Rem Set initial value for number of cuts and pieces
70 Let c= [1]
80 Let p= [2]
90 Print c, p
100 Let c = [c+1]: Let p = [p + c]: Rem repeat process
110 If p [ < 620] then [90]: Rem check if done
120 Print "It takes at least" c "cuts for [Lee] and [Pat]
   to produce [620] pieces of pizza."
130 End
```

Note 1: **Rem** stands for "remark" and tells the computer that this line is a note to the programmer--and that it should ignore the rest of the line.

Note 2: items in [brackets], were left blank on the student worksheet.

Note 3: there are many possible answers for lines 70, 80: c = 0, p = 1; or c = 2, p = 4.

Discretion...

(Continued from page 5)

the consumer and non-traditional algebra students, I discussed multiples instead.

The second part of the lesson brought another code to the students' attention. While examining their textbooks they discovered the ISBN (International Standard Book Number). The students were fascinated by the breakdown by number group: group identifier, publisher identifier, title identifier, check digit. (I also explained that the bigger publishers have small publisher-identifier numbers to allow the title identifier to be longer. This led to a discussion of the number of possible titles available given a certain number of digits in the title identifier group.) The check digit in the ISBN number works like the zip code check, except that the sum is taken mod 11 instead of mod 10. By examining several ISBN numbers, the students were able to figure this out on their own.

The calculus and advanced mathematics students accomplished these discoveries in one class period, although a second period was necessary to fully discuss modular arithmetic and equivalence. The consumer and algebra students were only able to handle the zip code on the first day: they needed quite a bit of guided practice to interpret the bar code. I gave them a worksheet with addresses on one side and the bar codes on the other and asked them to match them up. I also encouraged my students to do some exploration on their own. The section on codes in *The Mathematical Tourist* by Ivars Peterson [3] is accessible to interested students.

As it turned out, coding methods were not what I taught in this lesson. Nor did I teach counting methods. Though I did not expect it, the end result of the lesson, rather than an understanding of codes, was a true appreciation of algorithms. Although several of my students have studied computer science, only a few could explain that an algorithm is a "step-by-step procedure." I wheedled an understanding out of each class through examples: "I know you all know an algorithm for dressing each morning, or for starting the car. You learned an algorithm in third grade for long division." As a result the word has become part of my students' vocabularies.

FURTHER READING

1. Gallian, Joseph, "How Computers Can Read and Correct ID Numbers," *Math Horizons*, Winter 1993, 14-15.

2. Lefton, Phyllis, "Number Theory and Public-Key Cryptography," *Mathematics Teacher*, 84 (January), 54-63.

3. Peterson, Ivars, *The Mathematical Tourist*, New York: W.H. Freeman and Company, 1988.

How can errors be found in zip codes?

You may wonder why the particular set of 5-digit blocks were chosen to represent the digits 0-9 for the zip code. In fact, this system allows one to detect errors **within** the blocks. Notice that each block of five has exactly two 1's and three 0's. Thus, if any one bar is misread you can tell that an error has been made. Observe, however, that you might not be able to tell where the error occurred: if 11001 is read, the original could have been 11000 (0), 01001 (4), or even 10001 (7). Also, if **two** bars are misread, you might not even notice: if the correct block is 10100 (9), and you misread the first two digits, you would think that the number was 01100 (6). Thus, this is a "single-error-detecting code", but is NOT a "single-error-*correcting* code". Now suppose that a single error occurs in one of the blocks. The final check digit actually allows you to **correct** the error. Figure out how this works by trying the following example:(solution on p. 6).

You might also try to explain how you can sometimes correct **two** errors this way. For more information and other references, see:

Malkevitch, Joseph, "Have you seen ... (Codes)" and "Mini-Bibliography...Codes", *In Discrete Mathematics*, Issue 3 (Aug. 93), p. 1, 8-9.

COMAP, *For All Practical Purposes*, Freeman, 1994, Chapter 9.

Solving the TSP...

(Continued from page 3)

(This is an example of a "linear integer programming problem.") What makes the problem hard is that the number of constraints grows exponentially with the number of vertices. The strategy in using "cutting planes" is as follows. First find a vector which satisfies only **some** of the constraints, but does minimize the sum; this will be an approximate solution. Then, add new constraints (called "cutting planes") one at a time, so that each time you compute a new approximate solution it is closer to the true solution. To picture this, imagine that you want to find a diamond that has been baked into a cake, and that you're only allowed to make straight cuts. Certainly the method is effective only if the number of cutting planes needed does not grow too large. Sophisticated mathematical methods from linear algebra are needed both to find the approximate solutions and compute the cutting planes. So far, no one has found a better method for solving very large TS problems exactly.

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