IN DISCRETE MATHEMATICS

Using Discrete Mathematics in the Classroom

Premiere Issue *November 1991*

Speaking discretely...

by Joseph G. Rosenstein

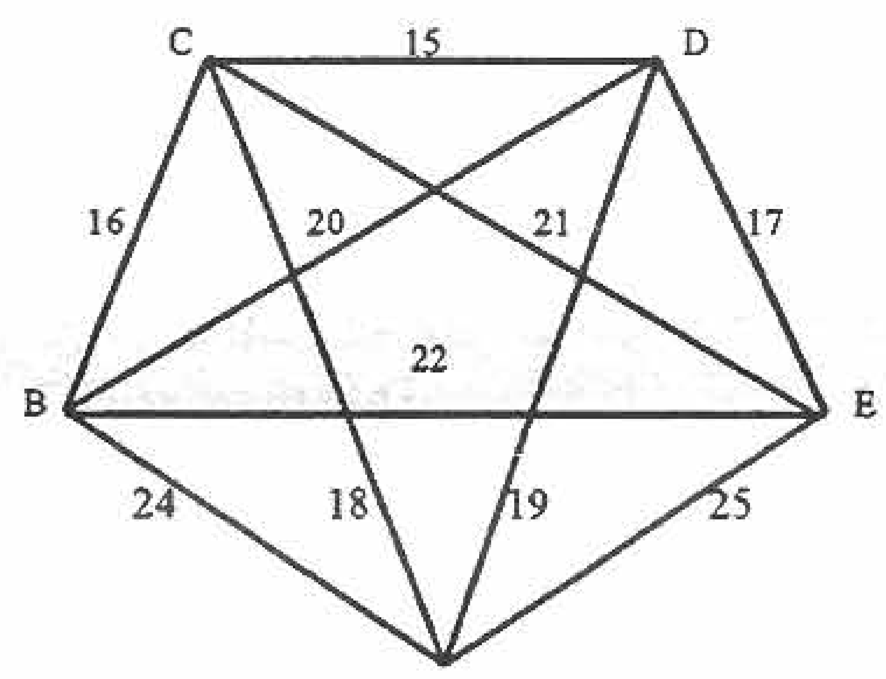
In the last few years a number of teachers have tried to introduce topics of discrete mathematics into their classrooms. If you are one of these teachers, then this newsletter is designed for you!

We hope to serve as a forum where teachers across the country can share their ideas, their classroom activities and experiences, their successes and failures, and their questions about implementing discrete mathematics in the schools.

We also hope to assist you by informing you about resources on which you can draw. Not too much is available about teaching discrete mathematics in the schools -- although that is changing -- and what there is may be hard for you to locate. We hope to be of some assistance in showing you where to look.

Many of you teach in schools where you are the only one that has become enthusiastic about discrete mathematics, and some of you have had to exercise much patience and perseverance in order to get to teach these topics. This newsletter is intended to provide you with a national network of teachers who have had similar experiences.

Find the shortest route that begins at city A, visits the other four cities and returns to A. Distances between cities are as indicated.



(Solution -- page 8)

Most of you, we hope, have found teaching discrete mathematics rewarding. It provides lots of opportunities to try the student-oriented instructional techniques advocated in the NCTM *Standards*; for example, many discrete mathematics problems (like the Traveling Salesman Problem above) lend themselves readily to experimentation and conjecture, to hands-on activities, and to group learning settings, and have many easily understood applications. We intend to use this newsletter to advocate using discrete mathematics to implement the *Standards*.

(Continued on page 2)

In This Issue

Have-you-seen...

by Joseph Malkevitch

... two recent articles in the New York Times and the Wall Street Journal dealing with the Traveling Salesman Problem.

When you use a public phone, you deposit a coin in a box; eventually the coin box fills up and the phone company must have an employee collect the coins. PROBLEM: Given a collection of phone booths to visit, design the most efficient route, visiting each phone booth site once and only once to pick up the coins, and starting and ending at the collector's place of work. See the example in the diagram at the left.

We can abstract the essential features of this situation. We are given a collection of sites which must be visited once and only once, starting and returning to a home base. To go from site i to site j requires the payment of a "cost," c(i,j). Often c(i,j) = c(j,i), but sometimes these two costs are not equal. (For example, in driving, the distance from i to j is usually different from the distance from j to i due to the presence of one way streets.) Thus, there are two versions of the problem to consider, depending on whether or not the cost function is symmetric.

The Traveling Salesman Problem (or TSP, the traditional name for this problem) calls for finding the route used to visit the collection of sites which involves the minimum total cost. The reason for the name is that a salesperson must solve a TSP in order to find the minimum cost of visiting his/her territory. Other situations requiring the solution of a TSP include picking up fish catch from sites where nets have been set, parcel post deliveries, gas meter reader routes, meals-on-wheels routes, picking up kids to take them

(Continued on page 9)

Reports... The NCTM Annual Meeting

by Julia E. Magana

New Orleans. City of hot jazz, cold hurricanes, and discrete mathematics. At least that is how I viewed this dynamic city during the NCTM 69th Annual Meeting.

Many different topics on a variety of levels were discussed, but this was the first time in 69 years that the NCTM focused on discrete mathematics and its applications in K - 12 curricula.

There were many workshops with discrete mathematics themes, some giving just overviews and other dealing with specific topics. "Implementing the Discrete Math Standard in the Secondary School Classroom", a workshop conducted by Nancy Crisler, Gary Froelich, and Larry Spence, used John Dossey's *Discrete Mathematics and the Secondary Mathematics Curriculum* as a way of presenting different discrete mathematics topics to people of a variety of mathematical backgrounds.

After reviewing the Dossey materials, many teachers realized that some of the discrete math topics, such as combinatorics, probability, matrices, and linear programming are discussed in textbooks which they have been using for years. Other topics however, like graph theory, apportionment and fair division, difference equations, and fractals are new to the high school classroom.

The 1991 NCTM Yearbook, *Discrete Mathematics Across the Curriculum K - 12,* was released at the meeting, and its contents were reviewed by editor Margaret Kenney. A sample of other presentation titles includes "Applications of Finite, Discrete and Combinatorical Mathematics", "Graph Theory -- The Queen of Discrete Mathematics", "Activities in Discrete Mathematics: Backpacks, Yearbooks and Trees", and "Counting, Matching, Graphing: Discrete Mathematics in Elementary School".

A number of sessions at the conference dealt with specific discrete math topics such as chaos and fractals.

Heinz-Otto Peitgen spoke on "Fractals for the Classroom: The Fascinating Concept of Chaos and Fractals." He began with an in-depth discussion of the "chaos game", which is one of the key entrance points to the study of fractals. This then led to the idea of using limits as a way to describe self-similarity. Peitgen's reference material was a newly released NCTM publication, *Fractals For the Classroom*, which he coauthored (see complete reference on bottom of page 4).

Robert L. Devaney also gave a talk on chaos and fractals but he concentrated more on the use of iteration to create dynamical systems. "This is a branch of research mathematics that is accessible," he said. "We are talking about quadratic functions!". Devaney's book *Chaos, Fractals and Dynamics* provides an introduction to these three topics using a combination of hands-on computer experimentation and precalculus mathematics.

Besides going to the workshops and meeting mathematicians from all over the world, I also visited the Exhibit Hall which was full of displays on "the most current mathematics education products, publications, software, and services" including an increasing number of discrete math materials.

This year's NCTM Annual Meeting was definitely a great experience! For a discrete mathematician, it offered more than in past years and

Speaking discretely...

(Continued from page 1)

This is the first issue of the newsletter. It was assembled and largely written by teachers in the Leadership Program in Discrete Mathematics at Rutgers University. We hope that future issues will have your contributions as well.

We are particularly interested in hearing about topics that you have used in your classes, about how your students have responded to discrete mathematics, and about how discrete mathematics has affected your approach to teaching. We look forward to hearing from you.

In each issue...

... you will find a variety of articles under the following headings:

Teaching briefs...
suggestions for classroom activities
Spreading the word ...
communicating with teachers and administrators
Have-you-seen ...
recent articles about discrete mathematics in the news
Mini-bibliography ...
helping you find your way into a topic
Topics ...
articles to introduce you to various topics
Reports ...
on happenings and events
Announcements ...
of opportunities and events
Ask a discrete question ...
of the editors or other readers
Reader responses ...
letters to the editor

... and you are invited to submit your own comments, letters, and articles under any of these (or other) headings. Please use the Newsletter address on page 6.

Teaching briefs... Fractals in the Classroom

by Elyse Magram, with *quotes from an article in the school newspaper by Keith Knittel*

The classroom is a beehive of happy activity. Small groups sit clustered, eagerly measuring, talking quietly, contemplating the next generation of figures. The atmosphere is charged with the sounds of a video that shows a multitude of fractal colors and patterns. The computer program generating a fractal tree slowly adds branches to the varying trunk. Is this a scene from "Stand and Deliver"? No, a unit on fractals in one of my classes.

> *You know how excited she gets when she learns something new...she introduced virtually all of Smithtown West to the wonderful world of fractals.*

I would like to share with you the enthusiasm generated in four of my high school classes as I introduced this multifaceted topic -- which I learned about mostly on my own. I found that fractals could be used equally with the slower learners in a 10th grade class and the brighter students in precalculus. Is it possible that through playing with fractals, the slow learners can achieve brightness? I now believe so.

We began the unit with a discussion of these self-similar figures and their applications in nature.

> *Fractals contain the property of self-similarity. In each fractal there are shrunken, repeated versions of the same shape. In nature, coast-lines, ferns, clouds, trees, lungs, intestines, and popcorn all have repeated fractal shapes. The coastline of Ireland has been matched to a computer generated fractal. In police work, crimes have been found to follow fractal patterns. Meteorologists have used fractals to chart the paths of tornadoes.*

Then we proceeded to "make" fractals. Students loved doing "art" in math class, and to measure carefully and creatively. They found the Koch snowflake (see *Illustration* on page 4) particularly fascinating, and were intrigued with the lace-like effect of the fractal fern. The class worked cooperatively, sharing materials and ideas. The mathematics abounded, for we discussed a variety of topics, such as ratio of perimeters and areas, similarity, and the percent colored after each generation. In calculus classes, we discussed the limit of the perimeters and areas.

I highly recommend the topic as fascinating, colorful, a wonderful change of pace in a classroom, and one that will produce a magnificent outcome. Fractals get a high vote for one of the best math topics going.

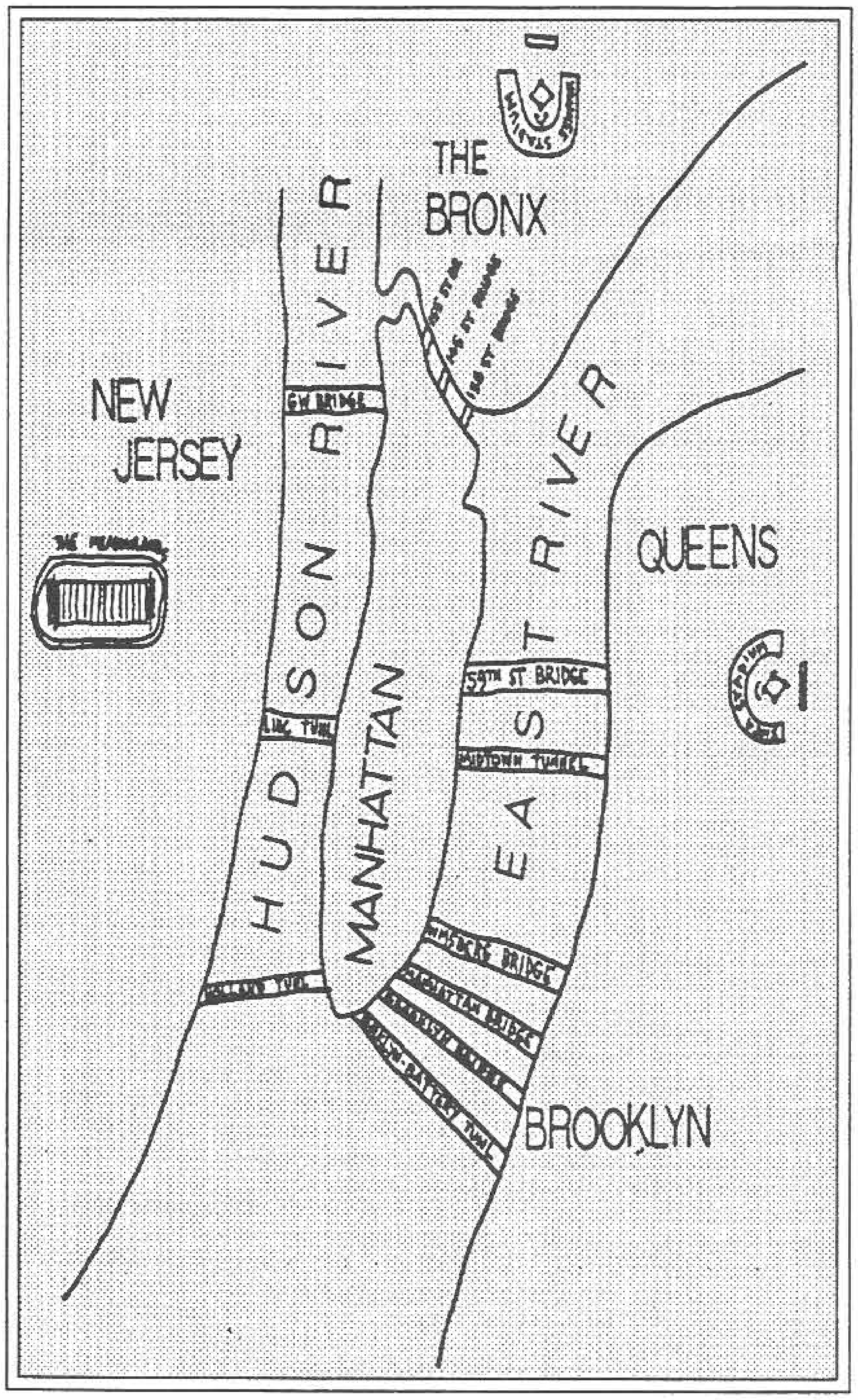
> *Senior Nick Mequia says that "fractals are by far the most interesting things in the world" and is reported to have devoted his entire life to fractals.*

Teaching briefs... Maps and Graphs

by Susan H. Picker

To extend the Konigsberg Bridge problem, and show students the applied nature of graph theory, it is very easy to construct maps with interesting Euler path/ciruit problems. I have found maps of cities with bridges such as New York, Paris, and Amsterdam to be particularly suitable, but any regional map can become the source of an imaginative problem requiring students to use their knowledge of the principles of graph and network theory. Below and on page 9 are two examples I have used with great success in both remedial and honors classes.

The map below depicts the bridges and tunnels connecting Manhatten with the other boroughs of New York City and with New Jersey. Is it possible to start at the Meadowlands in New Jersey, travel each bridge and tunnel exactly once and end at Shea Stadium in Queens? Is it possible to start at the Meadowlands and end at Yankee Stadium in the Bronx? Draw a graph and explain your answers.



Teaching briefs... Digraphs and Relations

by Ruth Ann Krayesky

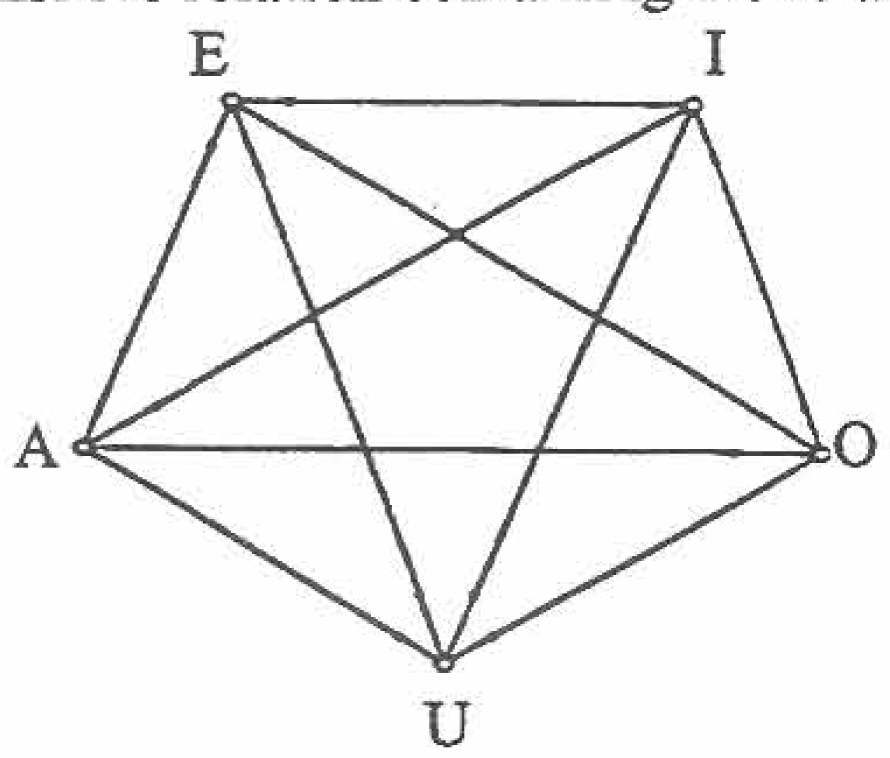
Have you ever considered using a *digraph*, or directed graph -- with arrows instead of edges, to explain relations and their properties? Fehr, Fey and Hill's *Unified Mathematics* presents a model that has been useful in helping my students to understand relations.

Starting with a finite set and a relation, arrange the elements of the set in a circular pattern. These elements become the vertices of our graph. Next draw an arrow from each element to every element of the set to which it is related. These arrows become the directed edges of our digraph.

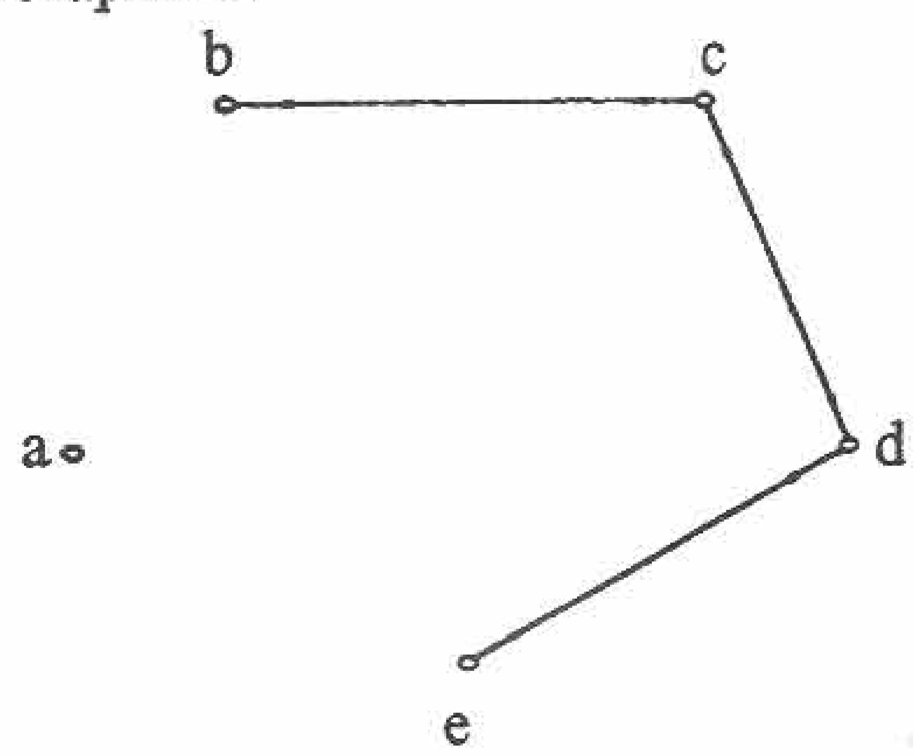
For example, the relation "X is older than Y" among five students in the class might result in Graph I.

If every element has an arrow to itself, then the relation is reflexive. If whenever there is an arrow in one direction between two elements, there is also an arrow between the two elements in the other direction, then the relation is symmetric. And if whenever there is a directed path between vertices made up of two edges there is also an arrow from the initial vertex to the terminal vertex, then the relation is transitive. (Note: In this case, there is an edge connecting the vertices whenever there is a directed path - of any number of edges -- between the vertices.)

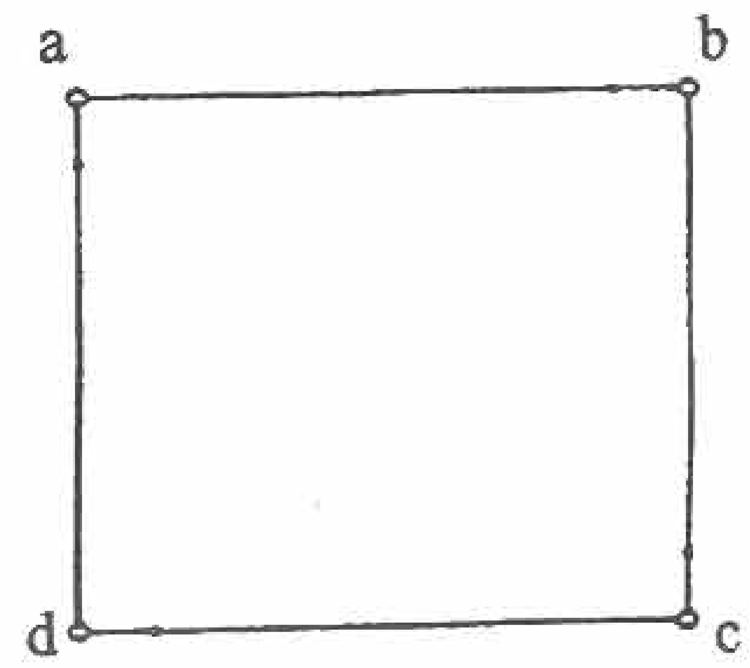
For example, the relation depicted in Graph II is symmetric and reflexive, but not transitive. Can you find the smallest transitive relation containing the relation in Graph III?



Graph I



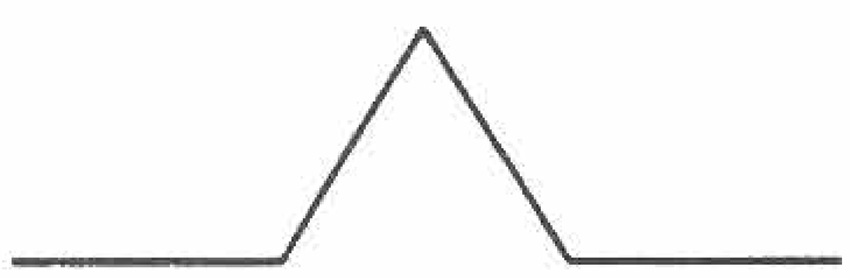
Graph II



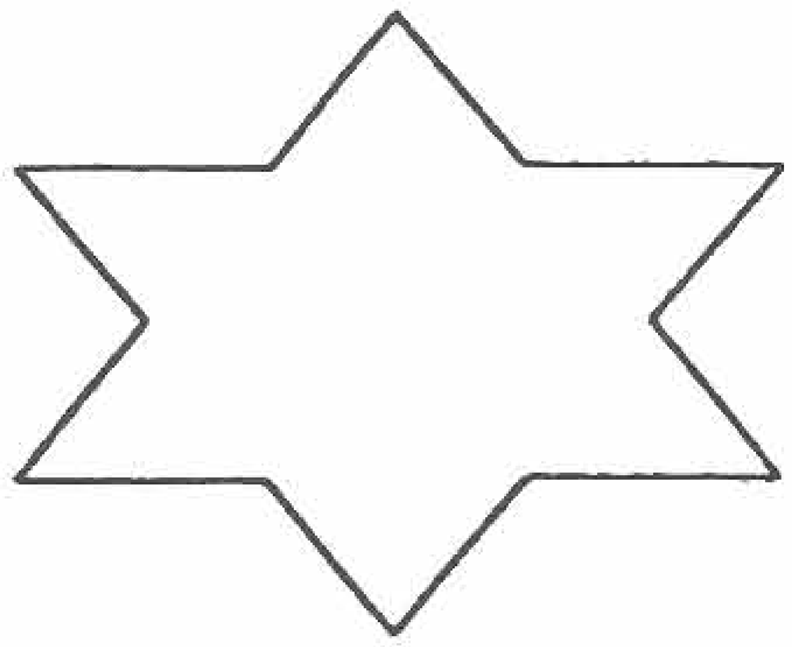
Graph III

Illustrations... Koch snowflake

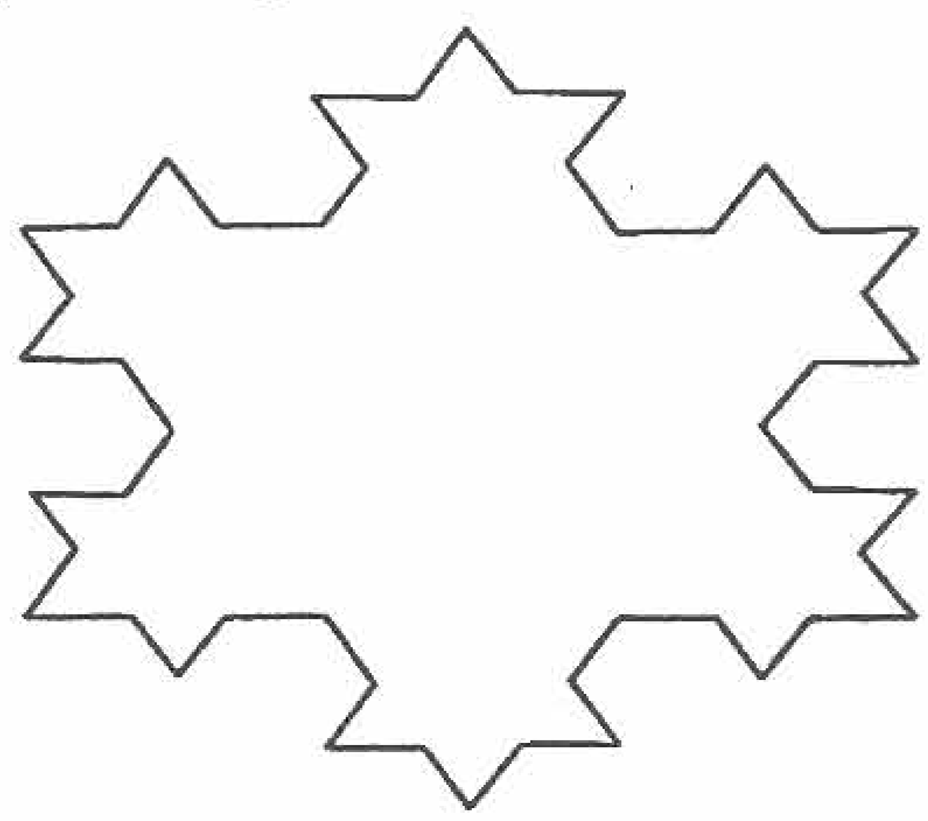
The Koch snowflake (referred to on page 3, column 1) is obtained by starting with an equilateral triangle and replacing each side by the pattern --



-- obtaining a six-pointed star.



Each of the twelve line segments of the resulting star is replaced by a similar pattern to obtain the third generation picture --



-- which begins to resemble the Koch snowflake obtained by repeated iterations of this "replacement procedure". To generate this snowflake, your students should first draw an equilateral triangle whose sides are all 27 centimeters. (When you try to draw it yourself, you'll see why it helps to start with a power of 3.

Worksheets for carrying out iterations for other fractals can be found in *Fractals for the Classroom: Strategic Activities Volume I*, by Heinz-Otto Peitgen et al., Springer-Verlag and NCTM, 1991.

Announcement...

To obtain a copy of a *Bibliography on Fractal Geometry and Chaos*, write to Hubert J. Ludwig, Department of Mathematical Sciences, Ball State University, Muncie, Indiana 47306.

Topics... What the Computer *Can* and *Cannot* Do

by Frances Marcello

In this age of seemingly limitless technology, one might assume that there is no job a computer can't do. But the real question is "Can we wait for the computer to finish?".

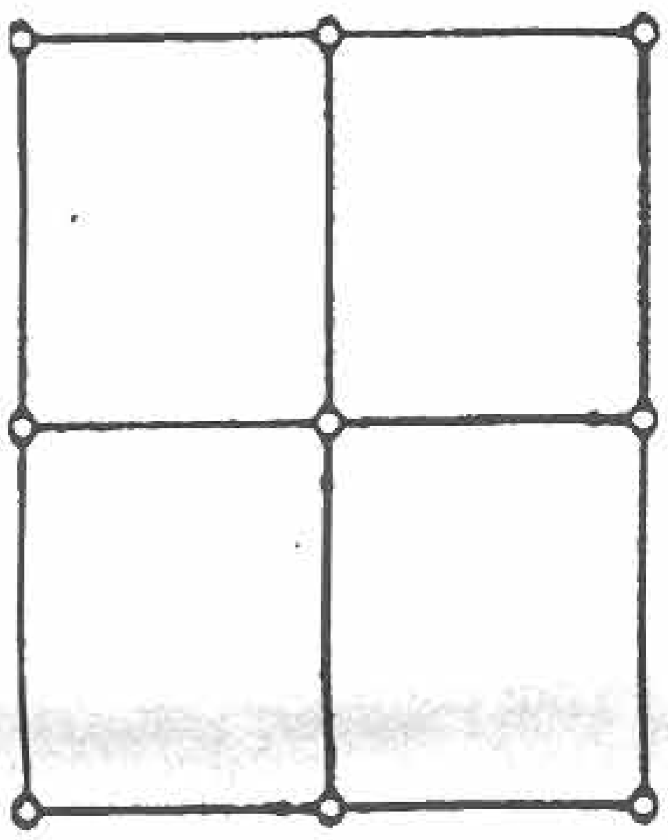
The problem used to discuss this question appears in *Dynamic Programming -- An Elegant Problem Solver*, by Cliff Sloyer et al., Janson Publications, Inc. (1987).

PROBLEM: You are given a 30 x 30 grid with a number on each edge representing the time required to travel that edge. Find the fastest path (consisting of North and East directions only) to get from point A to point B.

When I presented this problem to my students and asked for a possible solution, they immediately gave me the brute force approach -- simply find the lengths of *all* paths and then choose the smallest.

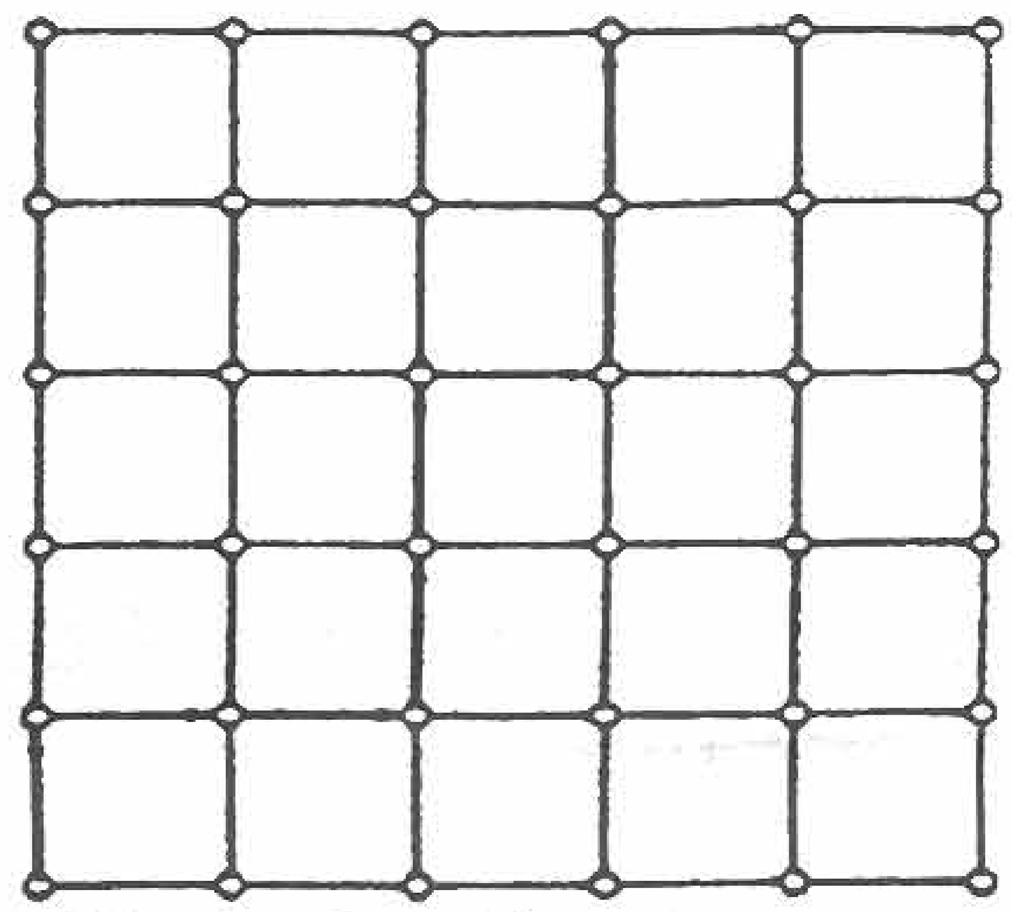
The question "How much time would you need to calculate the solution?" resulted in answers from 30 minutes to as long as 2 weeks. "How much time would a computer operating at 100,000 operations per second require?" "No time at all!!" was the unanimous opinion.

We decided to analyze the problem to confirm their opinion. Here are three simplified versions of the problem:



Consider the 2 x 2 grid at the left. A path here requires traversing 4 edges, e.g., ENEN. Since each path consists of 4 edges with 2 N's and 2 E's, there are ${}_4C_2 = 6$ paths to calculate. How many additions are necessary to determine the length of each path? E + N + E + N requires 3 additions. We need to perform ${}_4C_2 \cdot 3 = 18$ additions.

Now consider the 5 x 5 grid at the right. Here we traverse 10 edges, e.g., NEENNENNEE. The 10 edges of 5 N's and 5 E's produce ${}_{10}C_5 = 252$ paths. Each path requires 9 additions for a total of ${}_{10}C_5 \cdot 9 = 2268$ additions.



Now consider a 10 x10 grid. We are up to 20 edges of 10 N's and 10 E's resulting in ${}_{20}C_{10} = 184,756$ paths. With 19 additions per path we would have ${}_{20}C_{10} \cdot 19 = 3,510,364$ additions.

By this time students were astonished to see the number of paths and operations skyrocket as we went from 2 to 5 to 10 unit square grids. And we weren't finished. Now we had to select the shortest path!

"How many comparisons are necessary to find the shortest path?" Again we use brute force. We compare the 1st path to the 2nd, choose the smaller, compare that to the 3rd, choose the smaller, ... continuing until all paths have been compared.

Our 2 x 2 grid requires ${}_4C_2 - 1 = 5$ comparisons.
Our 5 x 5 grid requires ${}_{10}C_5 - 1 = 251$ comparisons.
Our 10 x 10 grid requires ${}_{20}C_{10} - 1 = 184,755$ comparisons.

Finally we sum the total number of operations required and find the time required for our computer to finish its job.

Total operations = (number of additions) + (number of comparisons)

Time required = (total operations)/100,000 seconds

The 2 x 2 grid requires a time of .00023 seconds, the 5 x 5 grid a time of .02519 seconds, and the 10 x 10 grid a time of 36.95119 seconds. At this point I can hear a sigh of relief; after all, even though the number of operations seemed large, the job can still be done in a feasible amount of time.

But then the class returned to the original problem, looking at the 30 x 30 grid. We have established some patterns we can use to help in the calculations. The grid has ${}_{60}C_{30}$ paths with 59 additions per path and will require ${}_{60}C_{30} - 1$ comparisons to find the shortest path. Carrying out the usual calculations yields the result that the 30 x 30 grid requires about 7.0×10^{13} seconds.

(Continued on Page 9)

Credits...

The contributors to the first issue of the Newsletter have been participants of the *Leadership Program in Discrete Mathematics* at Rutgers University, New Brunswick, New Jersey.

The editors of this issue of the Newsletter are Susan H. Picker and Joseph G. Rosenstein. This issue was composed on Pagemaker by Virginia Moore of DIMACS.

The *Leadership Program in Discrete Mathematics* is funded by the National Science Foundation (NSF) and is co-sponsored by the Rutgers University Center for Mathematics, Science, and Computer Education (CMSCE) and the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS), which is also providing the funding for the Newsletter. Joseph G. Rosenstein is Director of the *Leadership Program in Discrete Mathematics*.

IN DISCRETE MATHEMATICS... Using Discrete Mathematics in the Classroom

is published three times a year by the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS) at Rutgers University, New Brunswick, New Jersey. Communications should be sent to the following address:

CMSCE - DM NEWSLETTER
P.O. Box 10867
New Brunswick, New Jersey 08906.

PHONE: 908/932-4065
FAX: 908/932-3477
EMAIL: joer@math.rutgers.edu

Announcement...

The next issue of this Newsletter will describe programs in discrete mathematics planned for the summer of 1992. Readers are asked to submit information about such programs. The *Leadership Program in Discrete Mathematics* at Rutgers University is described in more detail on page 11, as is a program based at Boston College.

Authors...

L. Charles (Chuck) Biehl teaches at Thomas McKean High School in Wilmington, Delaware.
Ruth Ann Krayesky teaches at Eisenhower Middle School in Bridgewater, New Jersey.
Julia E. Magana teaches at Washington Township High School in Sewell, New Jersey.
Elyse Magram teaches at Smithtown High School West in Smithtown, New York.
Joseph Malkevitch is a professor of mathematics at York College of the City University of New York.
Frances Marcello teaches at Oceanside Senior High School in Oceanside, New York.
Susan H. Picker is a Staff Development Specialist at the Office of the Superintendent for Manhattan High Schools, New York.
Joseph G. Rosenstein is a professor of mathematics at Rutgers University, New Brunswick, New Jersey.

Subscriptions...

Please send us the name, address, phone number, and school of any teacher who should receive a copy of this Newsletter, and we will include him/her on our mailing list.

Encouraging words...

This is *your* Newsletter -- that means that its success will be dependent on the willingness of you the readers to share your discrete thoughts and classroom experiences -- your use of written materials and software -- your information about resources -- your questions and responses -- your cartoons and problems -- your articles and announcements.

So while you are going about your way in discrete mathematics, keep the Newsletter in mind, and if you notice something that might be of interest, write a few paragraphs to submit to the Newsletter.

You will be thanked profusely by the other readers of *IN DISCRETE MATHEMATICS... Using Discrete Mathematics in the Classroom.*

Feedback...

Your comments on this issue, however brief, will be valuable to us in planning future issues. Here are some questions to which you can respond:

Which articles did you find most (or least) useful or interesting?
What types of articles would you like to see more of?
Was there an appropriate balance between different types of articles?
What suggestions do you have for future issues?

DIMACS...

The Center for Discrete Mathematics and Theoretical Computer Science (DIMACS) is a national Science and Technology Center (STC) founded by the National Science Foundation (NSF); it was formed in 1989 as a consortium of four institutions -- Rutgers University, Princeton University, AT&T Bell Laboratories, and Bell Communications Research.

Topics... Two Problems Involving Graphs

by Joseph G. Rosenstein

Here are two problems which can be stated very simply; the first may be familiar to you but the second you have probably not seen before. What do they have in common, and how are they connected with discrete mathematics?

PROBLEM 1: Can you use 31 1x2 dominos to cover the 62 squares of an 8x8 chessboard obtained by deleting two diagonally opposite corners?

PROBLEM 2: A mouse eats her way through a 3x3x3 cube of cheese by tunnelling through all of the 27 1x1x1 minicubes. If she starts at one corner of the cube and always moves to an adjacent uneaten mini-cube, can she finish at the center of the cube?

You may want to think about the first problem for a few minutes, in which case you should probably stop reading this article for the time being. You may even want to find a chessboard and play with the problem a bit.

If you solved the problem, you realized first that the two unused squares have the same color, and then noticed that the two squares covered by any domino have opposite colors. (Unless you are really good at "visualization", you would probably not have discovered the first fact without using a real chessboard.) Thus 31 non-overlapping dominos must cover 31 squares of one color and 31 squares of the other color, leaving uncovered one square of each color! So diagonally opposite corners cannot both be uncovered.

If your students have learned a little about graphs, they should probably be able to discover the graph that underlies this problem. It is a graph with 64 vertices, namely the squares of the chessboard; two vertices are adjacent (in the graph theory sense) if the corresponding squares are adjacent (in the physical sense).

The normal coloring of the chessboard -- involving red and black squares -- provides what is called a coloring of the graph. In a coloring of a general graph, adjacent vertices must have different colors. Graph colorings have many applications, to problems as diverse as map colorings, scheduling committee meetings (see article to the right and box on page 10), traffic lights, and radio frequency assignments. If you are interested in learning more about colorings of graphs, one source is *The Mathematician's Coloring Book*, by Richard L. Francis, in the HiMAP Module Series published by The Consortium for Mathematics and Its Applications (COMAP).

Graphs which can be colored using two colors are often called bipartite graphs, since the vertices can be separated into two sets with adjacency occurring only between vertices in different sets. The chessboard graph is bipartite

(Continued on page 10)

Spreading the Word... Introducing Teachers to Discrete Mathematics

by L. Charles Biehl

On March 21-22, the Maryland State Department of Education held the third annual Dwight D. Eisenhower Mathematics and Science Conference in Baltimore, Maryland. The focus for mathematics teachers was the NCTM Standards, and I was fortunate to be selected to give a presentation in discrete mathematics, entitled *From Final Exams to Traffic Jams: Using Graphs to Resolve Conflicts*.

The audience consisted of more than forty math teachers from all over Maryland, only a few of whom were familiar with graphs. The purpose of this presentation was to give these teachers enough exposure to feel comfortable with the basic ideas of graphs and graph coloring, to provide them with access to additional materials for further study, and to enable them to teach a one or two day lesson on the topic in their own classes.

After laying the foundation for the topic, I showed that conflicts that arise in a variety of situations can be modeled with graphs; examples include scheduling meetings for people who had multiple responsibilities (see *Illustration* on bottom of page 10), assigning frequencies to mobile radio telephone relay stations, scheduling final examinations at a small college, and sequencing green lights at an intersection. To resolve the conflicts in each situation, we had to define what the conflict was and assign the minimum number of "colors" (or meeting times, or relay towers or traffic light changes) to ensure that no one had to be two places at the same time, no radio frequencies interfered with each other, no cars collided, etc.

All the actual problem solving was done by the participants; I encouraged them to work in pairs or groups. Once the ice was broken and the first problem had been solved (in more than one way, I should add) the workshop continued as a lively problem-solving session, with participants convincing themselves of the correctness of their solutions by comparing notes with neighbors. The final ten minutes of the hour was spent discussing the underlying concepts, their applications to other and more diverse situations, and a plethora of potential classroom activities.

This presentation was a big step for me. It was one thing to present this material to a small group of colleagues in my district; but to present to a group twice the size, and to strangers no less, filled me with apprehension and a fear of being asked questions I could not attempt to answer. This must be the feeling that we all had when we first entered the profession. After the first five minutes I was completely at ease, and I felt that the participants were too.

Discrete mathematics topics work extremely well in this type of environment, since they can be learned and appreciated at many levels. The same is true in the classroom,

(Continued on page 10)

Mini-bibliography... Graph Theory

by Joseph Malkevitch

One of the most appealing topics in discrete mathematics is graph theory. The subject is quick starting, geometric, rich in applications (e.g. robot motion planning, examination scheduling, snowplow routing, etc.), and abounds in easy to explain unsolved problems. Here is a bibliography describing six relatively elementary books on the subject, followed by three intermediate level books and three which are more advanced.

Elementary:

Biggs, N., Lloyd, E., and Wilson, R. ***Graph Theory 1736-1936****, Clarendon Press, Oxford, 1986.* This is a paperback reissue of an earlier hardbound history of graph theory. It includes excerpts from some of the major papers contributing to the early development of the field.

Chartrand, Gary, ***Introductory Graph Theory****, Dover, New York.* This paperback book covers digraphs, traversability problems, connectivity, and mathematical modeling.

Malkevitch, Joseph, and Meyer, Walter, ***Graphs, Models, and Finite Mathematics****, Prentice-Hall, Englewood Cliffs, 1974.* This book's beginning chapters illustrate how by using graphs to construct mathematical models various problems in operations research can be solved. Topics treated include traversability, the critical path method, and coloring problems.

Ore, Oystein, ***Graphs and Their Uses****, Mathematical Association of America, Washington, 1963. (Revised edition: 1991, R. Wilson).* This introductory paperback book covers coloring problems, puzzles and games, traversability problems, trees, and matchings. The original version is a bit dated, but the revised version, updated by Robin Wilson, includes significant new material.

Steen, Lynn (ed.), ***For All Practical Purposes****, (second edition) W. H. Freeman, New York, 1991.* The beginning chapters of this book apply mathematical modelling techniques to a variety of problems. Topics treated include traversability problems, minimum cost spanning trees, and the critical path method. Five video tapes from the TV series with the same title support the written materials.

Wilson, Robin, and John Watkins, ***Graphs: An Introductory Approach****, John Wiley and Sons, New York, 1990.* Topics covered include: planarity, trees, colorings, digraphs, and applications of these concepts.

Topics... Recursively Expanding Enthusiasm

by Elyse Magram

One can't help but be amazed at the number of rabbits that are predicted by the Fibonacci numbers, a recursive pattern which begets a combinatorial explosion -- 75,025 rabbits by the twentieth generation, and many more to come, all generated from the simple recurrence relation $r_{n+2} = r_{n+1} + r_n$, where $r_0 = 1$ and $r_1 = 1$. This reminds me of the enthusiasm that has steadily grown in me since meeting discrete mathematics.

After 25 years of teaching, with two children in college, it seemed to be a good time to expand my horizons. "Why pursue math?" asked my friends, "take something for fun, something light and colorful, something for joy." How could they understand that all this and more could be fulfilled for me in a math institute. This I found when I attended the Leadership Program in Discrete Mathematics at Rutgers University.

The work was overwhelming. The professional contacts were superior. The interrelationships between the participants, dedicated teachers, was incredible. We were exposed to distinguished speakers who introduced us to graph theory, algorithms, combinatorics, fair elections, and computational geometry. We worked together, alone, in twos and fours -- all day, in the evening, and on the weekends, too. We griped, we laughed, we sang, we burned the midnight oil. The contact and the sharing have been a true highlight in my teaching this year.

The material is exciting to use in class. The Tower of Hanoi was an excellent motivation for sequences and series in precalculus. Matrices were greatly enhanced by introducing the topic of secret codes. Map coloring and optimization were exciting to students at all levels. My students enjoyed doing an optimal time problem for lasagna preparation. Minimum spanning trees intrigued the slowest students, especially in planning condominium roads (see *Illustration* on top of page 10) and going through rat mazes. My students' intuition was awakened, and they found that math is really fun. There is such wonder in seeing the range of applications that mathematics has.

Teaching discrete mathematics can change your outlook as well; I urge you to search out a local program to add vitality and joy to your teaching.

Solution... TSP *(Continued from box on page 1)*

There are altogether 24 possible routes, corresponding to the 4! arrangements of B, C, D, and E; however, only 12 calculations are necessary since the 24 includes the reverse of each route. The shortest route is ACBEDA (or its reverse) with a total length of 92. Curiously, you don't travel from C to D which are the nearest pair of cities.

The Traveling Salesman Problem...

(Continued from page 1)

to camp, sequence of motions of a robot arm in working on a job, etc.

The TSP offers opportunities to study algorithms, to look at different kinds of distance, to use enumeration methods, to investigate mathematical modelling, and even to allow grade schoolers to practice arithmetic.

The Wall Street Journal article cited below gives an account of some new ideas developed by two employees of Dupont to solve the assymetric TSP. The New York Times article is concerned with recent work on TSP problems which involve a very large number of sites; it has little overlap with the other article. An elementary treatment of the TSP can be found in Chapter 2 of the book cited at (4). A history of the TSP can be found in the first article (by A. Hoffman and P. Wolfe) of the book cited at (5); this book consists of many excellent survey articles, parts of which are accessible to all readers.

Bibliography:

(1) *Mathematicians Find New Key to Old Puzzle*, The Wall Street Journal, February 15, 1991, pp. b1,b2.

(2) Kolata, Gina, *Math Problem, Long Baffling, Slowly Yields*, The New York Times, March 12, 1991, pp. c1,c7.

(3) Miller, D. L., and Pekny, J. F., *Exact Solution of Large Asymmetric Traveling Salesman Problems*, Science, 251 (1991) 754-761. (This is the article on which the Wall Street Journal article is based.)

(4) Steen, L., (ed.), *For All Practical Purposes*, (2nd edition), W. H. Freeman, New York, 1991.

(5) Lawler, E. L., et al., eds., *The Traveling Salesman Problem*, John Wiley, New York, 1985.

Topics... What the Computer *Can* and *Cannot* Do

(Continued from page 5)

"Is this a long or a short period of time?" My students are not sure. We calculate that there are approximately 3.15×10^7 seconds per year and so the job will require $(7 \times 10^{13}/3.15 \times 10^7) = 2 \times 10^6$ or *2 million years* to complete, and that's a conservative figure.

Student reactions ranged from a simple "Wow!!" to "I wouldn't want to pay that electric bill!". And the response I was looking for -- "What do we do now?". The following sessions covered short-cut algorithms, including dynamic programming (see reference cited above). And no one in class lost sight of the fact that time efficiency was a crucial element in any algorithm we analyzed.

Another good example of "computational explosion" or "computational infeasibility" is given in *Number Theory and Public-Key Cryptography*, Mathematics Teacher, January 1991. Here the time required for a computer to factor the product of two 100 digit primes is 3.8 billion years, making this cryptosystem reasonably secure.

Ask a Discrete Question...

Dear Euler, Having read the article on page 5 on the limitations of computers, I understand that building a computer which runs twice as fast will only cut the time in half. But one of my students suggested that if each year computers double in speed, then the "computation explosion" will eventually catch up with the "combinatorial explosion". Is that true? *Perplexed.*

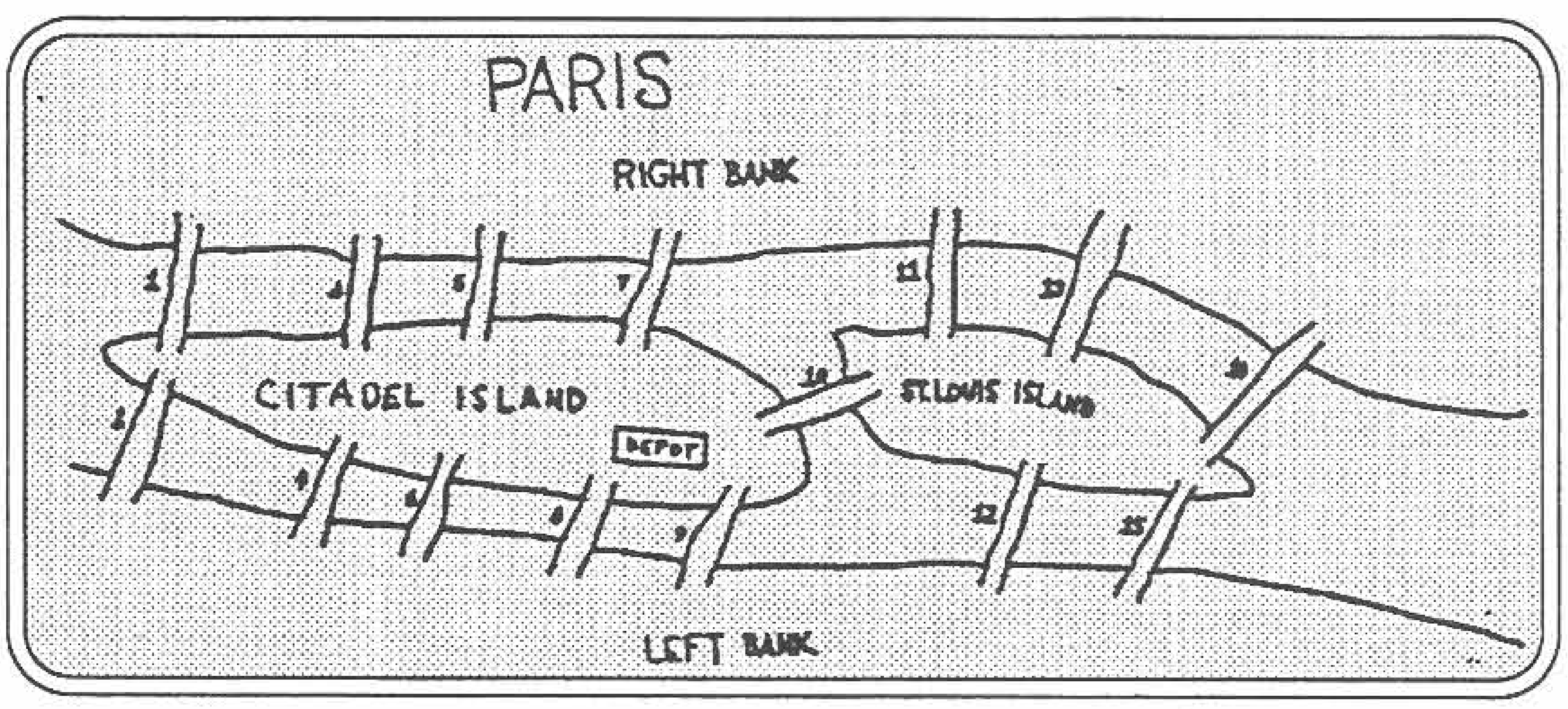
Dear Perplexed, First of all, that's a very big "if." Secondly, eventually can be a long time. If you remain perplexed, I suggested you seek out a mathematical counselor.

Illustration... Maps and Graphs

(Continued from article on page 3)

Is it possible for the Paris bridge sweepers to leave their depot on Citadel Island, sweep each bridge just once and return to their depot?

A huge potato spill has resulted in the closing of bridges 1 and 2. Can the sweepteam now start at the depot, sweep bridges 3-15 and return to their depot without repeating a bridge? Draw a graph and explain your answers.



Topics... Two Graph Coloring Problems

(Continued from page 7)

because red squares are adjacent only to black squares and conversely. Try drawing a few other bipartite graphs and verifying that their vertices can be colored using two colors.

Your assignment, if you accept it, is to figure out what all of this has to do with the second problem. (And, if the terms are familiar to you, how these problems involve matchings and Hamilton paths.) The solution will appear in the next issue. If you use these problems in class, please write a few paragraphs for the next Newsletter telling us what happened.

Spreading the Word... Introducing Teachers to Discrete Mathematics

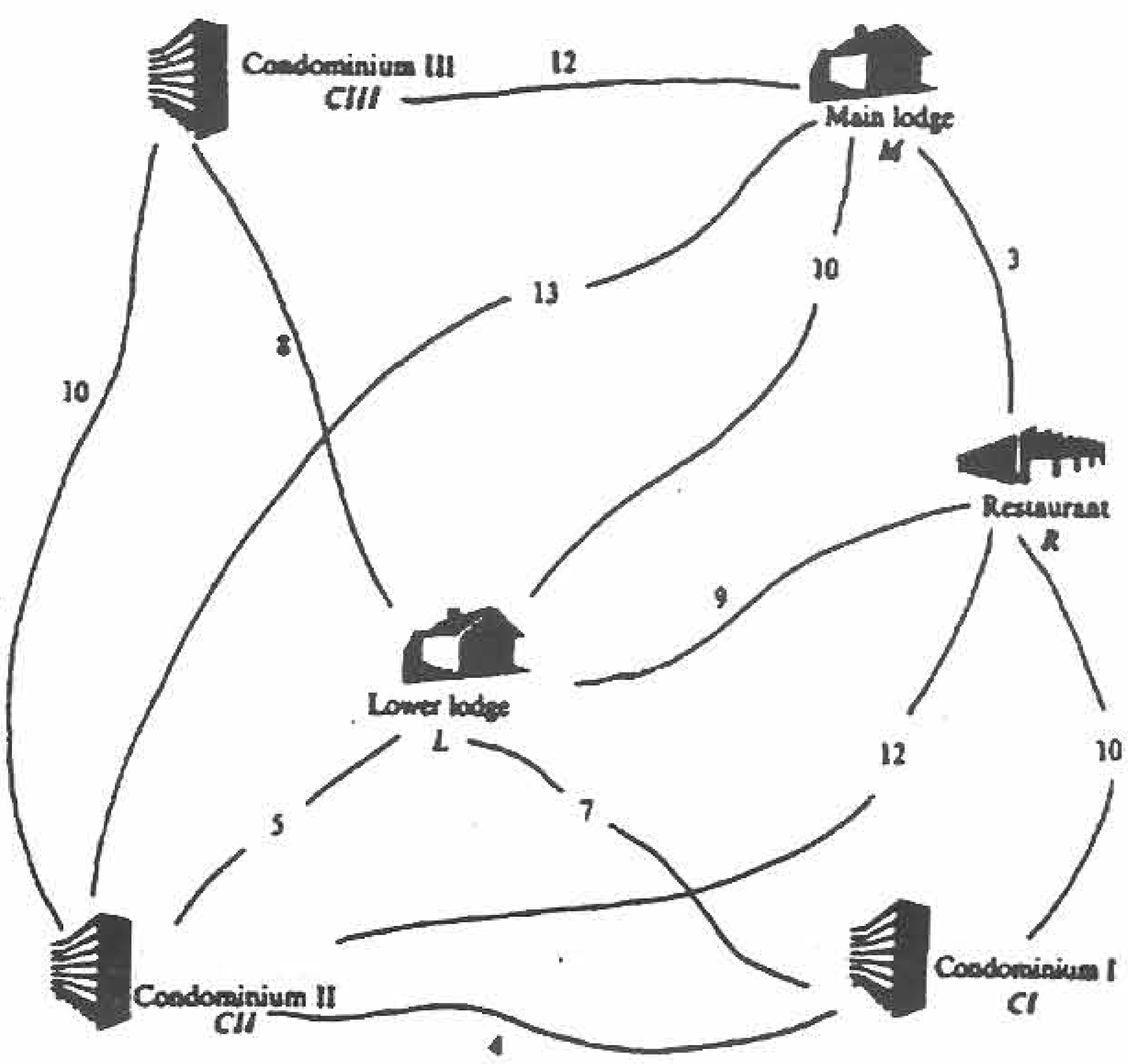
(Continued from page 7)

and I can say with assurance that the enthusiastic response received from teachers in workshops I have conducted has been matched, if not exceeded, by the response of the students in my own classes.

Illustration... Planning Roads

(Continued from article on right of page 8)

Which of the roads in the picture should be built if we want to connect all six locations at the smallest possible cost? (The cost to build each road is given in thousands of dollars.)



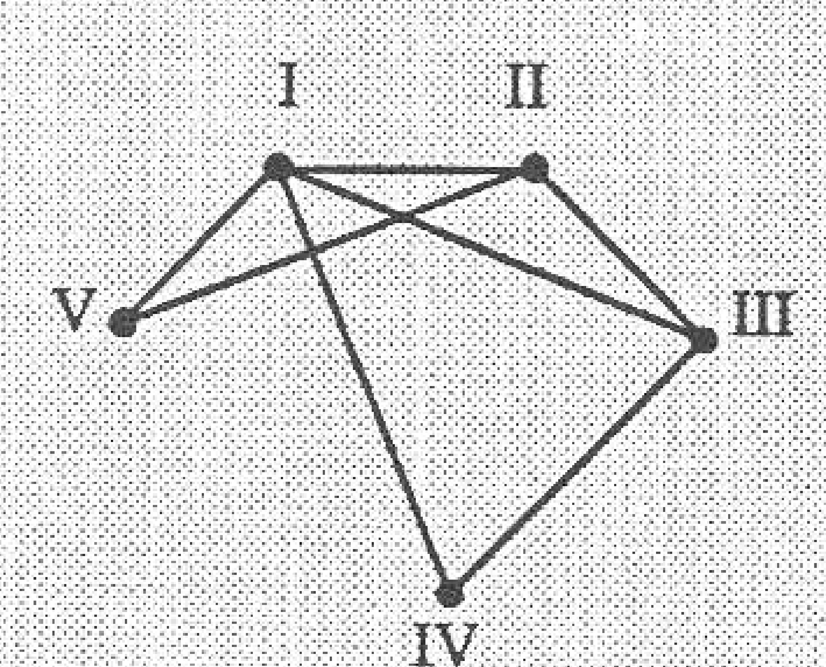
This problem is taken from HiMAP module 6, *Problem Soving Using Graphs*, by Margaret Cozzens and Richard Porter, COMAP, Arlington, MA (1987).

Illustrations... Scheduling Meetings

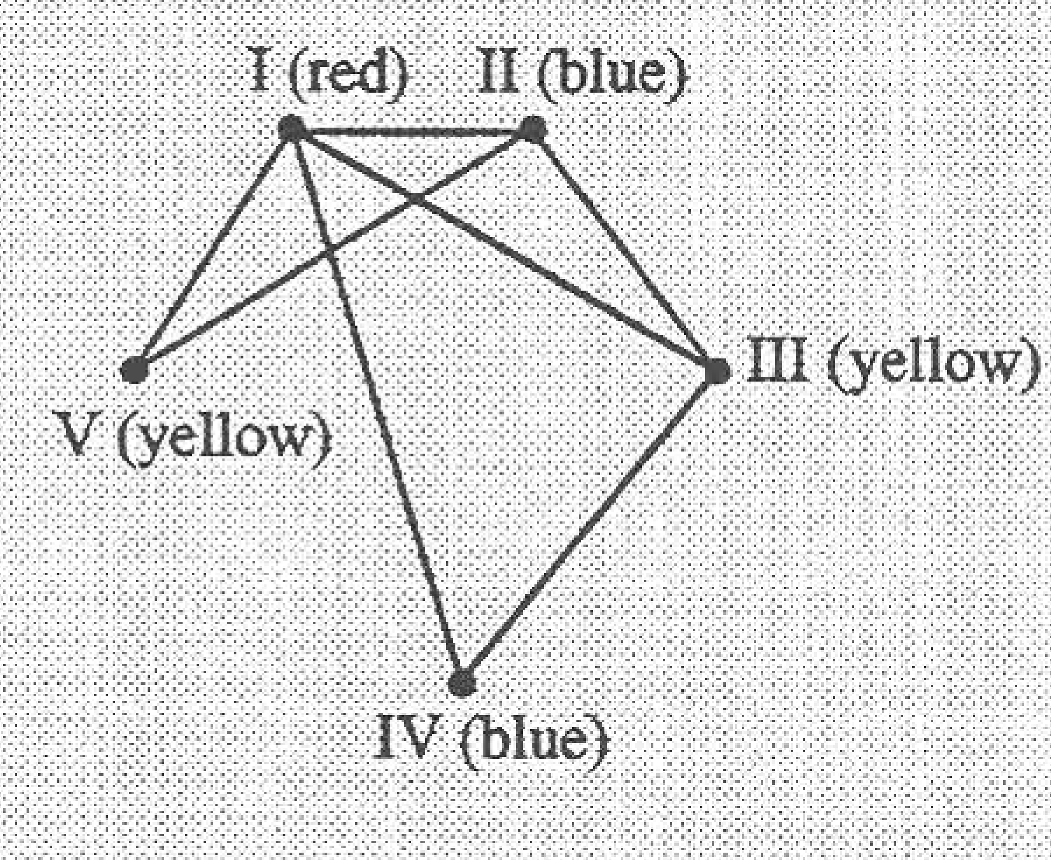
The general problem is to schedule meetings for a number of committees so that no committee member has a conflict.

For example: The membership of committee I is A, B, C and D; of committee II is A, C, E, and F; of committee III is A, B, F, H, and I; of committee IV is B, G, H and I; and of committee V is C, D, E, and G. Can all five meetings be scheduled if only three meeting times are available?

Construct a graph where the vertices are the committees and where an edge is drawn between two vertices (committees) if they have overlapping membership (conflict!)



A coloring of the vertices of this graph in which adjacent vertices are assigned different colors would provide an appropriate meeting schedule. Thus the coloring below



provides a positive answer to our question. Schedule committee I for one meeting time (red), committees II and IV for a second meeting time (blue), and committees III and V for the third meeting time (yellow).

Mini-bibliography... Graph Theory

(Continued from page 8)

Intermediate:

Barnette, David, ***Map Coloring, Polyhedra and the Four-color Problem****, Mathematical Association of American, Washington, 1983.* This book treats topics about graphs and polyhedra related to the four-color problem. Euler's polyhedral formula (V-E+F=2) is treated in detail.

Beineke, Lowell, and Wilson, Robin (eds.), ***Selected Topics in Graph Theory, 1, 2, and 3; Applications of Graph Theory****, Academic Press, New York, 1978, 1983, and 1988.* These volumes contain a collection of survey articles which cover a tremendous amount of the graph theory landscape. Topics covered include hamiltonian ciruits, chromatic polynomials, communications networks, applications to architecture, etc. Although nominally designed for researchers in graph theory, these books can be looked at for ideas by relative beginners.

Capobianco, M., and Molluzzo, J., ***Examples and Counterexamples in Graph Theory****, American Elsevier, New York, 1978.* This book includes a rich variety of graph examples that show that certain theorems are best possible.

Advanced:

The last three books are popular advanced undergraduate and graduate texts. However, since graph theory is so relatively accessible, parts of these books will be appealing to relative newcomers to the subject.

Bondy, J.A., and Murty, U.S.R., ***Graph Theory with Applications****, American Elsevier, New York, 1979.*

Chartrand, G., and Lesniak, L., ***Graphs and Digraphs (second edition)****, Wadsworth, and Brooks/Cole, Monterey, California, 1986.*

Harary, F., ***Graph Theory****, Addison-Wesley, Reading, Massachusetts, 1969.*

Implementation of NCTM Discrete Mathematics Standard Project

This three year project based at Boston College gets underway July 13, 1992 with a three week summer leadership workshop in which teachers with prior experience teaching discrete math are trained to become members of leadership teams that will instruct groups of teachers in summer workshops at six sites in years two and three of the project. Information and applications are available from the Project Director, Dr. Margaret Kenney at the Boston College Mathematics Insitute, Chestnut Hill, MA 02167.

Leadership Program in Discrete Mathematics -- Summer 1992

During the summer of 1992, the fourth annual *Leadership Program in Discrete Mathematics* will take place at Rutgers University, New Brunswick, New Jersey. Two three-week residential institutes are scheduled for **June 29 to July 17, 1992.**

One institute will be designed primarily for high school teachers; a second parallel institute will be designed for middle school teachers and elementary mathematics specialists. Middle school teachers may attend either institute. Participants will be expected to attend follow-up sessions during the 1992-1993 school year and a one- or two-week program in the following summer.

Participants will also be expected to develop materials and activities for incorporating discrete mathematics topics in their classes, to play leadership roles in introducing these topics into their schools and curricula, and to conduct workshops on these topics in their schools and districts.

The three main topics in the three-week program for high school teachers will be applications of graphs, algorithms and graphs, and combinatorics. In the following summer, additional topics in discrete mathematics will be covered during a two-week program.

The three-week program for middle school teachers will deal with applications of graphs, combinatorics, probability, geometry, and fractals. In the following summer, a one-week program will be designed to help participants consolidate their knowledge of these topics.

Also offered will be a seven-day leadership training program for teachers who are experienced with discrete mathematics. Participants will develop materials for in-service workshops and will be expected to offer these workshops in various schools during the 1992-93 school year. The dates of the program are May 15-16 and July 27-31, 1992.

Anticipated funding from the National Science Foundation will pay for participants' room and board, and a stipend of $300 per week of the program. The *Leadership Program in Discrete Mathematics* is sponsored by the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS) and the Center for Mathematics, Science, and Computer Education (CMSCE) at Rutgers University.

Applications will be due by March 13, 1992. To receive an application form, please call Stephanie Micale at 908/932-4065, or write to CMSCE - Leadership Program, P.O. Box 10867, New Brunswick, New Jersey 08906.

What Do Mathematicians Do?

If one asks the "person on the street" what plumbers, electricians, chemists, or geologists do, they are likely to give you a reasonable answer. Put in more dramatic terms, when home-owners see water cascading through the ceiling, they do not call a carpenter or a mathematician. But few people on the street know when to call a mathematician.

One thing that we can do about mathematics' image problem is to discuss how mathematics affects people's lives, even if we cannot always do proper justice to the mathematics involved. For example, we can say that mathematicians (not chemists or plumbers) study waiting lines, and show that this can be applied at banks, airports, and in computers. Or, we can say that mathematicians find shortest paths and networks, and show that this can be applied to travel arrangements and telephone connections.

The above paragraphs are adapted from "Mathematics' Image Problem" by Joseph Malkevitch (see address on page 11). The cartoon on the right was drawn by Joe Pipari one of whose colleagues at Thomas McKean High School (Wilmington, Delaware) participated in a discussion with Malkevitch at the 1990 Leadership Program in Discrete Mathematics.

"HON-EEEEE ...CALL A MATHEMATICIAN!"

CMSCE - DM NEWSLETTER
P.O. Box 10867
New Brunswick, New Jersey 08906

Non-Profit Org.
U.S. Postage
Paid
New Brunswick, NJ
Permit No. 157