IN DISCRETE MATHEMATICS

Using Discrete Mathematics in the Classroom

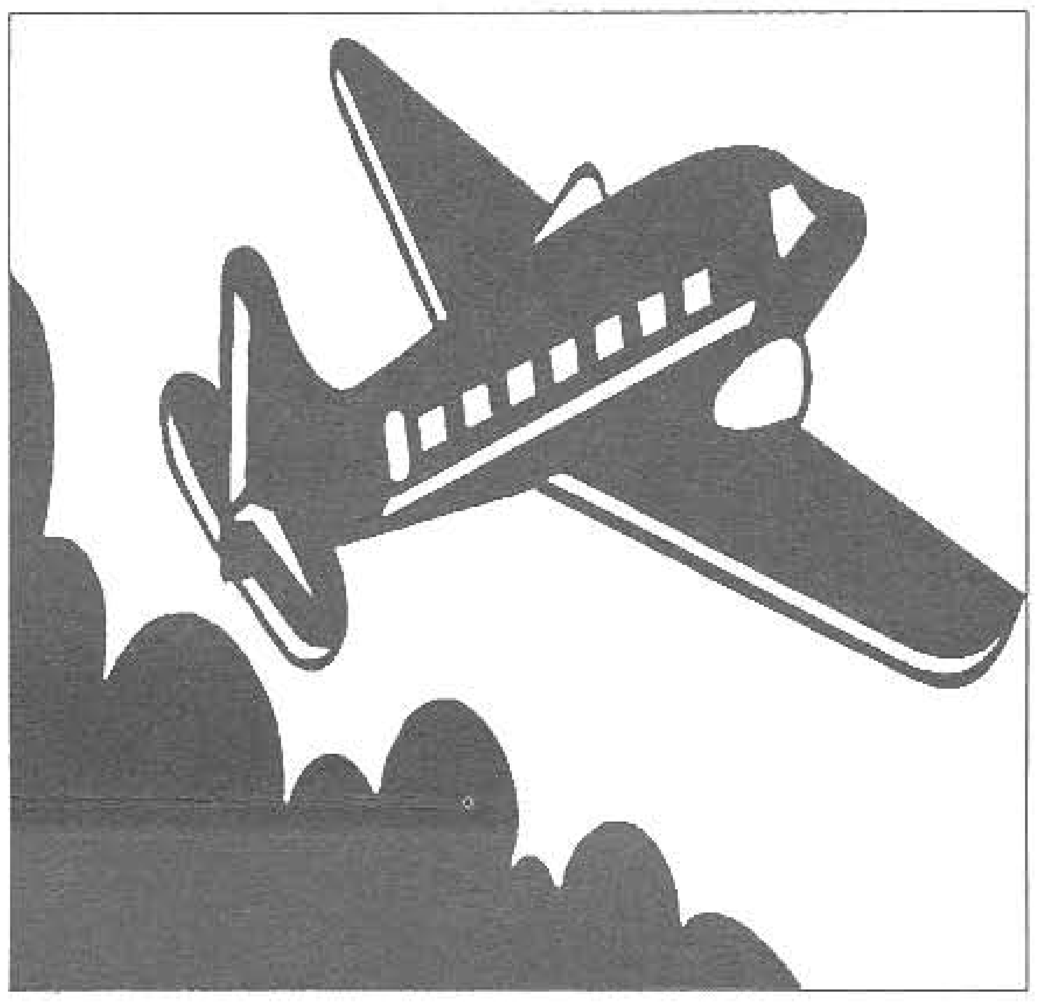
Issue #5 *November 1994*

Speaking discretely...

by Deborah S. Franzblau

First, some news on the *Leadership Program in Discrete Mathematics*. Beginning next summer, we plan to introduce new programs for K-8 teachers, and to continue our in-service workshops. Please tell your colleagues, and circulate our flyer (p. 11) widely!

If you've ever tried scheduling final exams, or organizing a conference, you'll appreciate the focus in this issue on discrete scheduling problems. In the lead article (p. 1) and Minibibliography (p. 10), Joseph Malkevitch discusses the mathematics of scheduling, and sketches the useful "Critical-Path" method for planning complex projects. L. Charles Biehl (p. 4) explains the use of graph coloring to resolve conflicts when scheduling meetings or exams. Kevin DeVizia (p. 2) describes a class project to find the best arrangement of songs on a cassette tape---which turns out to be equivalent to a well-known scheduling problem!



If you are looking for ideas to enliven the class around election time, you should look at the article by Michael Ecsedy (p. 5), showing how a method of voting influenced a real election, and the teaching brief by Sherida Hare (p. 4).

Diane Amelotte (p. 7), takes a fresh look at a familiar problem from algebra and calculus: that of creating a garden plot with minimum cost. She turns the problem into a Thanksgiving story suitable for students at many levels. William Bowdish (p. 3) shares his experience on bringing the concept of "fractal complexity" into an algebra class.

In This Issue

Is It on Time?

by Joseph Malkevitch

For many years Denver has been looking forward to the opening of a new international airport, which would not only expand operations, but would feature a state-of-the-art baggage system with unprecedentedly fast and accurate handling of baggage. Unfortunately, "there is often a slip twixt the cup and the lip..." After several postponements, the opening of the airport has once more been delayed: in the most recent test of the baggage handling system, pieces of baggage were not only ripped, cut in half, and tossed in the air, but they often arrived at places other than those intended [1].

Contrast this with what happened in Los Angeles after the recent earthquake. That earthquake knocked out of commission the largest and most heavily used freeways in the city, including the legendary Santa Monica Freeway, one of the highest-volume roads in the world. Yet despite predictions that the road would not be returned to service quickly, it was reopened earlier than scheduled, with people tripping over each other to receive credit [2].

Efficient scheduling is often critical in modern-day life. For example, if surgical operations at hospitals can be scheduled more efficiently, millions of dollars for the construction, maintenance and staffing of additional operating theaters can be avoided. If the production of new cars can be organized and carried out more efficiently, then the cost of cars can be cut and the United States can become more competitive with other car producers.

Scheduling used to be a trial-and-error procedure. Someone tried out a schedule and tinkered with it to see if it got better. If no improvement occurred, someone else would tinker in a different way. The advent of computers has vastly increased our ability to search for improved schedules. In addition, computers have created a

(Continued on page 9)

A Musical Packing Problem

by Kevin DeVizia

I find it more challenging to make math seem important to my unmotivated Algebra I students than to any other group that I teach. In order to convince such students that mathematics is useful, I've often found that the best examples are all around me in everyday life, just waiting to be noticed. I found myself in the car one day stuck in a traffic jam, when the cassette tape to which I was listening stopped playing. After a long wait, the tape still was not playing. Finally, switching the tape to the other side revealed that there was a great disparity between the total lengths of songs on the two sides of the tape. Could there be a better way to organize the songs so that the "dead time" on the tape could be minimized? Great! Another application for my kids to think about.

Listed below are the times of the songs on the two sides of this cassette (in minutes:seconds).
Side 1: 4:10, 2:39, 3:59, 3:47, 3:20, 3:11 (Total = 21:06).
Side 2: 3:21, 4:38, 2:32, 3:49, 1:47, 3:40 (Total = 19:47)

The data above gives a disparity of 1:19 and a completion time of 21:06. Essentially, this is a bin-packing problem, in which the songs are items to be packed in two variable-sized bins. (See the sidebar on **Packing and Scheduling**.) While my Algebra I students had never heard of bin-packing, they could certainly relate to this application.

I asked my students how to arrange the songs to achieve the minimum disparity, and a lively discussion ensued, with students offering different methods of attack. Occasionally, a student would be sure that the perfect solution was found--only to find an error in computation or someone with a better answer. Eventually the students wanted to know, "Well, what is the answer?" Of course, in my excitement over the problem, I never did decide on a solution, and this was all for the best--it was up to the students to check whether we had found the best solution. Could we be sure? As one student pointed out, we could list all possible ways to place the songs on the tape and then be sure. No problem, except that there are 2048 different ways to do this (this includes ridiculous arrangements, like all 12 songs on side one). By hand, the best solution my class found has a disparity of only 3 seconds. Of course, this does not take into account any aesthetic concerns for arrangement of songs. An interesting variant would be to classify songs as "slow" or "fast" and require that each side have an alternating sequence of these two types. This problem inspired my class to share a wide variety of creative strategies, and allowed students to interact in a meaningful way with mathematics and with each other.

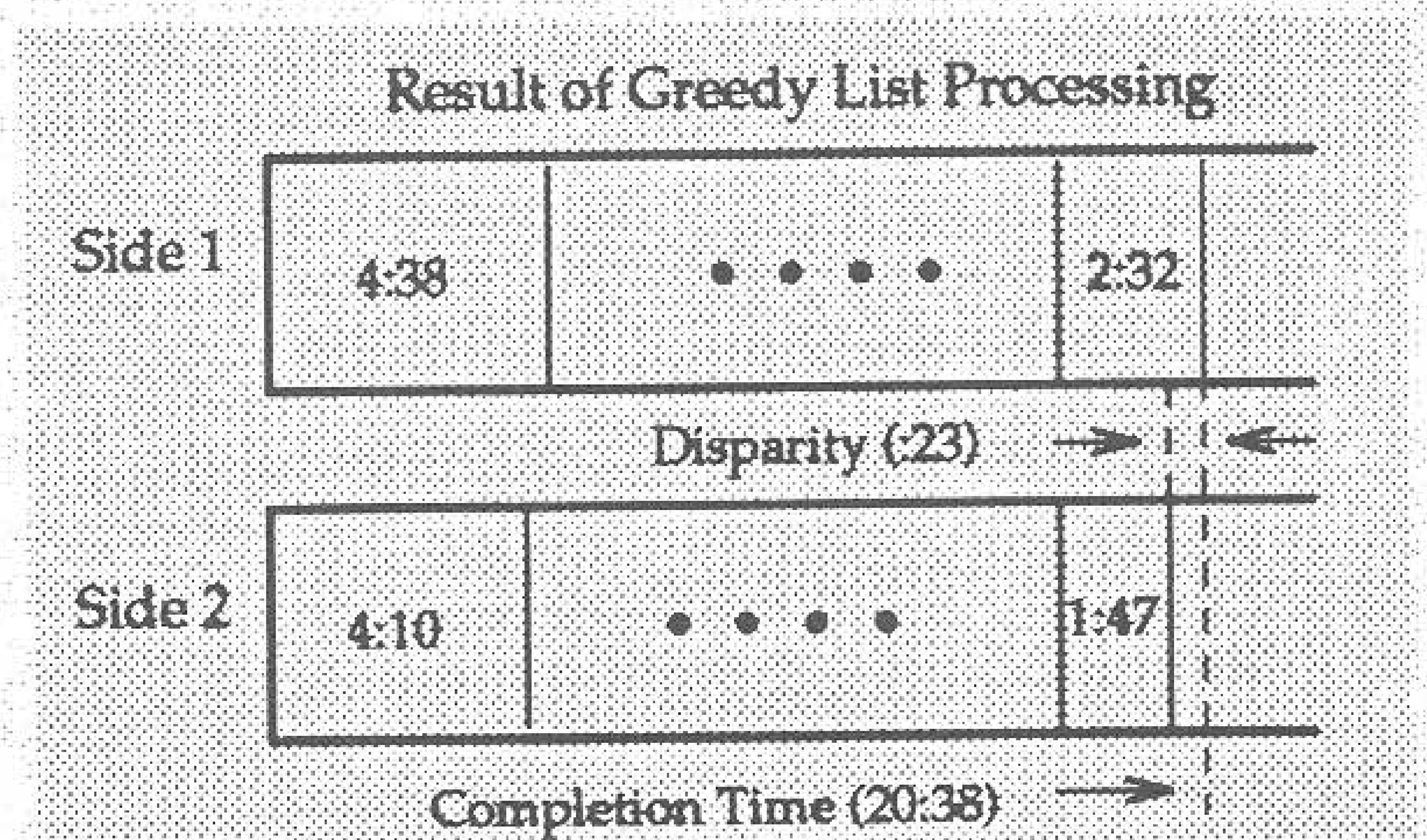
Note: an earlier version of this article appeared in the Newsletter of the Pennsylvania CTM in Spring, 1993.

Packing and Scheduling

It may not be obvious that the musical packing problem has any relation to scheduling. However, the problem is essentially the same as "Two-processor Scheduling" [1]. In two-processor scheduling, you are given a list of tasks (in this case a list of songs to be played), along with the time needed to complete each task. You have two equivalent machines (one side of a cassette in this case) which can process the tasks; the problem is to assign the tasks to the machines to minimize the completion time. The two-processor scheduling problem is known to be NP-hard [1, 2] (which tells us that there is probably no fast algorithm to solve the problem exactly for large inputs). Thus, heuristic strategies, such as letting the class compete to get the best answer, may be quite practical!

It turns out that there is a simple, greedy "list processing" strategy which yields a completion time which is at worst 7/6 (1.17) of the best possible completion time (see [1, 2]):
(1) sort the tasks from longest to shortest;
(2) each time a machine becomes idle, assign the next task on the list.

If you use this strategy on the data above (converting to decimal notation first) you end up with two sides of approximately 20.63 and 20.24 minutes, giving a disparity of .39 min. (about 23 sec.) and a completion time of 20.63 min. The best possible completion time must be



at least half of the total time (40.87 min.), or 20.435 minutes; The ratio 20.63/20.435 ≅ 1.01, which is less than 7/6 as claimed.

References

1. Garey and Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness,* W.H. Freeman, 1979, p. 65, 238.
2. COMAP, *For All Practical Purposes*, 3rd Ed., W.H. Freeman, New York, 1994, pp. 83-88.

Finding the Fractal Complexity of a Coastline

by William L. Bowdish

Veteran teachers are fully aware that 14 year olds know everything that there is to know about mathematics! However, when I gave this lesson on fractal dimension (complexity) to my accelerated 9th-grade honors-algebra class, they were truly amazed. I got the idea for this lesson from Terry Perciante, who taught an excellent one-week course in the Leadership Program in Discrete Mathematics at Rutgers during the summer of 1993. To prepare this three-day unit, I needed only a few hours of preparation and reading [1, 2].

I decided to ask my class to compute the complexity of the coastline of Martha's Vineyard, a gorgeous island off the coast of Massachusetts, and focus on using the "compass method" to calculate its fractal dimension. Each student received a photocopy of a hypothetical irregular coastline (see below), a map of Martha's Vineyard, and a (drawing) compass. I used transparencies to explain the calculation.

I drew a 4-inch straight line segment on a transparency, and showed, by "walking" a compass along the line, that the line is 8 units long if the units are 1/2 inch (the distance between the compass legs is 1/2 inch), and 16 units long if the units are 1/4 inch: i.e.,

$$4 = 8 \times 1/2 = 16 \times 1/4.$$

This helped motivate a procedure to compute the dimension of any curve by walking a compass along the curve for different lengths. We let X be the number of units counted for compass opening 1/2 inch, and Y the number of units for compass opening 1/4 inch. (Note: 1/2 and 1/4 are chosen for convenience, other pairs like 1/8 and 1/16 would also work in theory--but no one would have the patience in practice!)

If the dimension were one (like the straight line), we'd expect $Y = X \times 2^1$ (above, $16 = 8 \times 2^1$). By analogy, letting D be the dimension, we set

$$Y = X \times 2^D, \text{ or } 2^D = Y/X.$$

To approximate D, we need only compute X and Y by counting compass "steps." For example, if X is 10.8 and Y is 23.2, we get $2^D \cong 2.148$, or $D \cong 1.1$. The main drawback with this method is that it can be a struggle to compute X and Y using compasses.

I gave each student a copy of an imaginary coastline similar to that below (but larger), put students in groups of 2 or 3, and asked them to find the dimension.

The students found X and Y, then using a guess-check-and-revise strategy, the students solved for D. Then, with their calculators, they checked the answer using the formula

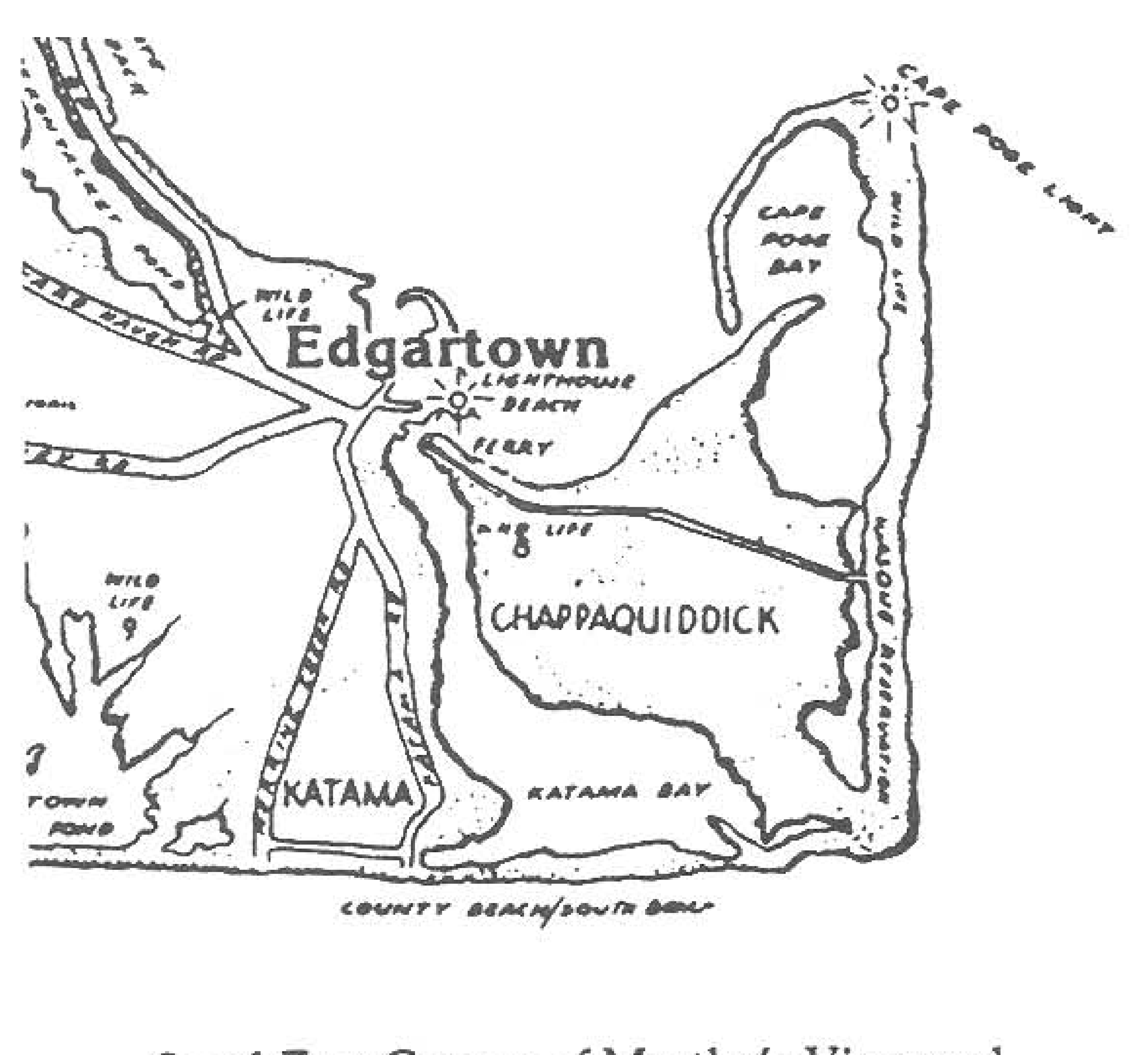
$$D = \log(Y/X)/\log 2 \quad \text{(i.e., log base 2 of } Y/X\text{)}.$$

Lastly, I passed out the maps of Martha's Vineyard, and directed the groups to calculate its coastline dimension. According to my class, D is approximately 1.44.

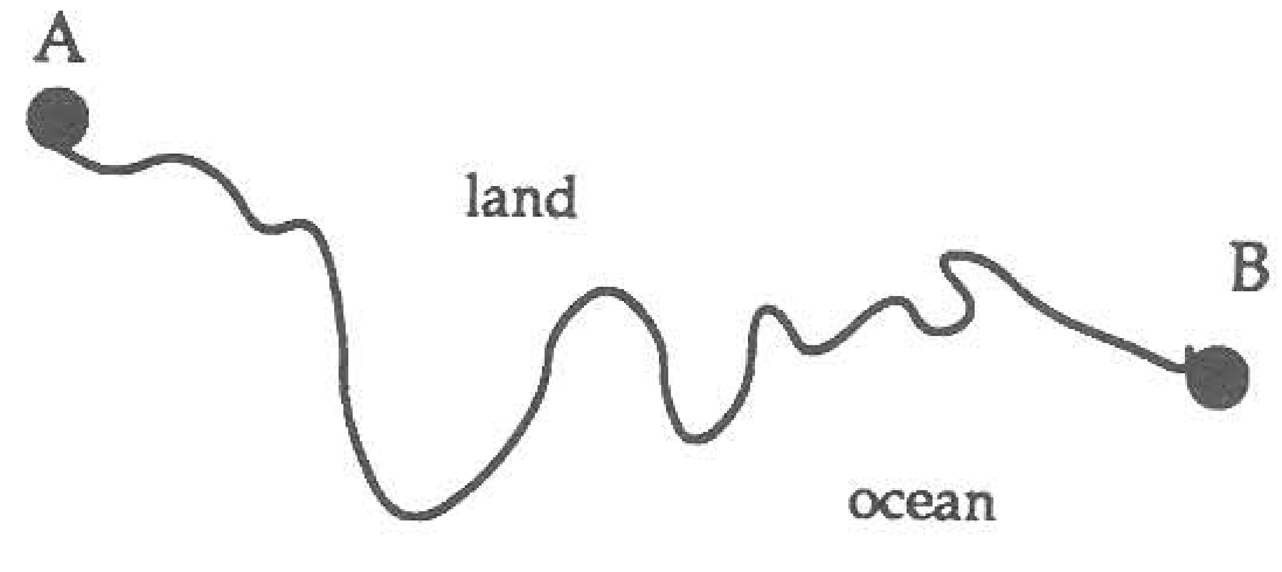
References

1. Peitgen, H., Jurgens, H., Saupe, D., Maletsky, E., Perciante, T.,Yunker, L., *Fractals for the Classroom,* Volume One. New York: Springer-Verlag, 1991.

2. Senk, S., Thompson, D., Viktora, S.. Rubenstein, R., Halvorson, J., Flanders, J., Jakucyn, N., Pillsbury, G.. & Usiskin, Z., "Dimensions and Space," in *Advanced Algebra* (pp. 857-860). Scott, Foresman and Company, Glenview, IL: 1990.



SouthEast Corner of Martha's Vineyard



Imaginary Coastline

Scheduling and Graph Coloring

by L. Charles (Chuck) Biehl

Consider a school that requires its staff members to serve on various committees which all hold regular meetings. Trying to schedule the minimum number of meetings so that no one has to be in two places at the same time can be a difficult task, especially if the scheduling is done by trial-and-error. Here is a situation in which using a graph model can greatly reduce the difficulty of the problem.

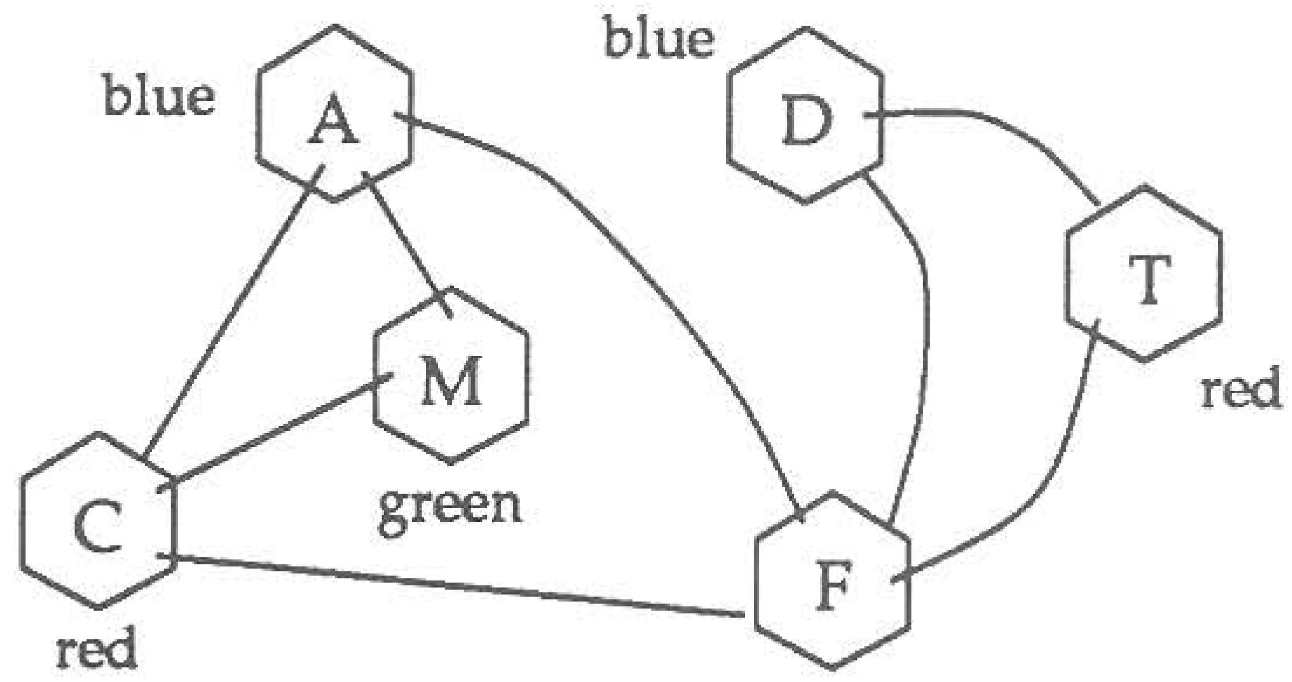
Recall that a graph is a set of points, called vertices, connected by a set of lines, called edges. In the case of a scheduling problem such as the one above, the vertices represent the committees, and the edges connect those committees which cannot meet at the same time because of a membership conflict. Such a graph is called a conflict graph.

The following is a sample problem whose solution is given below. The committees and their respective memberships are:

(C)urriculum: Davis, Franks, Grover
(D)iscipline: Bennett, Edwards, Hill
(T)extbooks: Bennett, Edwards, Isaacs
(A)ssessment: Alamos, Chavez, Davis
(F)acilities: Alamos, Edwards, Franks
(M)anagement: Chavez, Grover, Johnson

The conflict graph is shown in Figure 1. It is clear from the graph that there have to be at least three different meeting times; for example, committees M, A, and C all have conflicts with one another, as do committees F, C, and A and F, D, and T.

So where does coloring come in? Suppose that the vertices of the conflict graph are colored so that vertices



Conflict Graph

joined by an edge have different colors. Then a set of vertices which are the same color represent committees that can meet at the same time. Notice that it is possible to color C and T red, D and A blue, and M and F green. This means that a possible solution to the problem is to let the first meeting time be Curriculum and Textbooks, the second meeting time Discipline and Assessment, and the third meeting time Facilities and Management.

The origins of conflict graphs are in coloring maps, where regions which share a border must be different colors. However, the idea can easily be extended to cover other kinds of conflict as well, such as animals which cannot be placed in the same habitat, chemicals which cannot be stored in the same room, or school courses whose final examinations cannot be given at the same time. (Note: the zoo habitat problem appears on the video *Geometry,* available from COMAP.

An Election Followup Activity

by Sherida Hare

This is an activity that I used successfully in a precalculus class. I presented material and worked examples similar to those in [1] and [2] which took about 5 days. The presentation covered not only different voting methods, but also some of the paradoxes in voting.

After the students had a working knowledge of how voting works, we put together a questionnaire on the 1992 presidential election asking readers to rank the three candidates (Bush, Clinton, and Perot). We then looked at six vote-counting methods discussed in class: Majority-Rule, Plurality, Condorcet, Borda Count, Sequential, and Sequential Run-off [1, 2]. We polled the entire school and looked at the results of each method. Clinton won in all cases except for the 11th grade Borda Count in which Perot won. Bush came in last place in all of the results.

References

[1] COMAP, *For All Practical Purposes*, 3rd Ed., W.H. Freeman, New York, 1994, Chap. 11.

[2] L. Charles Biehl and Joseph G. Rosenstein,"Calling that *Mathematician...",* In Discrete Mathematics, No. 2 (October 1992), pp 11-12.

The Last Shall Be First: a Historical Illustration of Sequential Run-off

by Michael Ecsedy

In a multi-candidate election, one way to determine the winner is by the sequential run-off method. The voting proceeds in phases; after each phase, if there is no majority winner, the candidate with the fewest votes is eliminated. Voting is continued until some candidate has a majority of the votes. It is known that insincere voting can influence the outcome, although we rarely see this in practice. This article is about such an election which occurred four years ago [1].

The Republican party of Connecticut's fifth Congressional district was to nominate a contender for the seat being vacated by the incumbent, who was making a run for governor. The district was blessed with 5 candidates for the position, whom we shall label A, B, C, D, and E.[1]

The delegates began the voting at 7:00 PM. The first three rounds of balloting produced no winner and little shifting of candidate strength between rounds. The voting went as follows:

Round 1:	Round 2:	Round 3:
A 36	A 37	A 36
B 33	B 35	B 37
C 30	C 30	C 32
D 22	D 19	D 16
E 21	E 21	E 21

Convention rules dictate that no candidate can be chosen without receiving votes from the majority of the 142 delegates assembled. If only a plurality were required, Candidate A would have won on the first ballot.

After the third round, candidate D noted his vote totals decreasing, read the handwriting on the wall, and dropped out of the race, releasing his delegates. His support was scattered among the four remaining candidates through the next three ballots.

Round 4:	Round 5:	Round 6:
A 39	A 39	A 38
B 43	B 42	B 42
C 36	C 33	C 33
E 24	E 28	E 29

At this point (1:15 AM), the exhausted and exasperated conventioneers decided to adopt the sequential run-off procedure to break the deadlock. This is where strategic insincere voting played a role. Candidate B knew he wouldn't be eliminated on the next ballot and instructed some of his delegates to vote for Candidate E, fearing that Candidate C would ultimately be his strongest foe in the final rounds and knowing that he would not get any of candidate E's delegates. He felt that if he could knock out Candidate C at this point, he could ultimately win the nomination. The totals for Round 7 read:

Round 7:
A 37
B 36
C 34
E 35

and Candidate C was eliminated. It is worth noting that on this ballot any one of the 4 remaining candidates could have been eliminated with a switch of just a few votes.

Candidate C was incensed at Candidate B's gamesmanship, and instructed his delegates to vote for Candidate A. However, 14 of the 34 decided to support Candidate E instead, enough to eliminate B rather than E. (Was their motive to exact revenge on Candidate B?) The 8th ballot totals read:

Round 8:
A 57
B 38
E 47

and Candidate B was eliminated. B threw his support to E, and at 2:45A.M. the balloting was completed, with the final totals showing:

Round 9:
A 61
E 81

and E won the nomination.

Candidate E was Gary Franks, a formerly obscure Waterbury alderman who went on the win the election and rose to fame as the only black Republican in the 1991-1995 House of Representatives. He was frequently cited by the Bush administration as a black man who could win running as a Republican. The remarkable distinction of this election is that the man who was preferred by the least number of delegates on the first ballot became the eventual winner. He who would have been last ... under the plurality method ... finished first!

1. Steve Watson (A), Alan Schlesinger (B), Warren Sarasin (C), James McLaughlin (D), and Gary Franks (E).

References

[1] *The Waterbury Republican* and *The Danbury News-Times newspapers,* July 19-20, 1990.

Credits...

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Call for Authors...

This newsletter is written entirely by you, our readers. *In Discrete Mathematics* welcomes articles (even in rough form), letters, math problems.... We especially want to hear about your classroom experiences with discrete mathematics. Please send all contributions to the Editor, Deborah Franzblau, by email: franzbla@dimacs.rutgers.edu; or regular mail: DM Newsletter, P.O. Box 10867, New Brunswick, NJ 08906.

Subscriptions...

Please send us the name, address, phone number, and school of any teacher who should receive a copy of this Newsletter, and we will include him/her on our mailing list. (See the last page for a subscription form.)

Minibibliography

(continued from page 10)

7. Lenstra, J.K. and A.H.G. Rinnooy Kan, "Sequencing and Scheduling", in *Combinatorial Optimization,* ed. by M. O'hEigeartaigh, J. Lenstra, and A.H.G. Rinnooy Kan, Wiley, 1985, This article is an annotated bibliography of books and important papers in scheduling theory. Reading through it gives an idea of the range and scope of modern scheduling theory.

8. Malkevitch, J. and W. Meyer, *Graphs, Models and Finite Mathematics,* Prentice-Hall, 1974. An elementary account of the critical path method is given, as well as how graph-coloring algorithms can be used to help with scheduling.

9. Rinnooy Kan, A.H.G., *Machine Scheduling Problems: Classification, Complexity, and Computation,* Nijhoff, The Hague, 1976. The state of the art in 1976.

Classroom Ideas...

A certain number of people are in a room. If each person shakes the hand of every other person how many handshakes will occur? By physical experimentation we found that the first person shook hands with one less person than the total number of people in the line. Each person thereafter shook one less hand than the person before. Another method that led to the same pattern was to use a polygon which had the same number of sides as the number of people. When we added the number of diagonals to the number of sides it gave us the number of handshakes.

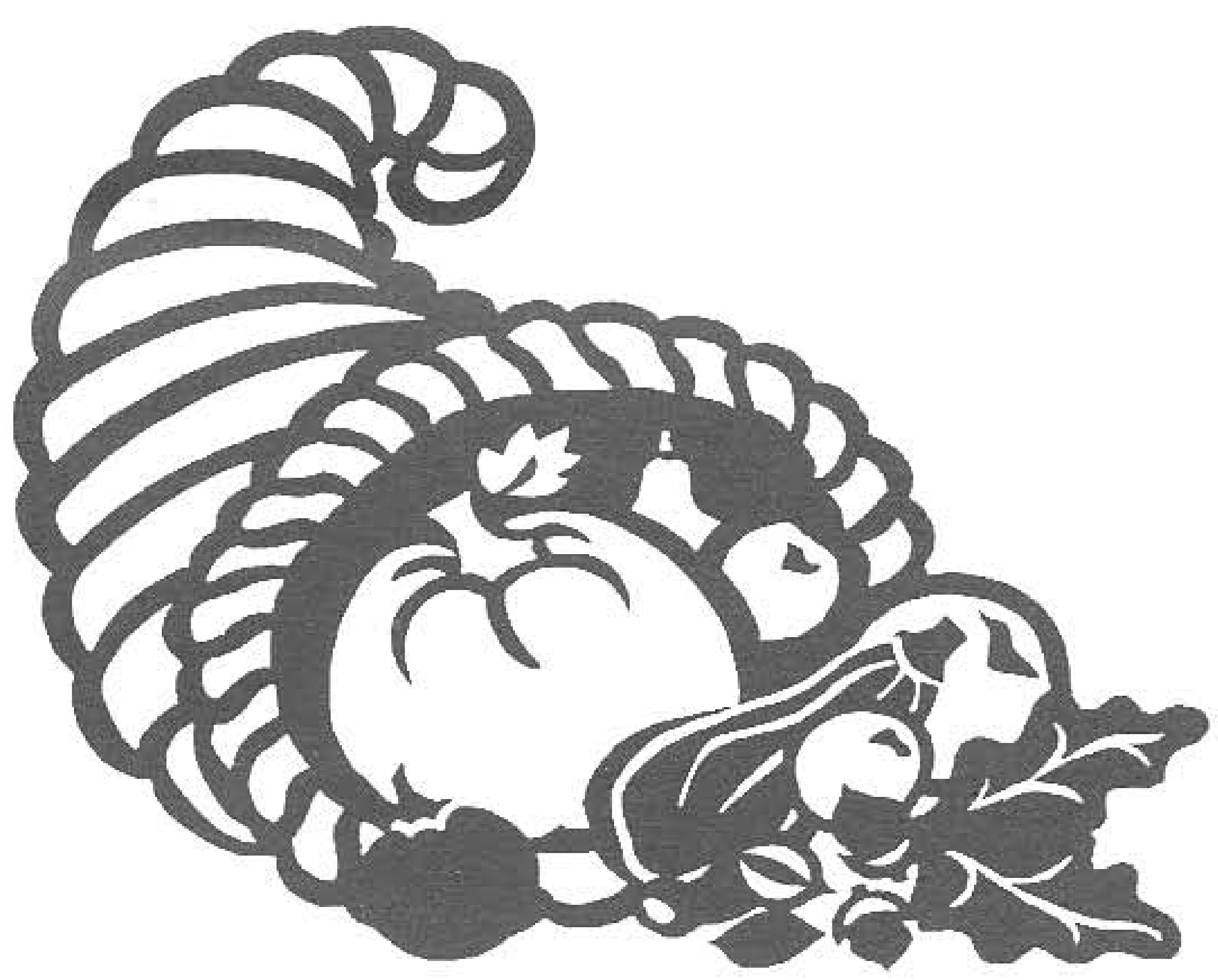
Marion Gorman (LP '92)

Pre-Thanksgiving Project

by Diane Amelotte

This is a lesson I designed for the three school days prior to Thanksgiving to hold the attention of my eighth grade pre-algebra classes. It is based on the following situation.

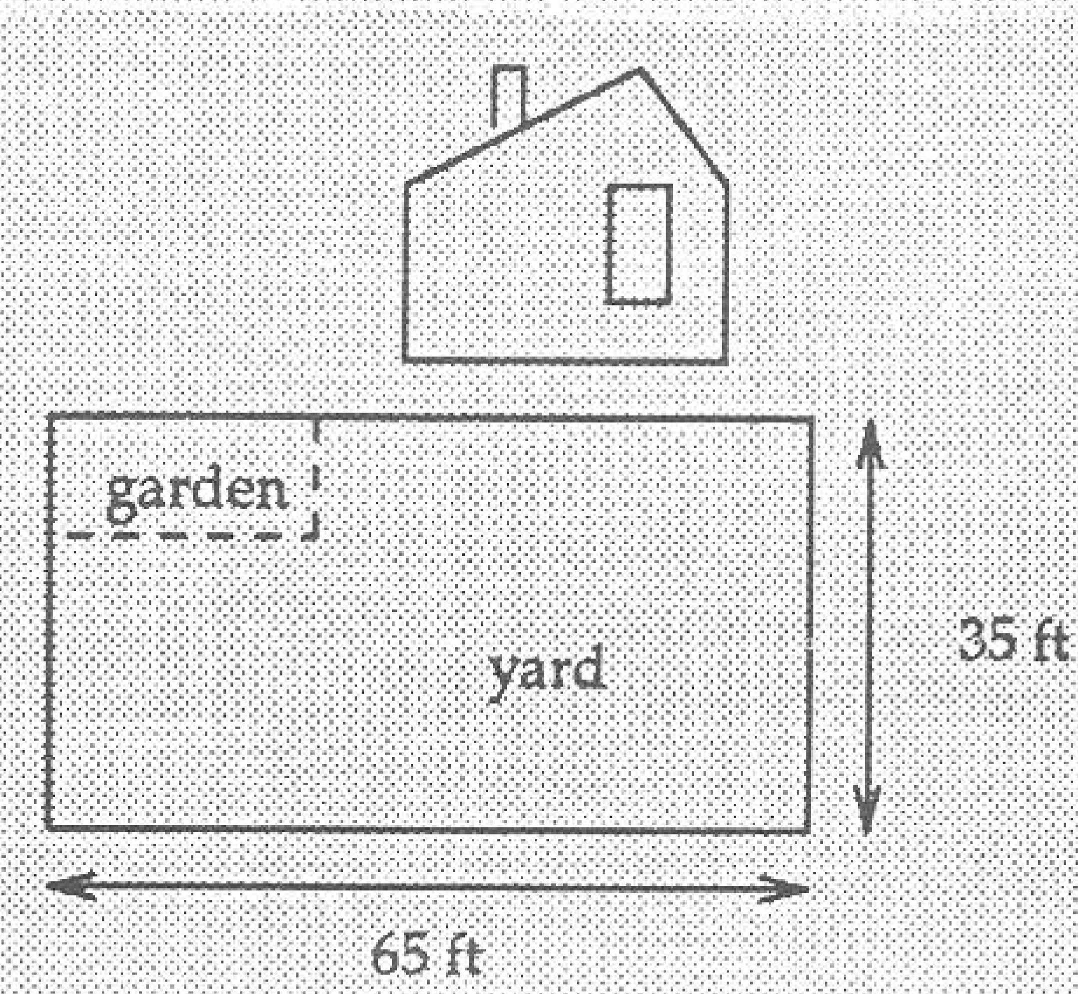
THANKSGIVING PROJECT: A new scientifically developed fertilizer called Harvest Plenty Soil Enhancer costs $175 for a bag that will fertilize an area of 240 square feet. Miles Standish IX wishes to fence in a garden with this area in a portion of his yard. The yard itself is a rectangle 65 feet by 35 feet. He must purchase the fertilizer, fence posts and the fencing. Pilgrim Hardware offers the best deal for fencing. They sell fencing from a huge roll of fencing, charging $3.95 per linear foot. To prevent the fence from sagging, it must be attached to fence posts which are at most 8 feet apart as well as a post in each corner of the garden. Each post costs $7.50.



The project involves experimentation with different garden layouts. On the first day, we warm up by reviewing the concept of factoring and how to count the factors of a given number (see p. 8). On the second day, we review counting factors, then students are divided into groups of four (I do this by handing out playing cards and grouping by face value). Each group of four has one "recorder" (I arbitrarily designate the student with the "club" as the recorder), who receives the description above and an instruction sheet for the group (see below). (Note: I vary parts of the given information. For example, some teams are asked to place posts no more than 12 feet apart and others have different prices for fencing and posts. This reduces interest in the results of neighboring teams.)

Instruction Sheet

Part I. Suppose that the following requirements must be met:
* the garden must be a rectangle;
* each dimension must be a nonzero whole number measured in feet;
* the area of the garden must be 240 square feet;
* two sides of the rectangle must be the north and west boundaries of the yard.



How many different layouts are possible?

Sketch each layout on your worksheet marking a dot wherever a post should be placed, using as few posts as possible. Record the length, width, and perimeter of each one and indicate how many posts are needed.

Part II. Determine which layouts are most expensive and least expensive. Justify your answer, for example by preparing an itemized list of the expenses including quantity and cost of each purchase for each layout, and finding the total cost of each layout.

Part III. Suppose that Miles has a budget of $525; which layout should he choose? (Give the dimensions.) What might be a valid reason for his not choosing the least expensive layout?

In Part 1, since $240 = 2^4 \times 3 \times 5$, there are 20 possible factors, giving 20 possible pairs for the length and width. However, because of the restrictive dimensions of the yard, only 11 of these give possible layouts.(I assume 8 x 30 and 30 x 8 are considered different layouts.)

(Continued on page 8)

Pre-Thanksgiving...

(Continued from page 7)
Each student is given a worksheet; the recorder collects the worksheets and hands them in at the end of the period. To create the worksheets, I divide an 8.5 x 11 sheet of 1/4" graph paper into 12 equal parts, letting 1 unit = 5 feet. Each part contains a 7 x 13 unit grid (representing the 35 ft x 65 ft backyard). I give each student one 2-sided worksheet.

Some students will mistakenly assume that on the grid, 1 unit = 1 foot. I ask these students to estimate the ratio of the area of the garden to the area of the yard to determine if their layouts are reasonable. The ratio is 240/2275 which is approximately 240/2400 or 1/10. Revisions usually follow.

Students may also need to be reminded that determining the number of posts is not a matter of simply dividing the perimeter by 8. The team needs to discuss this carefully. Hint: ask them if there can be an odd number of posts. (A few days after Thanksgiving can be given to developing an algorithm for determining the number of posts using divisibility and the greatest integer function.)

To finish the project before Thanksgiving, Part I of the project should be completed on the second day. Lend assistance as needed if teams are not drawing sound conclusions after a reasonable time. Students can go on to Parts II and III on the second day, if there's time.

For Part II, the cost of implementing a layout includes the fixed cost of the fertilizer ($175) as well as the variable cost of the fencing and posts, which are determined by the perimeter and dimensions. Students should figure out that although a 12 x 20 layout may be distinguishable from a 20 x 12 layout in the yard, the cost to implement each is the same. In fact there are actually only 7 layouts with different costs. Students may guess or reason correctly that the layouts with the greatest and least perimeters have the greatest and least cost, respectively. Using a team strategy can reduce the amount of work needed in this step.

On the third day, students should complete parts II and III. If a team finishes early, give the recorder an additional problem:

Part IV. Suppose the four requirements listed in Part I are no longer necessary (for example, the dimensions need not be whole numbers and the shape need not be a rectangle), except that the area of the garden must still be (approximately) 240 square feet (to use only one bag of fertilizer). Sketch additional possible layouts. Creativity is encouraged! Label the dimensions and determine the costs. Are any of these less costly than the least-cost layout in Part II? Should cost be the only factor considered?

With the restrictions removed, a few students may realize that a circular garden with a radius approximately the square root of ($240/\pi$) only requires 7 posts. Others may see that of all rectangular layouts, a square (or nearly square) garden will have the least cost for a given area.

During the project, students may ask whether there is a need for some type of a garden gate. I suggest that a gate could be figured into the cost and the fencing perimeter and number of posts could be recalculated. Most students respond with the recommendation that, for the time being, Miles can easily hurdle the fence.

Finding all the Factors

I first review vocabulary: factor, divisor, multiple, prime and composite numbers, prime factorization. Then we work on the following problems as a class in preparation for the project.

List and count the factors of:

(1) **54** [Ans: 1, 2, 3, 6, 9, 18, 27, 54; 8 factors].
(2) **156** [Ans: 1, 2, 3, 4, 6, 12, 13, 26, 39, 52, 78, 156; 12 factors].

Now try:
(3) **16** [1, 2, 4, 8, 16], **32** [1, 2, 4, 8, 16, 32]
Observe: $16 = 2^4$ and has $5 = 4+1$ factors

(4) Generalize to $\mathbf{2^n}$ [Ans: $1, 2^1, 2^2, \ldots, 2^n$; n+1 factors]

Now observe: $54 = 2^1 \times 3^3$,
So the possible factors are:

$2^0 \times 3^0 = 1$ $\quad$ $2^1 \times 3^0 = 2$
$2^0 \times 3^1 = 3$ $\quad$ $2^1 \times 3^1 = 6$
$2^0 \times 3^2 = 9$ $\quad$ $2^1 \times 3^2 = 18$
$2^0 \times 3^3 = 27$ $\quad$ $2^1 \times 3^3 = 54$

and so 54 has $8 = (1+1)(3+1)$ factors.

Now try:
(5) $156 = 2^2 \times 3^1 \times 13^1$.
[Ans: $(2+1)(1+1)(1+1) = 3 \times 2 \times 2 = 12$]

Generalize to:
(6) $N = 2^a \times 3^b \times 5^c \times 7^d$
[Ans: $(a+1)(b+1)(c+1)(d+1)$]

On Time

(Continued from page 1)

new world of scheduling problems, in particular, how to meet the needs of the many users of a time- or resource-constrained computer system.

Mathematics has fundamentally changed scheduling. First, mathematical analysis introduced taxonomy. For example, some scheduling problems are deterministic or static (all necessary information is known), others are dynamic (information changes over time) and may involve stochastic processes (such as customers arriving at a bank). In developing this taxonomy, among the many questions that have been considered are as follows:

1. What tasks have to be accomplished, and by which machines? (E.g., the tasks could be financial transactions, and the "machines" could be either human or automated bank tellers.)
2. Are there restrictions on the order in which tasks must be accomplished? (E.g., you cannot put a roof on a new house before the foundation is laid.)
3. Are there other priorities given for the tasks? (E.g., you want to make sure the guest room is done before Grandma comes for her yearly visit.)
4. Are there penalties when tasks are completed after the due dates (or premiums for tasks completed early, such as in [2])?
5. Once a machine has started work on a task, must it complete the task, or can it interrupt its work if a higher priority task appears?
6. What goals are involved in the scheduling? Minimizing idle time of machines? Completing the tasks as quickly as possible? Using as few machines as possible?

Beyond taxonomy, modern mathematics has offered a broad array of optimization techniques and other tools to assist with the solution and understanding of scheduling problems. One of the early pioneers in the mathematics of scheduling was NASA. Carrying out the Apollo project to land on the moon required careful use and timing of resources. NASA still must schedule carefully for efficient operation of its Shuttle Fleet. Another pioneer was Bell Laboratories (now AT&T Bell Laboratories and Bellcore), where mathematicians such as Ronald Graham, David Johnson, and Edward Coffman made many advances in the mathematical theory of scheduling. Scheduling is now an established sub-area of mathematics within broader areas of mathematics concerned with operations research, management science, and computer science. Tools that have been used to study scheduling come from combinatorics and number theory, as well as other parts of mathematics, and include structures such as undirected and directed graphs and partially-ordered sets, and techniques such as mathematical programming, and packing and coloring algorithms. (See **Critical Path Scheduling**, below, as well as the articles by C. Biehl and K. DeVizia in this newsletter.)

References [see also the **Scheduling Minibibliography** on p. 10 of this newsletter]:

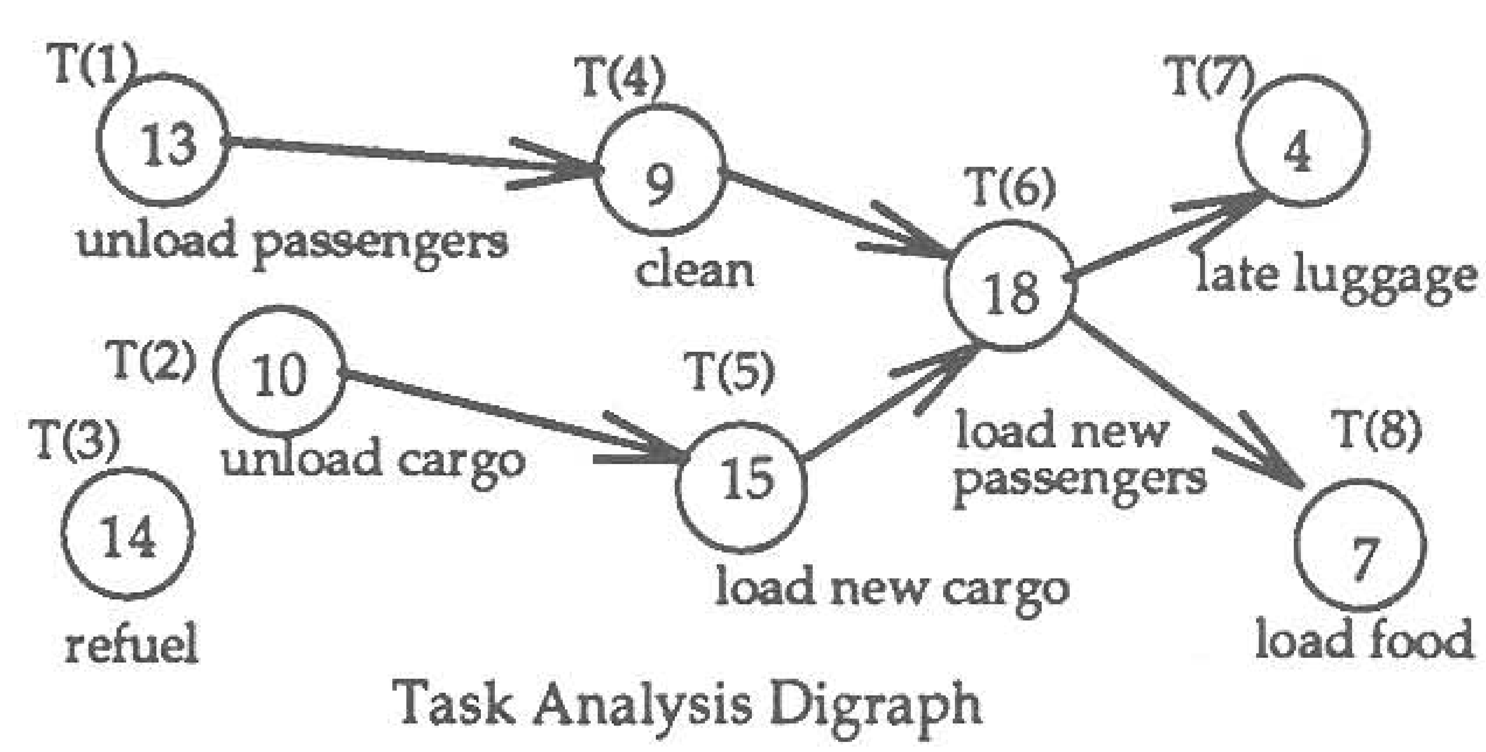
[1] Johnson, Dirk, "Denver May Open Airport in Spite of Glitches", *NY Times*, Wednesday, July 27, 1994, p. A14. This is one of a series of articles appearing over several months about delays in the planned opening of the new Denver airport.

[2] Margolick, David, "Quake-Damaged Freeway Reopening Ahead of Time", *NY Times,* Tuesday, April 12, 1994, p. A12. This article recounts the reconstruction program to reopen the Santa Monica Freeway after it was closed by the Los Angeles earthquake. The contractor responsible for the reconstruction finished the work 74 days ahead of schedule! The reconstruction cost $29.4 million (which includes a $200,000 bonus per day for each of those 74 days, or $14.8 million!)

Critical Path Scheduling

To give you some flavor of a modern scheduling problem, consider the question of efficiently turning around a shuttle plane providing service between two cities. Among the tasks that must be completed to do this are: refueling the plane, putting new drinks and food aboard, cleaning the cabin, unloading current cargo and loading new cargo, unloading current passengers and loading new ones. Clearly, some of these tasks must be done before others; one cannot clean the cabin before the current passengers are deplaned. One can use a "task analysis digraph," i.e., a graph with vertices (circles) representing tasks, and directed edges (arrows) representing precedence constraints (see figure). The length of time (in minutes) to perform a task is indicated inside the circle representing the task. An arrow from Task i to Task j means that Task i must be completed

(Continued on page 10)



Task Analysis Digraph

Critical Path Scheduling...

(continued from page 9)

before Task j can begin. The digraph shown in the figure is for illustration only; the real digraph for such a project may be quite different.

Our goal is to construct an early-start/early-finish table (Table 1), and a late-start/late-finish table (Table 2). Table 1 gives for each task the earliest time that work on that task can begin and end. Table 2 gives for each task the latest time that work on that task can begin and end, given the constraint that the last task(s) are completed by the earliest possible completion time. Essentially, the late-start/late-finish table identifies those tasks that have some flexibility in their scheduling. To perform the entire job efficiently, each task must begin on or after its early-start time but on or before its late-start time.

Table 1

Task	T(1)	T(2)	T(3)	T(4)	T(5)	T(6)	T(7)	T(8)
Early Start	0	0	0	13	10	25	43	43
Early Finish	13	10	14	22	25	43	47	50

Table 2

Task	T(1)	T(2)	T(3)	T(4)	T(5)	T(6)	T(7)	T(8)
Early Start	3	0	36	16	10	25	46	43
Early Finish	16	10	50	25	25	43	50	50

Here are some hints to show how Table 1 was constructed. First, scan through the digraph in Figure 1 from left to right. Since tasks T(1), T(2), and T(3) have no predecessors, they can start at time 0, and their early-finish time is their early-start time (0 here) plus the time the task takes. How early can task T(6) start? It can not begin until both T(4) and T(5) are done. Hence, the earliest start time or T(6) is the maximum of the early finish times of T(4) and T(5), or 25 minutes. Other entries are found analogously. Observe that if each task begins right on its early-start time, and is completed in the time shown, the whole job can be completed by the largest time given in the table (50 minutes here). Notice that there is a path, T(2), T(5), T(6), T(8), such that the sum of the task times is equal to the early-finish time; such a path is called a "critical path." The tasks on this path require 50 minutes, so that 50 minutes is the best possible completion time.

Once Table 1 is complete, Table 2 can be filled in. This time, we scan through the digraph from right to left. Since the latest completion time of any task is 50 minutes, this is also the earliest completion time for the whole job. Since T(3), T(7), and T(8) have no tasks which must come after them, their late-finish times are all 50. The late-start time for Task 7 is

50(late-finish time) - 4(task time) = 46 minutes.

Other entries in Table 2 are found in a similar way. Examining the two tables, one can see that there is no flexibility in scheduling a task if and only if it lies on a critical path.

Scheduling Minibibliography

by Joseph Malkevitch

1. Coffman, E.G., *Computer & Job/Shop Scheduling*, Wiley, New York,1976. Excellent survey but now a bit dated.

2. COMAP, *For All Practical Purposes*, W.H. Freeman, 3rd Ed., 1994. An account of the critical path method and of the list-processing algorithm for scheduling machines is described. Some of the paradoxical behavior that occurs in scheduling theory is described.

3. Graham, Ronald, *The Combinatorial Mathematics of Scheduling*, Scientific American, 238(3), March, 1978, pg. 124-132. A very readable account of machine scheduling, and some of the paradoxical behavior scheduling theory sometimes produces (e.g., adding more machines can sometimes make things take longer).

4. Graham, Ronald, "Combinatorial Scheduling Theory", in Steen, L.A.(ed.), *Mathematics Today*, Springer-Verlag, 1978. A survey of elementary results about scheduling.

5. French, Simon, *Sequencing and Scheduling*, Wiley, New York, 1982. Technical but locally readable account of scheduling theory.

6. Lawler, E., "Recent Results in the Theory of Machine Scheduling", in *Mathematical Programming: The State of the Art*, Ed. A. Bachem, M. Grotschel, and B. Korte, Springer-Verlag, New York, 1983. A technical but relatively accessible survey as of about 1980.

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RUTGERS UNIVERSITY

LEADERSHIP PROGRAM IN DISCRETE MATHEMATICS

Summer institutes ... for K-8 teachers ... in discrete mathematics

WHAT? Two-week residential and two-week commuter institutes at Rutgers University.

WHO? For teachers of K-8 students, as well as mathematics supervisors and specialists.

WHEN? The two-week institutes will run during the period from June 26 to July 28, 1995. Participants will be expected to attend four Saturday follow-up sessions during the 1995-1996 school year and a one-week institute the following summer.

STIPENDS? Anticipated funding from the National Science Foundation will provide each participant with a stipend of $250-$300 per week, as well as meals and lodging (double occupancy on weeknights) for the residential institutes.

PARTICIPANTS ... will be expected to:

* introduce discrete math into their classrooms
* develop materials for use by other teachers
* present workshops on institute topics

DATES: Applications are due by March 20; applicants will be notified by April 26, 1995.

QUESTIONS: For further information, or to receive an application form, call Stephanie Micale at 908/445-4065 or write to the address below.

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BY WHOM? Experienced teachers of the Leadership Program in Discrete Mathematics who have participated in a training program on preparing and presenting workshops.

COST? The district will be expected to pay only direct costs -- expenses and honoraria for the workshop leaders and instructional materials for the participants.

QUESTIONS? For further information, call Michelle Bartley-Taylor at 908/445-4065 or write to the address below.

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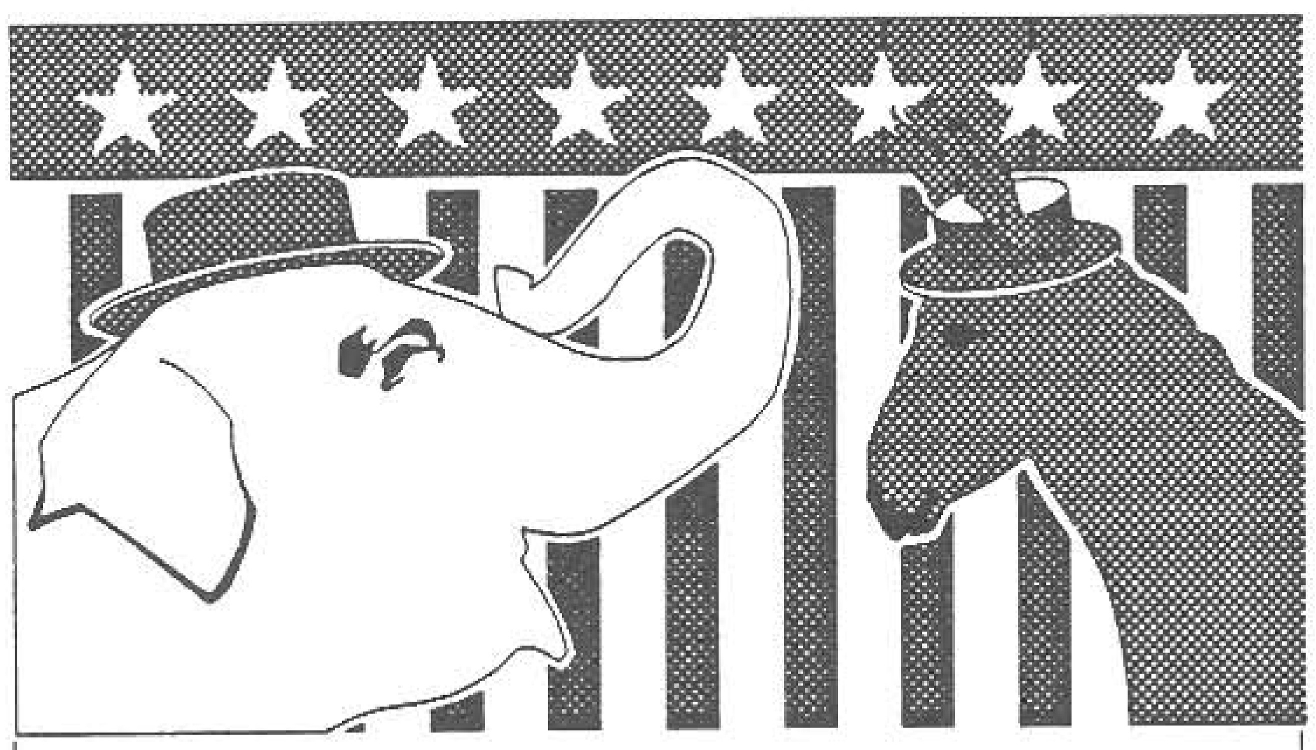

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