IN DISCRETE MATHEMATICS
Using Discrete Mathematics in the Classroom

Issue #6 *Spring/Summer 1995*

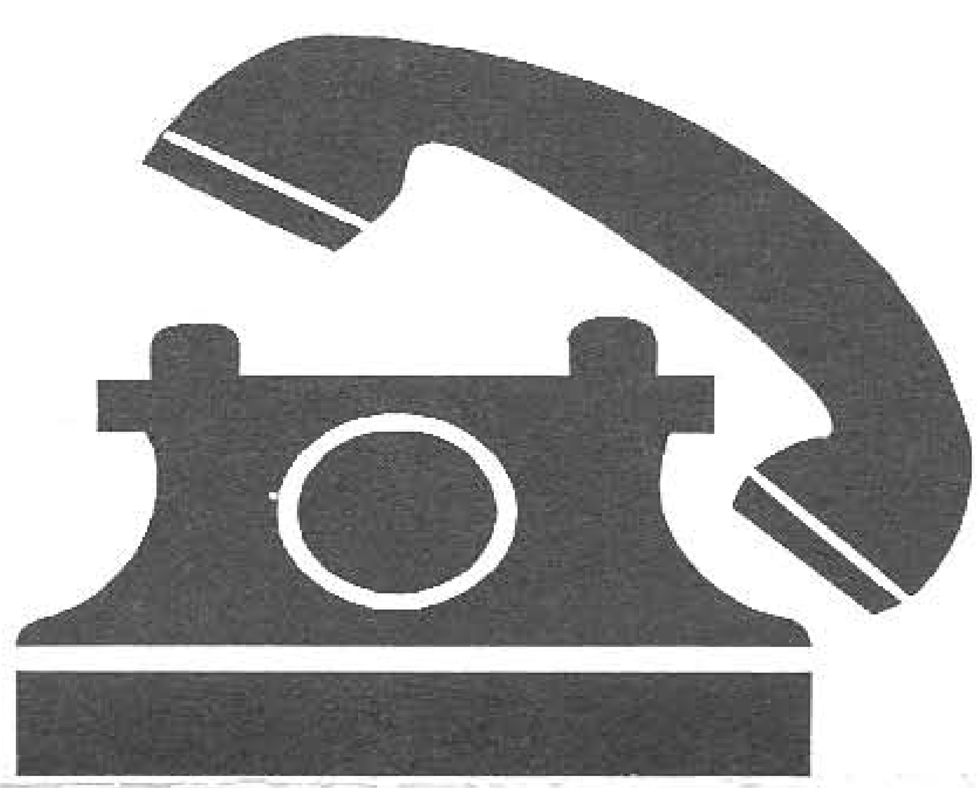
Speaking Discretely...
by Deborah S. Franzblau

This summer marks the beginning of a new phase of the *Leadership Program in Discrete Mathematics*, which will focus on K-8 teachers. We hope you will encourage K-8 teachers from your district to join the program next year (see flyer on p. 11).

The lead article in this issue (p.1) is on the branch of mathematics known as "game theory", which has many applications, not only to actual games of strategy such as checkers or bridge, but to political "games" such as arms treaty negotiations or elections. A list of books for further reading is included on p. 9. The classroom activity on p. 2 is a nice introduction to paradoxes in cooperative games with many players.

I am pleased to announce a new column, *The Discrete Reviewer* (p.7), edited by Janice Kowalczyk, containing capsule reviews of teaching resources (books, videos, software, etc.). The focus in this issue is on graph coloring, complementing an article giving the perspective of four teachers on using coloring in the classroom (p. 4). Coloring is also the theme of one of the picture puzzles on the back page (another new feature in this issue).

On p. 5, a teacher describes her novel method for teaching students the idea of an algorithm. Rounding out the issue is a report on implementing fair division in middle school (p. 3), and an interesting discussion on "decoding" area codes (p. 5).



In This Issue

Articles
Game Theory in the News 1
Area Codes: Are They Really Codes? 5
Bibliography...Game Theory 9
Teaching Briefs
A Classroom Dilemma 2
Fair Division in the Middle School 3
Graph Coloring...Starting the Year Off Right 4
Marty The Martian 5
The Discrete Reviewer 7

Game Theory In The News
by Joseph Malkevitch

I just heard on the radio that the midtown tunnel is jammed—should I use the uptown bridge to get where I am going? Or should I head for the midtown tunnel because everyone else will head for the bridge? Should I vote for my favorite candidate among the three running for President even though the polls say that my candidate can't win? Or am I just throwing away my vote? Public health officials now recommend that all children get vaccinated against measles, but I know that some children have an allergic reaction to the vaccine. Can't I assume that, since so many other people will vaccinate their children, any one child has little chance of getting the measles, and that I can safely keep my children unvaccinated?

We all face such strategic questions daily as part of modern living. In fact, questions of this kind can be analyzed from a mathematical point of view. Each of these questions involves a group of individuals, each of whom can take different courses of action, who are trying to decide which action is best or optimal for themselves, given some knowledge of the consequences—which depend on the actions taken by others. Such situations turn out to have many things in common with such games as chess or poker; the branch of mathematics concerned with these problems is called *game theory*. The three problems mentioned in the introduction can be modeled as many-person games which are "paradoxical" in that "rational behavior" can lead to irrational results.

Although mathematical game theory has been developed at least since the 1920's, its application to social and economic problems was spurred by the work of Oskar Morgenstern and John Von Neumann in the '40s [1], which inspired a great deal of further activity in the '50s.

(Continued on page 8)

A Classroom Dilemma

by Reuben Settergren

In the summer of 1994, I taught a course called Applications of Contemporary Mathematics (ACOM) at the Johns Hopkins University Center for Talented Youth (CTY), at its Los Angeles site. A new course, ACOM was aimed at the CTY's youngest students (who were about 12 years old). I used *For All Practical Purposes* [1] as a text, and included a unit on game theory.

My favorite activity was a game inspired by Douglas Hofstadter's article "Dilemmas for Superrational Thinkers" [2]. The purpose of the game would be defeated by cooperation, or even communication, between the players, so each of my students received a private letter from me telling them that they were selected to compete in a game, with rewards to be paid in real money. They were told the number of players, but not who the players were. Here is how the game works:

(1) each player secretly writes down a single "move": either a "C" (cooperate) or "D" (defect);

(2) to determine the payoff for each player: the moves of each pair of players are compared, and their winnings are augmented according to the payoff schedule below.

C vs. C: each gets 5 cents
D vs. D: each gets 1 cent
C vs. D: D gets 9 cents, and C gets nothing.

Notice that the total payoffs in this game vary according to the number of players. For example, consider a game with 16 players. If everybody cooperates, everybody gets 5 cents, so everybody gets 15 x 5 = 75 cents. However, if everybody defects, everybody gets only 15 x 1 = 15 cents. With 8 cooperators and 8 defectors, cooperators each receive 7 x 5 + 8 x 0 = 35 cents, and defectors each receive 8 x 9 + 7 x 1 = 79 cents. And—the holy grail of defectors—if only one player defects, he or she gets 15 x 9 = 135 cents, almost twice as much as the payoff for the cooperators: 14 x 5 = 70 cents each.

If you try this with your students, you can expect that well under a fourth of your students will choose to cooperate, thus limiting the expenditure of precious school (or personal) funds. The defectors will be disappointed with their classmates' greed, and some might be ashamed of their own!

Experienced game theorists will recognize that this game is simply a many-way "Prisoner's Dilemma". For any set of moves, the defectors will always earn more than the cooperators, and any one player will always earn more against any set of opponents' moves by defecting. The "dilemma", however, is that if everybody is greedy and defects, everybody loses. My hope is that the students will discover this and begin to think about what the best strategy is in playing the game.

Now, for the game to work properly, it is essential that the students not communicate. I suggest that you use the letter in Hofstadter's article (modified, of course, to fit your class size and budget): it does an excellent job of explaining rational decision-making, and the possible rewards or hazards of the different moves. I distributed my game letters very secretively, outside of class, and required students to give me their responses personally (along with an explanation and a complete chart of payoffs, to make sure they understood the game). After the results were in, I handed out the money, and we all discussed our reactions to the game and its results.

We then read Hofstadter's article [2], and discussed possible applications. This game is in fact very similar to many everyday situations: traffic ("I can slow down to rubberneck at this accident"), art ("I'd really like to touch this Van Gogh; it's a good thing nobody else ever would"), pollution ("If I alone disregard the polluting effects of my company, I can get an edge on my competitors"), etc. We also speculated what the students' moves would be in the various other games described in the postscript to Hofstadter's article, or how their strategy would change if the game had payoffs in dollars, or millions of dollars, instead of just cents (an excellent demonstration of the non-linearity of the utility or value of money).

My kids spent two hours on a Friday afternoon exploring these issues, and even forgot to rush out the door to their sunny Los Angeles weekends at the end of class! That might be more time than you can afford to spend in your class, but perhaps the less time you spend, the more your students will come away intrigued and inquisitive.

References:

[1] COMAP, *For All Practical Purposes*, 3rd. Ed., W.H. Freeman, New York, 1994, Chap. 15.

[2] Hofstadter, Douglas R., *Metamagical Themas*, Basic Books, New York 1985.

"Dilemmas for Superrational Thinkers" (Chapter 30 of this book) originally appeared as a "Metamagical Themas" article in *Scientific American*, June, 1983. "The Tale of Happiton" (Chapter 32) is a story about cooperation that would make a nice reading assignment. See also Chapter 29, "The Prisoner's Dilemma Computer Tournaments", which explores the success of different player strategies using computer simulations.

Fair Division in the Middle School

by James Kinyon

After taking a summer discrete mathematics institute aimed at high school teachers, I set out to use fair division in my "Eighth-Grade Mathematics" course. I teach four sections to about 100 students total per year. The setting is a rural/suburban school, Roland-Story Middle School, near Ames, Iowa. I used the module, *Fair Divisions: Getting Your Fair Share* [1], as a resource.

I began by telling the students that, later in the term, several cakes would be delivered, which we'd have to divide fairly among the class. I defined a "fair division method" for a group to be any method that everyone in the group agrees (in advance) is fair. Some of the students knew of the "divide-and-choose" method for two, which we all agreed was fair. Then, for three days, groups of three or four students came up with and tried methods for fairly dividing pictures of round cakes, square cakes, and irregularly-shaped cakes. (See [1], and [2], p. 76-82.)

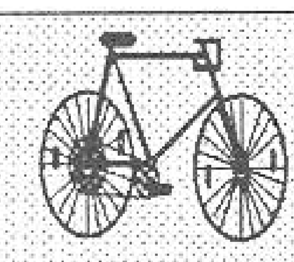
One Friday, our parent support group provided a rectangular and a round cake for each of my classes. The students suddenly became vitally concerned with the fairness of the division! In fact, one student decided that since HE was the biggest kid and HE had the knife, HE should take the biggest piece! (A brief intervention by the teacher was necessary to review the meaning of fairness.)

The students in my room sit at six tables in groups of three or four. We decided to first divide each of the cakes fairly into six pieces, one for each table. I suggested that we first elect two "designated cutters" to split each cake in half, then elect four cutters from the remaining tables to cut each half in thirds. When this was done, each sixth of a cake was cut fairly by the students at each table (using any method they had agreed on).

After the fair division of divisible (continuous) objects, we went on to dividing indivisible (discrete) objects. I first created a scenario in which a student from a math club left for Tibet, leaving some of his possessions to be divided among the rest (see sidebar). Students first discussed the problem in groups of two or three, and tried to devise their own methods. I then introduced the method of "sealed bids" (described in [1], p. 29-31 and [2], p. 51-58) and asked students to try the method on the problem. I found it necessary to provide a detailed form to help students organize their data, as in [1], p. 29. This problem worked well as a warm-up, since students tended to know the actual worth of objects like bicycles and stereos. For homework, I gave them an estate division problem with objects that they did not know the value of (a piano, mountain cabin, boat, etc.), and had them investigate typical monetary values as part of the assignment. I also gave students other similar problems (see [1] for ideas).

Students liked the chance of pace for the two weeks spent on the Fair Division unit. Perhaps the most important part turned out to be not the mathematics, but the resulting student discussions on their own understanding of fairness, and their own values. However, during this time, they solved problems, estimated, computed ratios, organized data logically, and did arithmetic with and without calculators; reinforcing many standard topics in the eighth-grade mathematics curriculum.

Fairly Dividing a Set of Gifts



Johnny's dad has taken a job in Tibet and is planning to move the family there. Since Johnny cannot take his larger possessions on the plane, he has decided to give some of them to you, the members of his math club at school. Here are the things that you need to divide fairly among yourselves.

1993 Encyclopedia.
Nearly-new 12 speed bicycle.
Collection of 100 Elvis Presley records.
400-Watt Sony stereo system.
Macintosh computer from Wal-Mart.

References:

[1] Bennett, et al., *Fair Divisions: Getting Your Fair Share*, HiMAP Module 9, COMAP, Arlington, MA, 1987.

[2] Crisler, Fisher, and Froelich, *Discrete Mathematics Through Applications*, COMAP, Freeman, NY, 1994.

Editor's notes: The method of sealed bids was described by the mathematician Hugo Steinhaus, and is often called the "Steinhaus Method". An example appears in "Dear Ann Landers", by Janice Ricks, *In Discrete Mathematics*, #3, Aug. 1993, p. 2.

Graph Coloring . . . Starting the Year off Right

by Richard Adkisson, Susan Howell, Steven Kepnes, and Lisa Soden-Winer

In the summer of 1994, we all attended the high-school institute of the Leadership Program in Discrete Mathematics. On the first day of the program, the instructor (and program director), Joe Rosenstein, introduced us to graph coloring. Since this worked so well for us as an introduction to discrete mathematics, we each decided to try it in our own classes in the fall. We wanted similar outcomes: a break from the monotony of "book work" or drilling fraction addition, more students confident in their own abilities, and a "proof" that math can be both useful and fun. We wanted students to get used to working in groups and to get to know one another. We found that graph coloring is a great topic for achieving all of these.

Richard, who teaches at a rural/suburban school in New Jersey, was teaching a remedial course for 17 ninth graders who had failed the state's 8th-grade "Early Warning Test". Susan and Lisa were each teaching sections of a course called "Algebra I, Part I", at a rural high school in New Jersey, each with 17-18 students. Their course enrolls students who have either done very poorly in mathematics or have special needs, and covers the standard Algebra curriculum at a much slower pace. Lisa's section had mostly ninth graders, while Susan's had mainly tenth and eleventh graders. Susan and Lisa each work with a second teacher, who has experience with special-needs students, who assists them and works with students individually during activities. Steve Kepnes was teaching a mainstream eighth-grade mathematics course.

Each of us followed a similar outline, although the students in the remedial courses needed more time to complete the activities (Steve used two days, while Lisa needed two weeks). We began by handing out maps with markers or crayons, and asking students to find a good way to color the maps so that no two regions sharing a border had the same color. We then introduced the problem of finding the minimum number of colors needed; some of us motivated the problem by observing that in map-making, each new color costs extra.

After the students had colored maps successfully, we showed them how to represent the problem as coloring the vertices of a graph, and defined the chromatic number of a graph (the minimum number of colors needed). Through experimentation, students observed that a graph with a triangle required 3 colors, and several conjectured the 4-color theorem for maps.

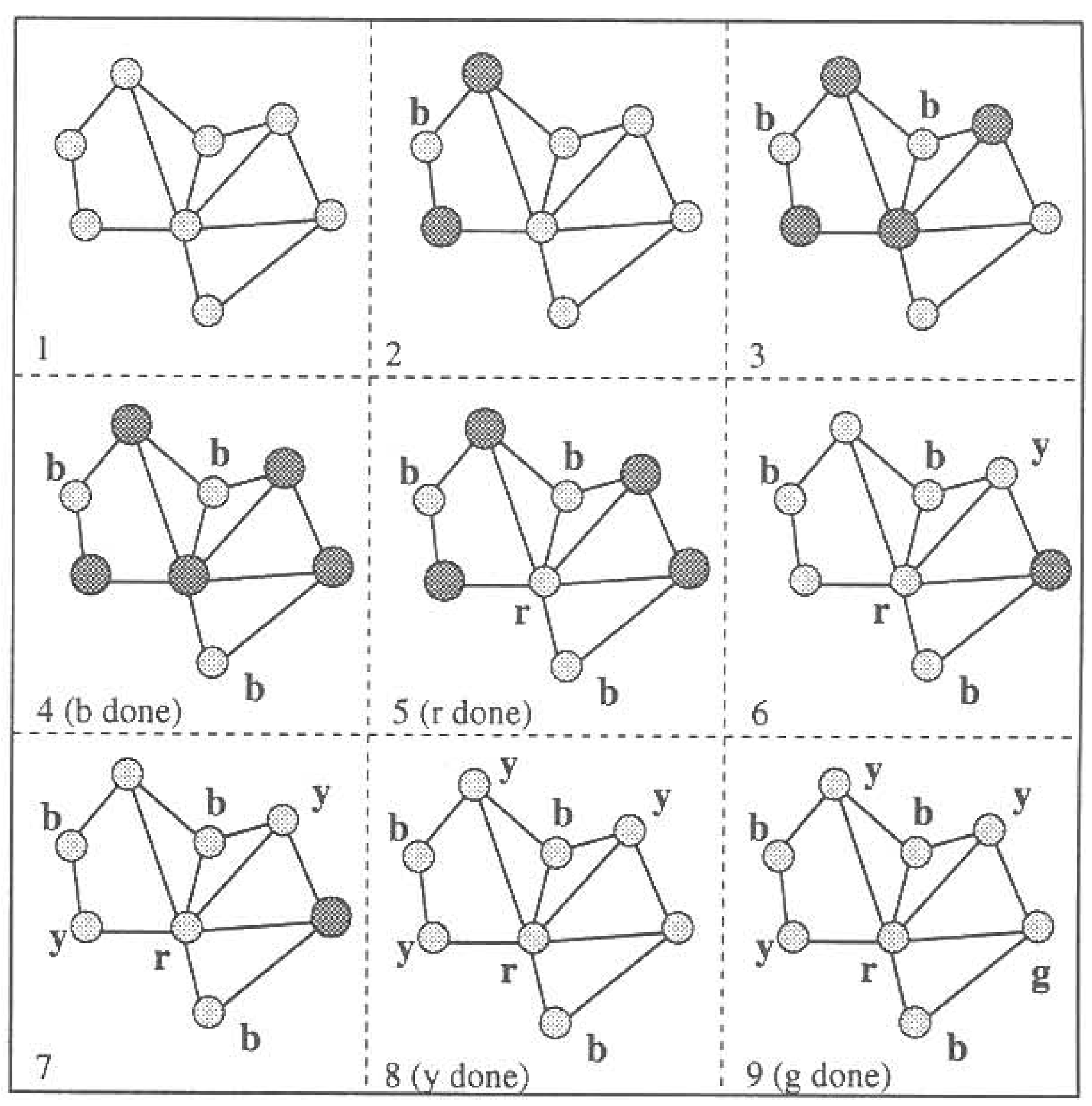
Finally, we introduced conflict problems which can be solved by graph coloring, such as the problem of scheduling committees with overlapping membership (see [1]), or that of assigning frequencies to mobile radio telephones ([2], p. 108, problem 8), as shown in the sidebar on page 10.

Steve asked students to present their solutions to the map-coloring problems to the class, letting them discuss whether the solutions were correct, and if better solutions (with fewer colors) were possible. He says that "if my students had the same enthusiasm every day as they had that day in wanting to present their solutions on the overhead projector, I probably wouldn't have to show up—the students could conduct the class themselves!"

Lisa found an interesting way to color a graph. She placed a graph (which the students had created for homework, whose vertices represented the townships of Monmouth County) on an overhead projector. She started by coloring one vertex blue, then putting pennies on the vertices attached to it by edges. She repeated the process, coloring a penniless vertex blue, and putting pennies on the vertices attached to it, continuing until all vertices were either colored blue or covered with a penny. She then removed the pennies and started over, coloring a penniless vertex red, continuing (adding new colors when necessary) until all the vertices were colored. At the end, she placed a transparency of the map on the graph so that the students could see that the coloring of the vertices truly represented a coloring of the townships.

Susan had her students present their solutions to the conflict problems that they were given for homework, and was impressed by the care the students took in their work

(Continued on page 10)



Lisa's graph-coloring method

Marty the Martian
by Melanie Drozdowski

In the fall of 1992, I volunteered to teach the General Math class which consists of 20 students. Approximately 12 of the students are classified as "special education students" in some way; some are physically challenged or brain damaged, and one girl cannot speak. The rest of the class are "general curriculum" students, some of whom have failed Algebra or General Math in the past. I have the advantage of working cooperatively with a special education teacher; however, the only time we have together is during class, so any cooperative planning is minimal.

The class has had no curriculum, and has a rather poor reputation, so I set out to develop a class which was reality-based, yet interesting for the kids. My colleague and I developed an introductory program in which we spent approximately one week on discrete mathematics, one week on the computer, and one week on mathematics in the "real world".

Shortly after the start of the school year it had become obvious that these students very often didn't "get it" only because they often did not understand what was wanted from them. We began talking about giving and receiving instructions, which very nicely set us up for a discussion of "algorithms." The word itself gave them some difficulty until someone suggested linking the term with the name of the vice-president (a candidate at the time): Al Gore. . . .Algorithm!

I introduced the idea of an alien being, Marty the Martian, who had been monitoring the Earth, and could understand simple things about us and our language. However, he could only follow very simple instructions. Things on his planet were very different, so we would have to teach him many of the simple tasks we took for granted. The ground rules were established: verbal instruction only, and in very simple terms.

Our first task: to teach Marty how to open a door. (To make this initial task a bit easier, the door was unlocked, but closed). I assumed the role of our alien visitor, and the class began their instructions: "walk to the door. . ." at which time I walked to and into the door! The students soon understood that directions had to be very explicit. We decided that "left" and "right" could not to be used; the kids came up with using the light from the windows (which make up one side of the room) as a reference point, since light was something that was universal, using "move towards the light", instead of "turn right".

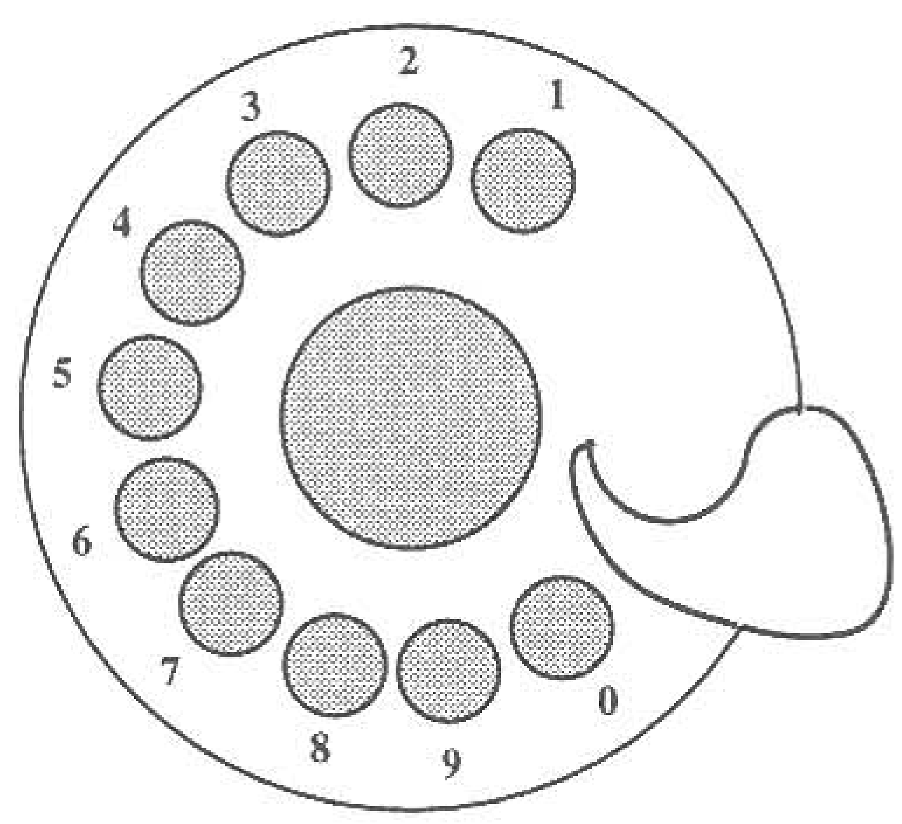
The kids began rather slowly—what was this strange woman doing? This was not the type of mathematics class they were used to. However, within half a class period, giving instructions was becoming natural, and they were beginning to understand the importance of being simple and specific.

Area Codes: Are They Really Codes?
by Laura Scerbo

Have you ever wondered how your area code number was chosen? As a child I lived in northern New Jersey, where the code was 201. My cousin Darcy lived in a suburb of Pittsburgh, with area code 412. I used to tease her and say that more people would want to live in my area because my area code came first!

Later on, during a course in college, our professor challenged us to to figure out the "code" behind area codes. My group started by listing all of the area codes we could think of. We came up with some major cities: New York (212), Pittsburgh (412), Los Angeles (213), Chicago (312), and Philadelphia (215), as well as a few from less populated places: Williamsburg, VA (804) and Sedona, AZ (602). We noticed that no area code started with a 0 or a 1, since these were special codes for the operator and long distance. We noticed also that every second digit was a 0 or a 1, which we thought was significant, but could not figure out why. We observed that the two largest cities in the U.S., New York City and Los Angeles, had consecutive area codes (212 and 213), and that Philadelphia came shortly after (215); in other words, the codes seemed to be in order of population.

We then came to a stand-still until our professor explained that when the codes were assigned, everyone had rotary phones (remember those?!): the 1 took the least time to dial, while the 0 took the longest. Since area codes cannot begin with a 0 or a 1, this explained why the codes for the major cities begin with 2 or 3. The second digit



was either a 0 or a 1, so naturally the largest cities have a 1 as the second digit. Thus, New York City has the fastest code, 212, while Alaska has a very slow code, 907.

When I told Darcy, she laughed and teased that more people would want to call her area than mine: "She who laughs last, laughs best!", she said. I guess she's right; but I'd still rather live here than in Pittsburgh.

Credits...

This Newsletter is a project of the *Leadership Program in Discrete Mathematics* (LP). The LP is funded by the NSF and is co-sponsored by the Rutgers University Center for Mathematics, Science and Computer Education (CMSCE) and the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS).

Joseph G. Rosenstein is Director of the LP and Founding Editor of the Newsletter. **Valerie DeBellis** is Associate Director of the LP.

DIMACS is a national Science and Technology Center, funded by the NSF, and formed in 1989 as a consortium of four institutions—Rutgers University, Princeton University, AT&T Bell Laboratories, and Bell Communications Research.

This issue was composed on *Microsoft Publisher* by Jason Werner. Figures were created by the Editor using *xfig*.

Subscriptions...

Just send us your name, address, phone number, and school.

IN DISCRETE MATHEMATICS:
Using Discrete Mathematics in the Classroom

Published by the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS) at Rutgers University

Communications should be sent to the following address:

DIMACS-DM NEWSLETTER
P.O. Box 10867
New Brunswick, NJ 08906

PHONE: 908/445-4065
FAX: 908/445-3477
email: micale@dimacs.rutgers.edu

Editors...

The Newsletter is edited by **Deborah S. Franzblau**, Research Assistant Professor and Education Coordinator at DIMACS. She has taught at Vassar College and in the *Leadership Program*. The editor of the column,"The Discrete Reviewer", is **Janice Kowalczyk** (LP '92). She is an Educational Mathematics Specialist at Rutgers and on the staff of the LP. She is Director of the Rhode Island School of the Future project, and teaches part-time at Lesley College (MA).

Authors...

Richard Adkisson (LP '94) teaches at Oakcrest High School (NJ).

Melanie Drozdowski (LP '92) teaches at Hanover High School (MA).

Susan Howell (LP '94) and **Lisa Soden-Winer** (LP '94) teach at Allentown High School (NJ).

Steven Kepnes (LP '94) is teaching at Soehl Middle School (NJ).

James Kinyon (DM Institute, Normal, IL, '93) teaches at Roland-Story Middle School (IA).

Joseph Malkevitch teaches at York College (CUNY), and is an instructor in the Leadership Program (LP).

Laura Scerbo (LP '94) teaches at Randolph High School (NJ).

Reuben Settergren is working towards a Ph.D. in Operations Research at Rutgers University. He has taught at the Johns Hopkins Center for Talented Youth.

Submissions...

In Discrete Mathematics welcomes contributions from readers, especially short articles about your experiences while using discrete mathematics in the classroom.

Send descriptions and reviews of resources (books, articles, software, videos, etc.) to Janice Kowalczyk, jkowalcz@k12.brown.edu. All other contributions should be sent to the Editor, Deborah Franzblau by email at franzbla@dimacs.rutgers.edu. Or write us at the address on this page.

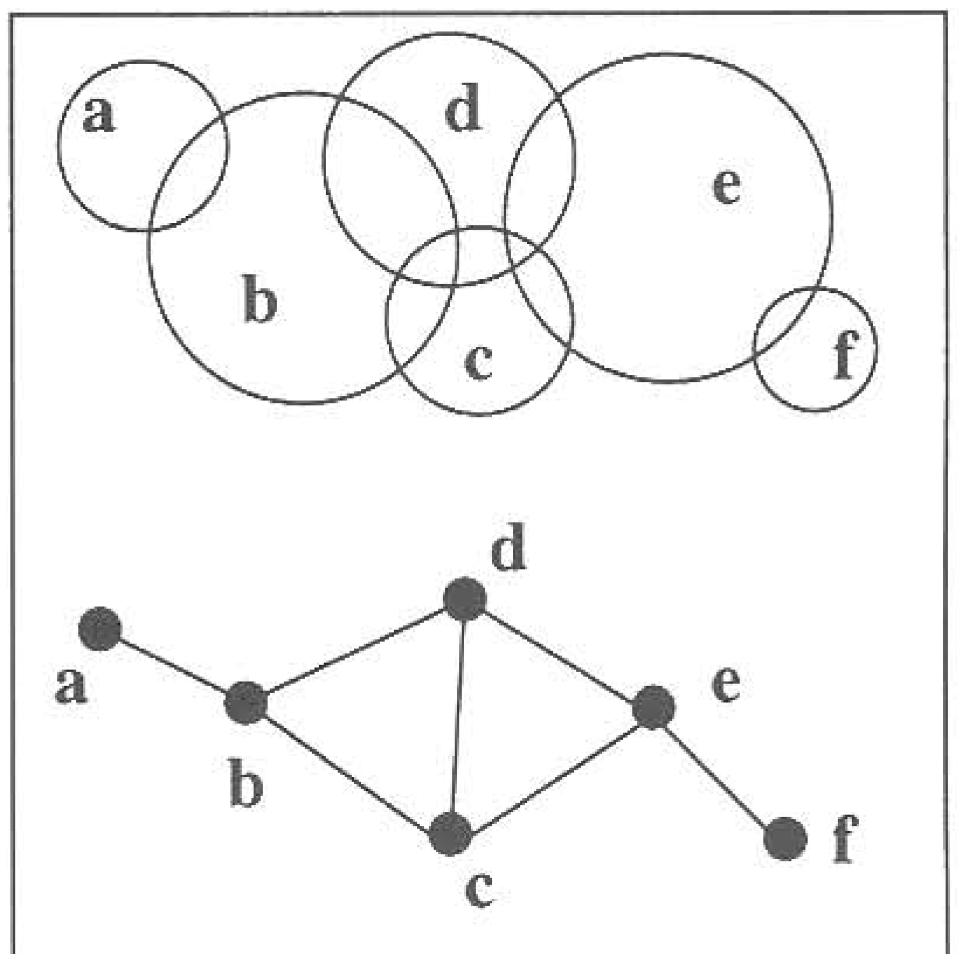
Picture Puzzles... (p. 12)

Kites, by **Judy Brown** (LP '92), who teaches at Pleasant Valley Middle School (PA). She creates her problems using PrintShop.
Our answer: 23 kites total. 7 kites in the sky + 16 paths through the triangular array that spell "KITES".

Map, by **Jake Moore**, a student at Mandarin High School (FL); his teacher, **Lillis Weber** (LP '90), said that her class was stumped for days trying to 4-color his map.

Solutions...

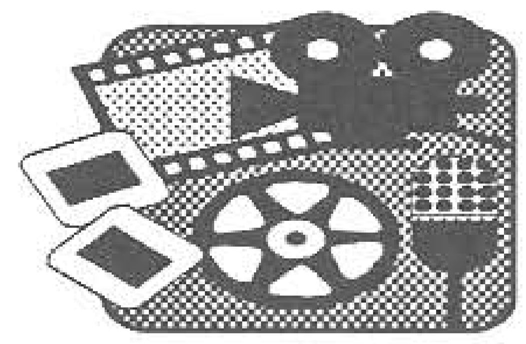
Radio Telephone problem on p. 10: Three frequencies are needed since three colors are needed to color the vertices of the "interference" graph below. The vertices represent the zones, and edges are drawn between zones that interfere.



Mobil radio zones (top) and "interference graph"

THE DISCRETE REVIEWER

by Janice Kowalczyk



This is a new column which will give suggestions for and reviews of discrete mathematics resources for the classroom. The original plan for this column proposed Siskel and Ebert as the reviewers, but upon review of our clip-art collection we realized we did not have one thumb up let alone two! Therefore, you and your fellow discrete math teachers will have to fill their shoes. We would appreciate your comments and recommendations in order to make future columns more useful to you.

Each column will have a special focus on a topic in discrete math as well as general recommendations. The special topic for this issue is graph coloring.

FAVORITE GRAPH-COLORING RESOURCES:

Activity (7-12)
NCTM Student Math Notes, by Thomas Dick, NCTM News Bulletin, November 1990.

I like this activity because it discusses the 4-color problem using the United States Map and is simple to use. It is perfect for 7th and 8th grade students.

—Judyann Brown, Pleasant Valley Middle School, Brodheadsville, PA.

Miscellaneous (3-12)
Giant U.S. Map, (approx. 4'x6'; heavy paper; B on W; no markings except borders), J.L. Hammett Co., P.O. Box 3106, 685 Liberty Ave., Union, NJ 07083
Phone: 1-800-333-4600
Order No: 87105 Cost: $10.59 + tax/shpg

Great for coloring activities with groups. —Editor

Video (9-12)
Geometry: New Tools for Technologies, COMAP Suite 210, 57 Bedford Street, Lexington , MA 02173.
Phone: 1-800-772-6627 Fax: 1-617-863-1202
E-mail: orders@comap.com Cost: $70

This well-done five-unit, one-hour video, complete with user's guide, illustrates the geometric tools of the 20th century: motion planning, error-correcting codes, Euler circuits, vertex coloring, and tomography. The unit "Connect the Dots" is a summary of graph-coloring applications.

—Ethel Breuche, Freehold High School, Freehold, NJ.

Learning Module (7-12)
The Mathematician's Coloring Book by Richard L. Francis, COMAP, Suite 210, 57 Bedford Street, Lexington, MA 02173.
Phone: 1-800-772-6627 Fax: 1-617-863-1202
E-mail: orders@comap.com Cost: $11.99

This entertaining unit on the 4-color theorem can be used for either general math or geometry students. It includes maps, pictures and worksheets to explore map coloring.—JK

OTHER RESOURCES:

Software (K-12)
Tabletop Jr. (K-6), and **Tabletop Sr**. (6-12), TERC, Broderbund Software, P.O. Box 6125, Novato, CA 94948.
Phone: 1-800-521-6263 Fax: 1-415-382-4419
Cost for each: School Edition, $89.95
IBM and Macintosh versions available.

Tabletop Jr. is an outstanding tool that enables students to sort, manipulate and create data sets. Students can develop theories concerning attributes, place value, and plotting while they work with engaging data on pizza or party hats. Accompanied by a teacher's guide with activities.

Tabletop Sr. is a database program designed for mathematics, science, social studies, or any subject in which students want to explore and analyze data. This program represents data as mobile icons that can arrange themselves into such forms as box plots, cross tabulations, histograms, scatter plots, and Venn diagrams. —JK

Resource Book (6-12)
Math Projects: Organization, Implementation, and Assessment, by Katie DeMeulemeester, Dale Seymour Publications, P.O. Box 10888, Palo Alto, CA 94303-0879.
Phone: 1-800-827-1100 Fax: 1-415-324-3424
Cost: $12.95

Katie DeMeulemeester, a 1991 participant in the Leadership Program for Discrete Mathematics, has written a book to help you navigate all phases of math projects. It contains sample student expectations, assessment and parent forms, plus suggestions for meeting the needs of students with different learning styles. —JK

Game Theory *(Continued from page 1)*

Recently, interest in game theory has been reawakened by the awarding of the 1994 Nobel Prize in Economics to John Harsany, John Nash (a mathematician), and Reinhard Selten [2, 3, 4] for work involving game theory. Moreover, new applications are emerging; for example, biologists have begun with some success to try predicting and explaining animal behavior using game models [5, 6].

An emerging subarea of game theory is the theory of voting. One important problem in this area is that of upholding the concept of "one person-one vote" within legislatures in which representatives represent parties with unequal population. One solution to this problem is to use what is called "weighted voting" in which the votes of different representatives have different weights (see [9, 10, 11]). Often, however, naive methods of assigning weights give unintuitive results (see sidebar), and mathematical analysis is needed. Examples in which weighted voting is used include the Electoral College, stockholder meetings, and the United Nations Security Council (see [11]). An example which has been in the news recently [7, 8], concerns the changes in voting within the legislative bodies of the European Union, necessitated by the addition of new countries with very different populations, size, and economic strength [7] (see also [10], p. 370-71).

Game theory has its limitations, of course: if (in trying to reflect reality) the game models become too complex, they are difficult to analyze; if the models are too simplified, the resulting solution may not address the original question adequately. However, as is apparent in the weighted-voting example, game theory can provide a framework in which relatively simple mathematical reasoning can be used to gain insight into real political situations.

References

(For further reading, see also the Game Theory Bibliography on page 9.)

[1] J. Von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton Univ. Press, 1944 (3rd Ed., 1953).

Classic, pioneering work on game theory and its application to economic and social problems.

[2] P. Passell, "Game theory captures a Nobel", *NY Times*, v. 144, Oct. 12, 1994, p. C1.

A description of the winners of the 1994 Nobel Prize in Economics and some of the work that led to the award.

(Continued on page 9)

A Paradox in Weighted Voting

Imagine a county legislature having one member for each of four towns, which at some time in the past had approximately equal populations. Now, thirty years later, the populations of the towns are:

Town A: 70,343 (41%)	Town C: 29,857 (18%)
Town B: 60,123 (35%)	Town D: 10,099 (6%)

Or, pictorially:

A B C D

Rather than redistrict or add more legislators, the town inhabitants agree to allow each member of the legislature to have a vote that has "weight" larger than one, if necessary. What is a fair way of assigning the weights? Given the data above (in which each population is near a multiple of 10,000), a natural assignment might be to add one to the "weight" of each legislator's vote for each group of 10,000 people in the district they represent (rounding to the nearest integer). If this is done the total weights assigned would be 7, 6, 3, and 1, respectively.

There is a total weight of 17; assuming that a majority is needed, there must be a weight of 9 to pass a bill. For example, if B and C both vote "yes", then the bill will pass. One can think of this as a game in which A, B, C, and D are players, who can only make one of two choices.

An analysis of the "power" that each player has in this game shows that this method of assigning weights is less fair than it seems. If you list the possible ways that the votes can be distributed, you will see that it doesn't matter what D does; the final outcome will be decided solely by the votes of A, B and C. In other words, D has positive weight but no power! (In fact, any assignment in which the weights are in proportion to the populations will have this feature.) In a slightly more complex setting, this situation actually occurred on the Nassau County Board of Supervisors in New York, and led to several lawsuits (see [9], and [10], p. 359-60, 372-73).

Is there a way to assign weights so that each representative ("player") has a fair fraction of the total "power"? A number of methods have been devised to try to answer this question, based on special "power indices" which are used to try to measure the power of players in a weighted-voting game [9, 10, 11].

Game Theory *(Continued from page 8)*

[3] S. Nasar, "The lost years of a Nobel Laureate", *NY Times*, v. 144, Nov. 13, 1994, p. F1.

A human-interest story about the game theorist John Nash, who suffered from mental illness at times during his career.

[4] L. Helm, "Nobel puts spotlight on game theory", *LA Times*, v. 113, Oct. 19, 1994, p. D1.

Discusses implications of game theory for the Federal Communications Commission auction of radio-wave licenses.

[5] R. Pool, "Putting game theory to the test", *Science*, v. 267, Mar. 17, 1994, p. 1591-93.

An ecological biologist tries to confirm game theoretic predictions of animal behavior.

[6] M. Nowak, R. May, and K. Sigmund, "The arithmetics of mutual help", *Scientific American*, v. 272, June, 1995, p. 76-81.

Describes attempts to use game theory to explain cooperation and non-cooperation within animal species.

[7] (Unsigned) "How to weigh a small nation", *NY Times*, Mar. 23, 1994, p. A11.

Discusses how the European Union deals with the member countries' differences in population and economic strength.

[8] C. Bolgar, "EU nations accept voting plan", *Wall Street Journal*, Mar. 30, 1994, p. 110.

Discusses changes in voting used by the EU, introduced when new members were added recently.

[9] W. Lucas, *Fair Voting: Weighted Votes for Unequal Constituencies*, HiMAP Module 19, COMAP, Lexington MA, 1992.

An introduction to weighted voting and the methods for weighting based on power indices.

[10] COMAP, *For All Practical Purposes*, 3rd. Ed, W.H. Freeman, NY, 1994.

Chapter 12 contains a detailed introduction to weighted voting. Chapter 15 discusses the theory of games in general.

[11] F. Roberts, *Applied Combinatorics*, Prentice-Hall, NJ, 1984, Sec. 2.16, "Power in Simple Games", and Sec. 4.7, "The Coleman and Banzhaf Power Indices".

An (advanced) introduction to weighted voting and power indices, with a number of examples.

Game Theory Bibliography
by Joseph Malkevitch

1. Friedman, J., *Game Theory with Applications to Economics*, 2nd Ed., Oxford U. Press, 1990.

This is a clear but mathematically sophisticated treatment of the theory of cooperative (and uncooperative) games. It includes exercises, with answers to some of them.

2. Gibbons, R., *Game Theory for Applied Economists*, Princeton U. Press, 1992.

Dynamic games (those which evolve over time), repeated games, and auctions are among the many topics treated here.

3. Luce, R. and H. Raiffa, *Games and Decisions, Dover*, 1989.

This reprint of a 1957 classic is still wonderful reading despite its age.

4. Lucas, W. *Fair Voting: Weighted Voting for Unequal Constituencies*, HiMAP Module 19, COMAP, Lexington, MA, 1992.

An introduction to weighted voting and the methods for weighting based on power indices.

5. Morris, P. *Game Theory*, Springer-Verlag, 1995.

An undergraduate text on game theory.

6. Ordeshook, P., *Game Theory and Political Theory*, Cambridge U. Press, 1989.

This book discusses such topics as utility (the value that individuals put on outcomes in a strategic situation), prisoner's dilemma (a paradoxical game involving decisions on whether to cooperate or not), and paradoxes assocated with voting games.

7. Straffin P., *Game Theory and Strategy*, MAA, 1993.

This is a wonderfully rich book about the theory of games. It covers most of the major ideas in a motivated and succinct way, and has many examples.

(Continued on page 10)

Graph Coloring *(Continued from page 4)*

(unlike her past experience in the course). She also gave the students a writing assignment for homework in which they had to explain how graph coloring was used to resolve conflicts.

As a fun concluding activity[1] her students formed human graphs, acting as vertices, and tried to three color themselves—they did not all succeed, but what an experience!

Not surprisingly, the students had the most difficulty in applying the concept of coloring to resolving conflicts. However, after completing a few examples, they were successful on these problems. Lisa had students write down what the vertices, edges, and colors represented before trying to give solutions, which greatly improved their understanding. Richard found that after working on a few conflict problems, his students became much better at solving other application problems. Susan noted that at the end of the unit, the students were almost disappointed that the "hard stuff" never came, so she pointed out that "Maybe it did, and you just rose to the occasion."

The students were also intrigued (some were even uneasy!) that it was possible to have many different, correct solutions to a problem. For example, in translating from a map (or conflict problem) to a graph, they found that their graphs could look different and still all be correct. It was also interesting to them that the maps could be colored differently and still be correct.[2]

We enjoyed this activity at the beginning of the year because it was a fun way to get all students actively involved, and gave students who normally do not excel in mathematics the chance to be successful. The students made connections not only within mathematics, but also with other fields, such as managerial science. The students were enthusiastic from the start, and this activity helped us establish a positive classroom atmosphere and good student-teacher rapport.

References

[1] L. Charles Biehl, "Scheduling and Graph Coloring", *In Discrete Mathematics*, No. 5, November 1994, p. 4.

[2] Fred Roberts, *Applied Combinatorics*, Prentice-Hall, Englewood Cliffs, NJ, 1984.

Assigning Frequencies to Mobile Radio Telephones [2]

In assigning frequencies to mobile radio telephones, each zone gets a frequency to be used by all phones while in that zone. Two zones that interfere (e.g., because of proximity or weather conditions) must get different frequencies.
How many different frequencies are required if
there are six zones, a, b, c, d, e, and f; where
zone a interferes only with zone b;
b interferes with a, c, and d; c with b, d, and e;
d with b, c, and e; e with c, d, and f; and f with e only?
(See page 6 for a picture and solution.)

Bibliography *(Continued from page 9)*

8. Taylor, A., *Mathematics and Politics*, Springer-Verlag, 1995.

This is an exceptionally clear account of issues related to voting, weighted voting, and the prisoner's dilemma, as well as a variety of other game-theoretic situations.

9. Young, H.P., Ed., *Negotiation Analysis*, U. Michigan Press, 1991.

This is an excellent collection of short articles by a variety of distinguished game theorists. The list of titles includes: Negotiation Analysis, Fair Division, and Arbitration Procedures.

10. Young, H.P., *Equity*, Princeton U. Press, 1994.

This book is destined to become a classic in the field. Equity issues, closely related to game-theoretic issues, are discussed in such contexts as bargaining, cost sharing, taxes, voting, and apportionent.

11. Zagare, F., *The Mathematics of Conflict*, HiMAP Module 13, COMAP, Lexington, MA, 1989.

An introduction to games for high school teachers, written by a political scientist who uses game-theoretic ideas in his research.

[1]*Editor's Note:* Based on an activity from the Leadership Program in Discrete Mathematics, July 1994, led by Joe Rosenstein, from a suggestion by Mike Fellows, Professor of Computer Science at the University of Victoria.

[2]*Editor's Note*: Other teachers, including middle-school teacher Jackie Faillace ("Graph Coloring and the Search for Multiple Solutions", submitted to LP *Greatest Hits* project, June, 1994), have also noticed that this a significant advantage of introducing graph coloring in class.

LEADERSHIP PROGRAM IN DISCRETE MATHEMATICS

SUMMER INSTITUTES FOR K-8 TEACHERS . . . IN DISCRETE MATHEMATICS

RUTGERS UNIVERSITY

WHAT? Two-week residential institutes at Rutgers University and two-week commuter institutes at sites to be determined (tentative sites for 1996 are in New Jersey, Rhode Island, and Virginia).

WHO? For teachers of K-8 students, as well as mathematics supervisors and specialists.

WHEN? The residential institutes are scheduled to run during the period from July 8 to July 19, 1996. Commuter institutes will be scheduled in July or August. Participants attend four Saturday follow-up sessions during 1996-1997 and a one-week institute the following summer.

STIPENDS Funding from the National Science Foundation provides each participant with a $300/week stipend; meals and weeknight lodging (double occupancy) are provided for residential institutes.

PARTICIPANTS ... will be expected to:

* introduce discrete mathematics into their classrooms
* develop classroom materials for other teachers
* present workshops on institute topics

DATES Applications are due by March 22, 1996; applicants will be notified by April 29, 1996.

QUESTIONS For further information, or to receive an application form, call Stephanie Micale at 908/445-4065, send email to micale@dimacs.rutgers.edu, or write to the address below.

DISCRETE MATHEMATICS . . .WORKSHOPS IN YOUR DISTRICT

WHAT? Full-day workshops can be scheduled in your district, for teachers of all grades, on topics in discrete mathematics which can be introduced into K-12 classrooms and curricula.

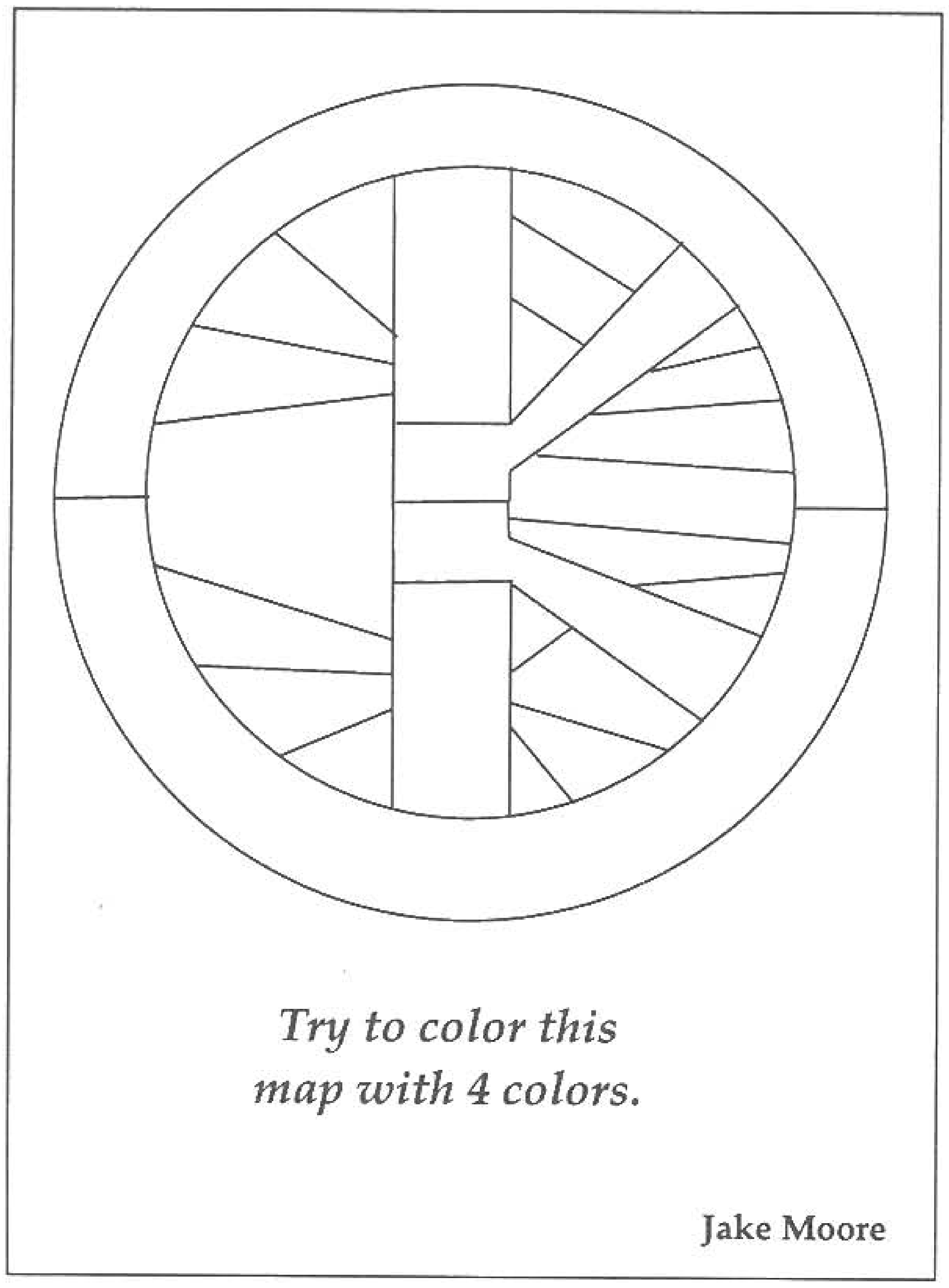
WHEN? Workshops will be scheduled during the school year (or during the summer) on an individual basis at the request of the participating district.

BY WHOM? Experienced teachers from the Leadership Program in Discrete Mathematics, who have participated in a special training program, prepare and present the workshops.

COST The district will be expected to pay only direct costs, that is, expenses and honoraria for the workshop leaders and instructional materials for the participants.

QUESTIONS For further information, call Michelle Bartley-Taylor at 908/445-4065, send email to mbt@dimacs.rutgers.edu, or write to the address below.

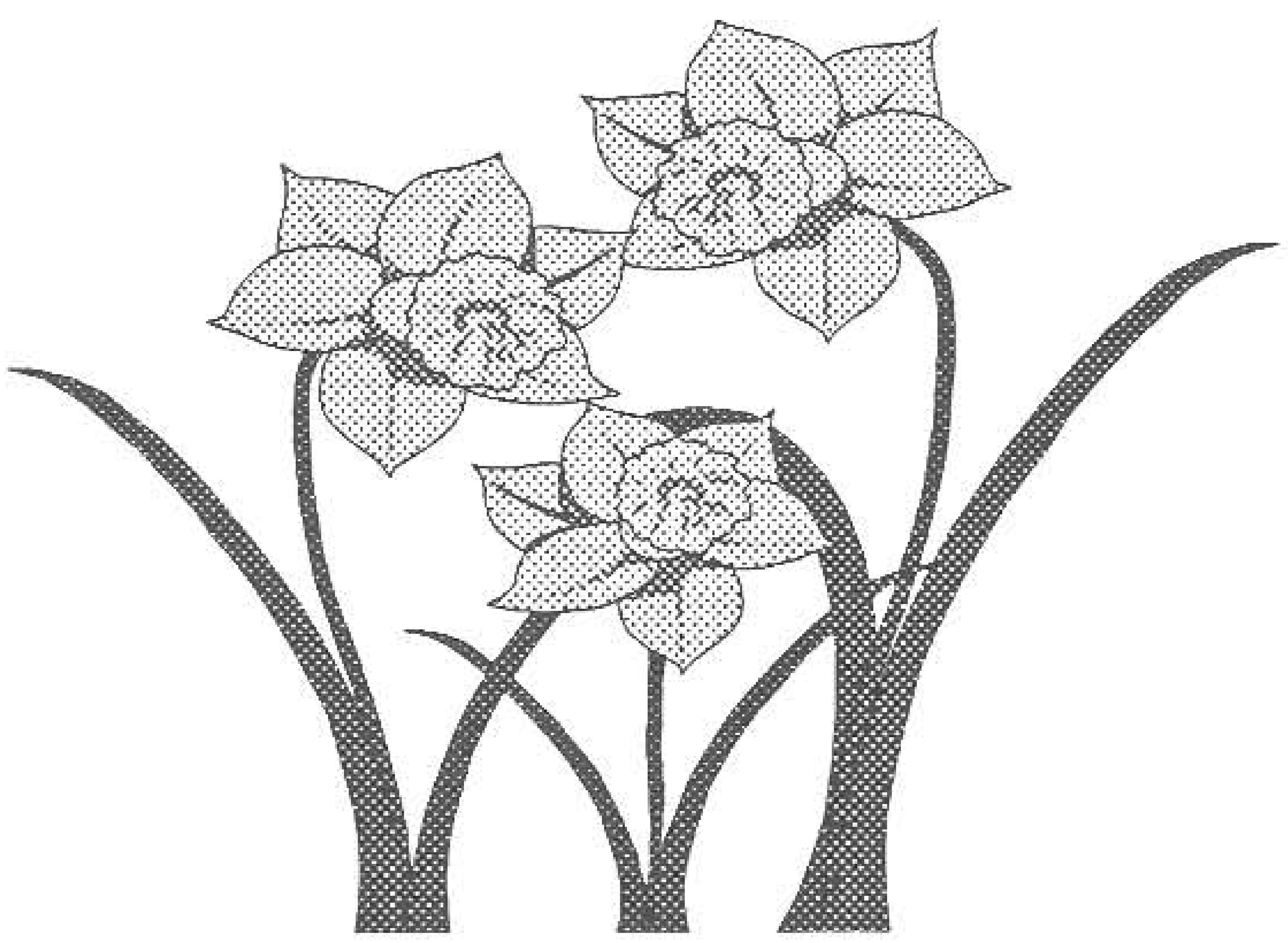
ADDRESS Leadership Program in Discrete Mathematics
P. O. Box 10867
New Brunswick, New Jersey 08906



Picture Puzzles (see page 6)

DIMACS - DM NEWSLETTER
P.O. Box 10867
New Brunswick, New Jersey 08906

Non-Profit Org.
U.S. Postage
PAID
New Brunswick, NJ
Permit No. 157



Inside this issue ...

Game Theory and Voting
The Discrete Reviewer
Graph Coloring