IN DISCRETE MATHEMATICS

Using Discrete Mathematics in the Classroom

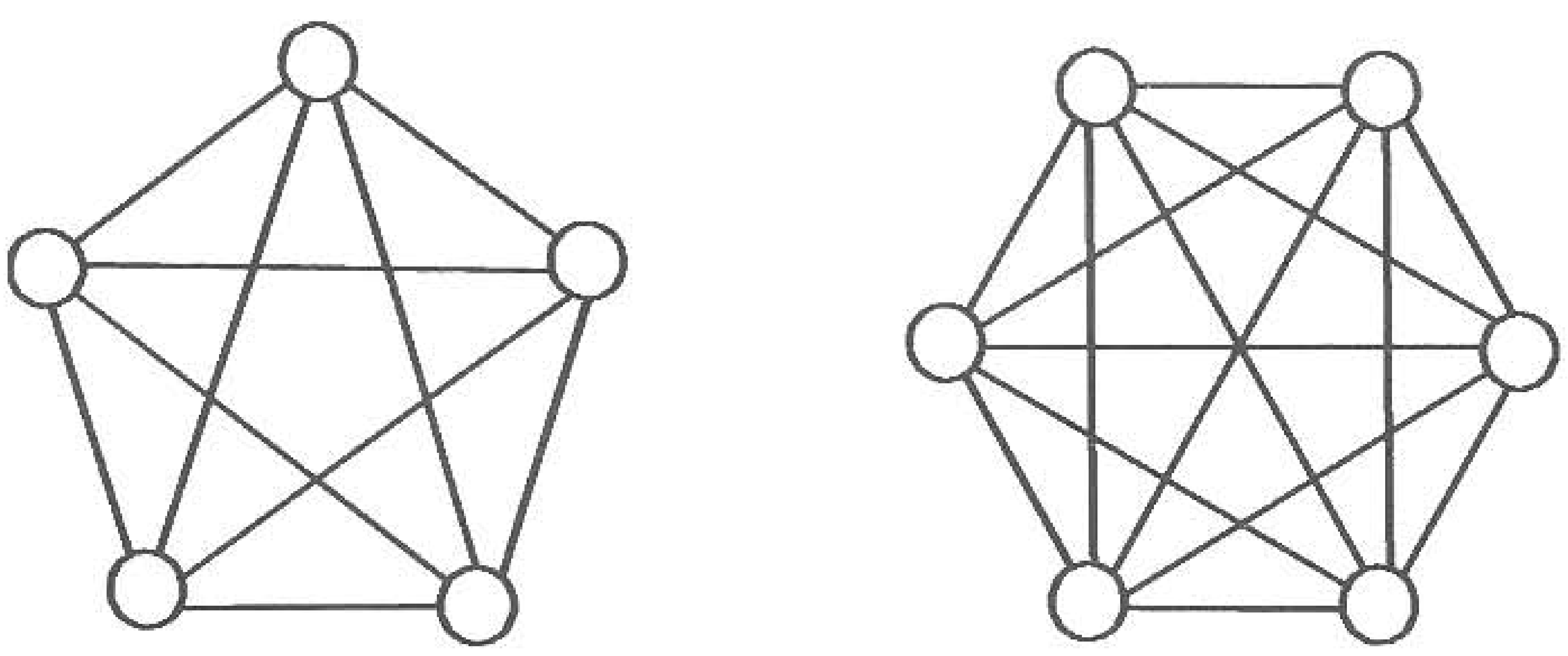
Issue #10 — *Winter 1999*

Speaking Discretely...

Robert Hochberg

I remember in graduate school when, in a certain class, we were stumped by the following question: "Can you color the edges of this graph, using just red and blue, so that no monochromatic triangles result?" (See diagrams and questions below, with answers on Page 6.) In the end, we solved the problem by imagining a game in which two players (red versus blue) took turns coloring the edges, and a player lost if he created a triangle, all of whose edges had his own color. Answering the original question then amounted to finding out whether a tie was possible in this game. That's when we discovered that there could be a lot of real mathematics in games.

The theme of this issue is "games." We have 3rd graders playing Nim, 7th graders inventing their own games, algebra students doing shortest path problems, and 5th graders solving mazes blindfolded. We also have a solution to the Farmer's Daughter Challenge from our previous issue, a new challenge, an article on prime and composite numbers, and our Discrete Reviewer presents a few books on games. Enjoy!



For the graph on the left, is there a way to give each edge a color (red or blue) so that no three edges that form a triangle all have the same color? How about the graph on the right?

In This Issue

Games are Math!

Elizabeth Kelsey Hicks

I teach at Algonquin Middle School in the Averill Park School District, a rural district on the outskirts of Albany NY. There are twenty four students in one of my 7th-grade classrooms. Although this is an accelerated class, I have discovered that it is a not-so-accelerated group; many students struggle to do any type of work that is not straightforward. The Game Theory Unit I did in this class helped students think differently and branch out from using solely traditional problem-solving strategies.

"Games are math?"

This was the question students asked when I introduced games in my 7th-grade class on the second day of school this year. Students were amazed to think that creating games and playing games was actually considered math. The students were able to then convince themselves and each other that there are many occupations that really use math and/or science to create games. They came up with occupations such as computer programmers, authors of mystery and drama stories, delivery people, map makers, and the list went on.

After discussing what a strategy is, I then asked, "Do all games involve strategy?" This question generated much discussion and as a class we decided that the answer was "No." We listed games that involve only dice or spinners where winning depends only on chance. The idea of strategy instantly had the students thinking because it grabbed their attention and was something they had not experienced.

My Game Theory Unit included some Nim-like games as well as the following game, involving both strategy and chance:

Remove One (See *Discrete Challenge* on Page 12)

Remove One
2 ______
3 ______
4 ______
5 ______
6 ______
7 ______
8 ______
9 ______
10 ______
11 ______
12 ______

Directions: Give each student a Remove One game board and 14 bingo chips. Explain to the students that they are to place their chips on the numbers; they may place as many chips as they like on each of the numbers, and they may decide to leave some numbers with no chips on them, but they must use all 14 chips.

Roll the dice and call out the sum. Every time you call out a sum, the students remove a chip from that number. The first student who removes all his chips wins.

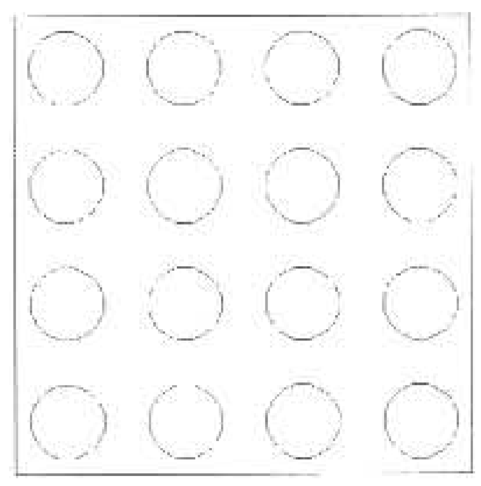
After we played one game we discussed the students' strategies for placing the 14 chips and then played again. After the second game we calculated the probability of the eleven possible sums and played again. The learning curve was dramatic from game to game!

Students' Games

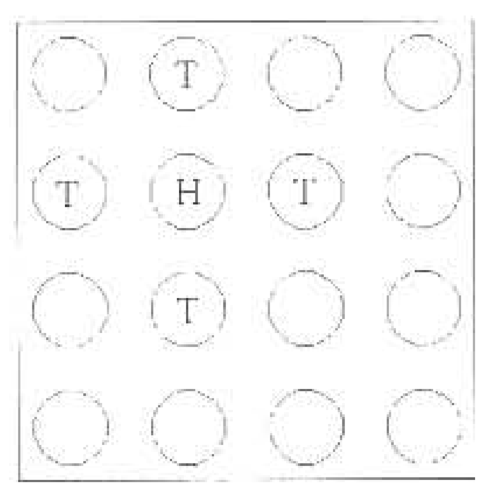
Throughout the unit students were assigned the task of creating a game, involving strategy, that they would play against one another. Many of the students came up with take-offs on games we had done in class. For example, one student invented "Make 35," which was obviously based on "Make 21." (In "Make 21," two students start with "0", and take turns adding "1" or "2" to the total; the student who "makes 21" wins.) But the students then added ideas, like using coins or dry pasta as markers, and then using the game to help teach younger students how to add.

The following is a description of three of the games my students came up with:

Coin Trap



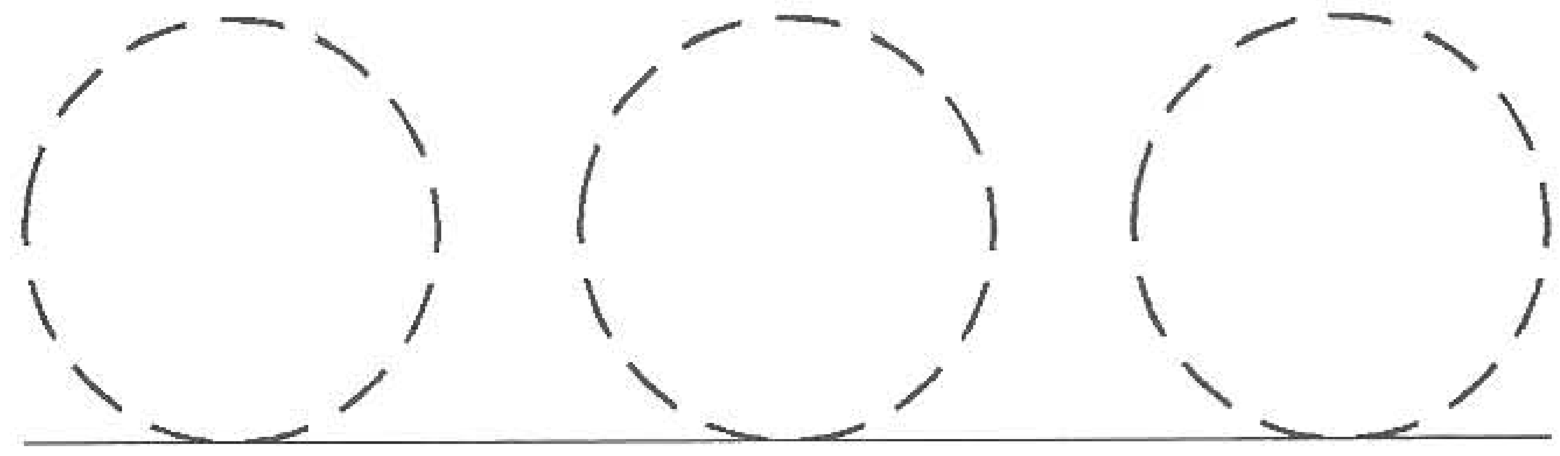
Coin Trap board



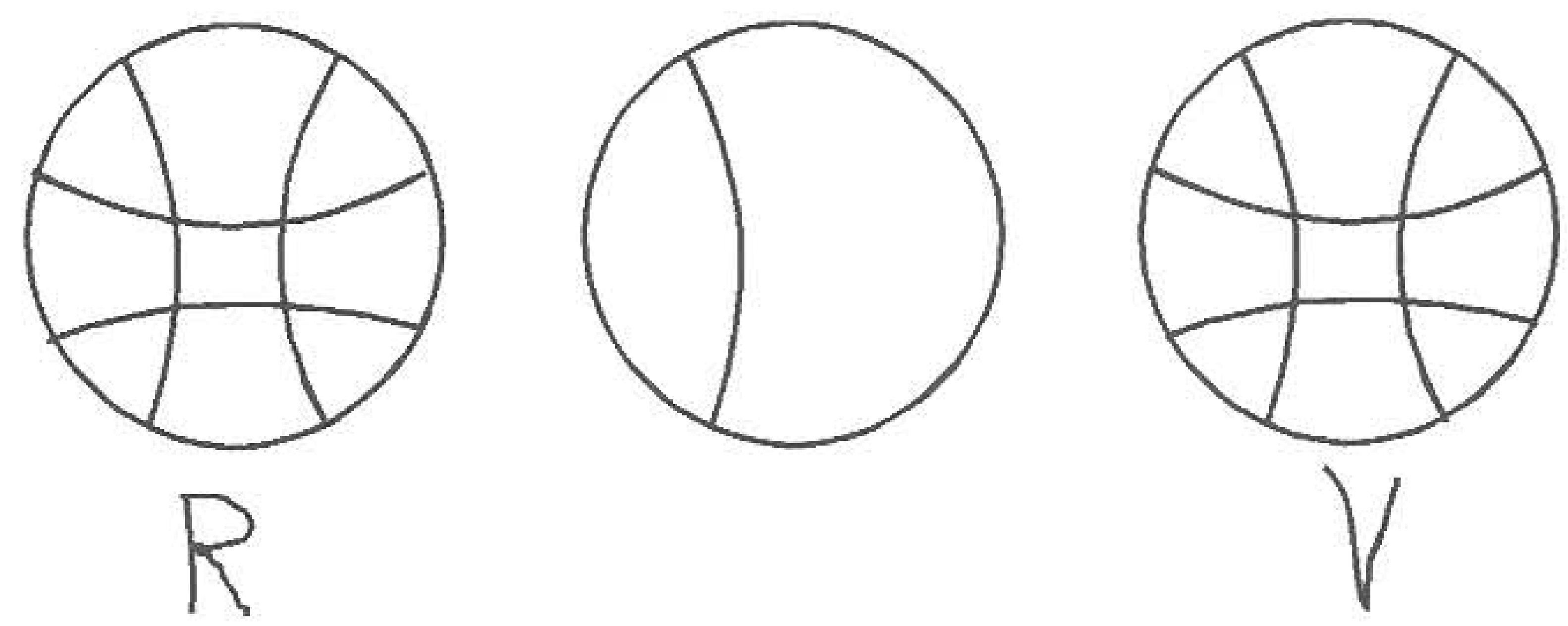
"H" is *trapped*

Directions: Player one (Heads) places a coin somewhere on the board. Player two (Tails) places a coin somewhere else on the board. Players then alternate placing coins on the board. The goal of the game is to try to trap the other player's coin inside four of your coins, as shown in the figure. (Coins along the edges cannot be trapped.) Once a player traps a coin he may flip that coin to his side. When all 16 coins have been placed, the player with the most sides up, wins.

This student had clear directions, gave examples, and even mentioned a strategy for winning which, although it didn't always work, demonstrated that the student had understood the concept of "strategy." Unfortunately, it is easy to avoid getting trapped, and the games tend to end in ties. Few coins were stolen in the games I watched. [*Ed. note: If you make the game board larger, say 6 by 6, and only require 3 of the 4 adjacent spots to be occupied by the opponent for a "trap," then the game becomes more complex to analyze, and thus more interesting to play.*]



The Volleyball Game Board



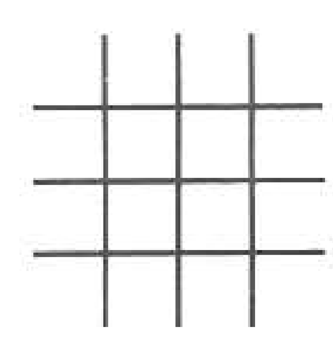
A Volleyball Game in Progress

Volleyball

Directions: Start with three dotted circles. Two players with different colored pencils play each other, alternately adding either one or two curves to one of the circles. A completed volleyball consists of 5 curves — the four curves on the inside and the circle itself, as shown in the figure. When a player completes a volleyball that player "wins" that volleyball, and gets to put his initial underneath it. The winner is the player who initials two volleyballs.

This game is a little like tic-tac-toe in its simplicity, but it was a thinker! [*Ed. note: One of the players has a winning strategy. Can you find it? Disguised hint: It is Player (10-5+3-8+2-7+6) who has the winning strategy.*]

Tic-tac-toe plus



Directions: This game is like tic-tac-toe, but a bit more challenging. Start with a 4x4 board (shown to the right). Players take turns putting their symbol (X or O) on the board. If a player makes 3 in a row, he draws a line through them. At the end of the game, whoever has more lines drawn is the winner.

Althought students can play this game for a while before noticing it, it turns out that Player I has an advantage in this game, and he will tend to win when beginners are playing. However, there is a strategy which Player II can use to always force a tie. Can you find it?

[*Ed. note: On larger boards (say, of size n×n) Player II still has a strategy to tie if n is even. But if n is odd, it may be that Player I is able to force a win. This seems like a difficult problem, and we welcome solutions.*]

My 3rd and 4th graders had finally identified the portion of the US map near Nevada which required 4 colors, but weren't sure if 4 colors were *really* needed. One group, containing the most precocious student, said, "I think it is possible to do it in three colors, but not by us. Probably by the smartest person in the world!" (Later, I made him become the smartest person in the world by disproving it himself!) – Callie Hershey, LP '99

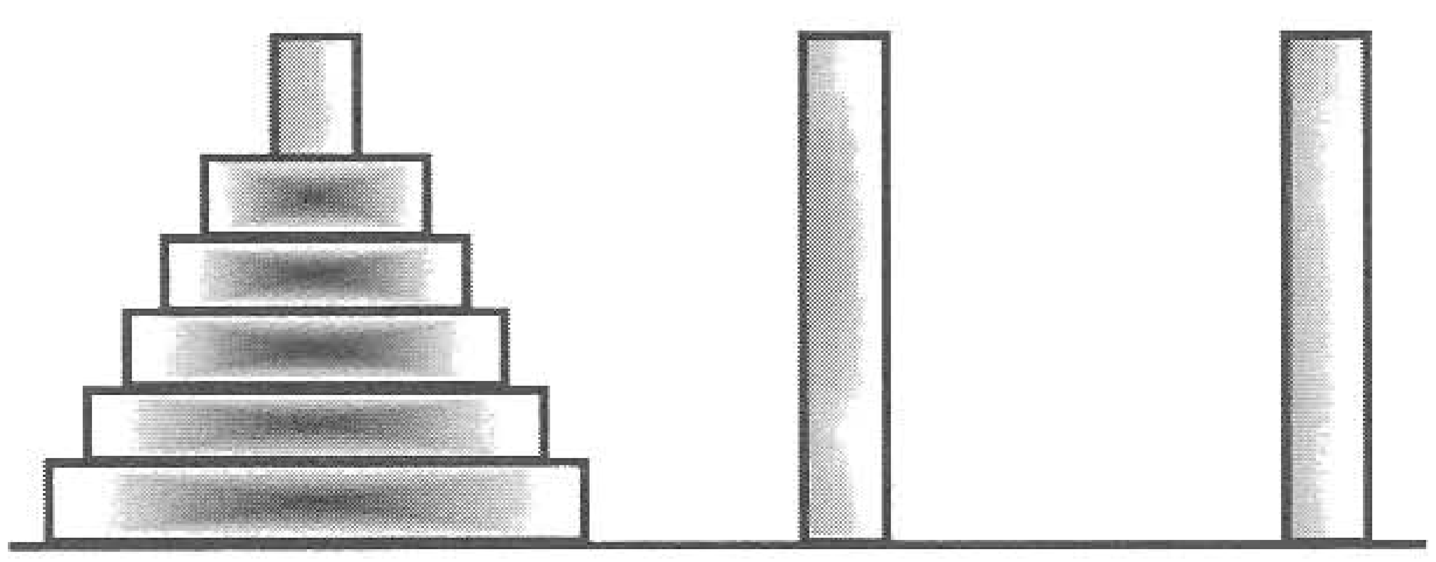
The Discrete Reviewer

Janice C. Kowalczyk

In keeping with the theme of this issue, I have put together a list of recommended "Games" resources for the classroom. The search for resources on this topic has not been an easy one, but with help from teachers in the Rutgers Leadership Program in Discrete Mathematics (LP) some valuable resources were found. While a number of games books do exist, there do not seem to be many that are written specifically for the classroom teacher. Materials on this topic were more likely to be found scattered in books one game at a time. I also found that many were not in keeping with our focus on problem solving, fair games or games with winning strategies. The following books therefore seem to be somewhat unique and therefore should be valuable resources to teachers who wish to incorporate this exciting topic into their curriculum.

As always, I would appreciate any comments or recommendations on this topic or this column. Your feedback on the resources that you try as a result of this column are also encouraged so that I can develop a better appreciation of the usefulness of these resources to classroom teachers of discrete mathematics.

Creating Nim Games *Grades 3 and up*
by Sherron Pfeiffer
Math Project Series
Dale Seymour Publications
Cost - Amazon.com $10.95
ISBN: 1-57232-272-1



This is by far the best resource I found for implementing strategy games into the classroom. It gives an interesting history of Nim, a rationale for using this topic in a standards-based curriculum, some nice assessment tools, and some delightful strategies on how to begin and how to create your own Nim games. The book has nine variations of Nim including Classic Nim, Array Nim, Calculator Nim, Pattern Block Nim, and Path Nim, and strategy sheets to help students explore and describe their game strategies in the game process. The final section of the book is a thorough teacher's guide for having students create their own Nim-game projects as well as examples of student created projects.

Math Around the World: Teacher's Edition *Grades 5-8*
by *Beverly Braxton, Jacqueline Barber, Linda Lipner, and Philip Gonsalves*
GEMS – Great Explorations in Math and Science
Lawrence Hall of Science
University of California at Berkeley
Cost – bn.com $25.50 (Barnes and Noble)
ISBN: 0-912511-94-X

This book contains eight multicultural games and gives detailed instructional strategies and materials for using them either in a series of class periods or at stand-alone learning stations. These classroom-tested discrete games are Nim, Kalah (also known as Mancala), Games of Alignment, Hex, Towers of Hanoi, Shongo Networks (Euler), Magic Squares, and Game Sticks. The first four on this list involve finding a winning strategy. Game Sticks involves probability and the notion of determining a fair game.

In my search for books about games a few other books were recommended by LP teachers. Although I have not had time to examine these resources personally, the descriptions and recommendations from LP teachers who have found them useful warrant their inclusion on this list.

Math Puzzle Mini-books *Grades 3-6*
by Michael Schiro and Rainy Cotti
Scholastic Book Clubs, Inc.
1-800-SCHOLASTIC - Item # 918092
ISBN: 0-590-918095
Cost - $8.95 plus $2.25 postage and handling

From Mary Kay Varley (LP '96) comes the following description and recommendation: "These are reproducible books of 8 pages each, the size of a gift card, which deal with One Line Drawings, Nim games, Paths and Circuits, Magic Squares, etc. My 3rd and 4th graders loved them."

Board Games Around the World: A Resource Book of Mathematical Investigations *Grade: 7*
by Robbie Bell and Michael Cornelius
Cambridge Press
Cost: bn.com $20.95 (Barnes and Noble)
ISBN: 0-521-35924-4

This book includes a selection of some 60 games, along with a brief history, description of rules, and suggestions for investigating strategies. Mary Reynolds (LP '97) states in her recommendation, "This book follows a game through the centuries and around the world. In addition, it also has the winning strategies in the back of the book. I am in the process of trying the Mancala games for a unit with the Social Studies teacher on Egypt. The students have already done the Morris game and found the strategy."

Nimomania

Patricia Thelander

After attending my second summer session of the Leadership Program at Rutgers, I decided to do a unit on the game of Nim with my 3rd-grade class. This is a class of twenty math students of different abilities in a suburban public school. It happens that math is the favorite subject of more than half my students, which is unusual.

We started the year with a review of addition. The chapter in their Addison-Wesley text emphasizes strategies for mentally adding numbers. By now, students should have facility with basic addition and subtraction facts, but this is not yet true of all of them. As I was teaching it this year, I was much more aware of the emphasis on strategies. Nim is introduced at the end of the chapter as a Critical Thinking Activity. We began with a simple game in which there is a single pile of eleven markers. Each player may pick up 1, 2, or 3 markers on a turn. To win, you must make *your partner* pick up the last counter.

The students were very excited to be able to play a game and began enthusiastically. I didn't give any suggestions for play the first few rounds, and basically they played without much planning. After a few games I told them to pay attention to their strategies and try to determine if there was a way to always win. I asked them to notice if it made a difference if they played first or second. After a few more rounds we recorded their Hints to Win on a chart.

I was surprised at what they discovered in such a short time. Their findings included: a) whoever reduces the pile to 5 wins, b) 5 is the "trap," c) player 2 should take whatever player 1 takes, and d) if player 1 takes an even number, player 2 should take an odd number and vice versa. Those last two strategies didn't always win, but they showed that the students had come to understand the concept of a strategy.

Several weeks later I introduced the game of 3-5-7 Nim. Here there are three piles and at each turn a player can take as many markers as he wants, but they must all be from the same pile. The winner forces *his opponent* to take the last one. I asked them to play and this time record what they did at each play, who won, and could they find a winning strategy. Later in the week I introduced variations on Nim that can be found in *Creating Nim Games* (see page 3).

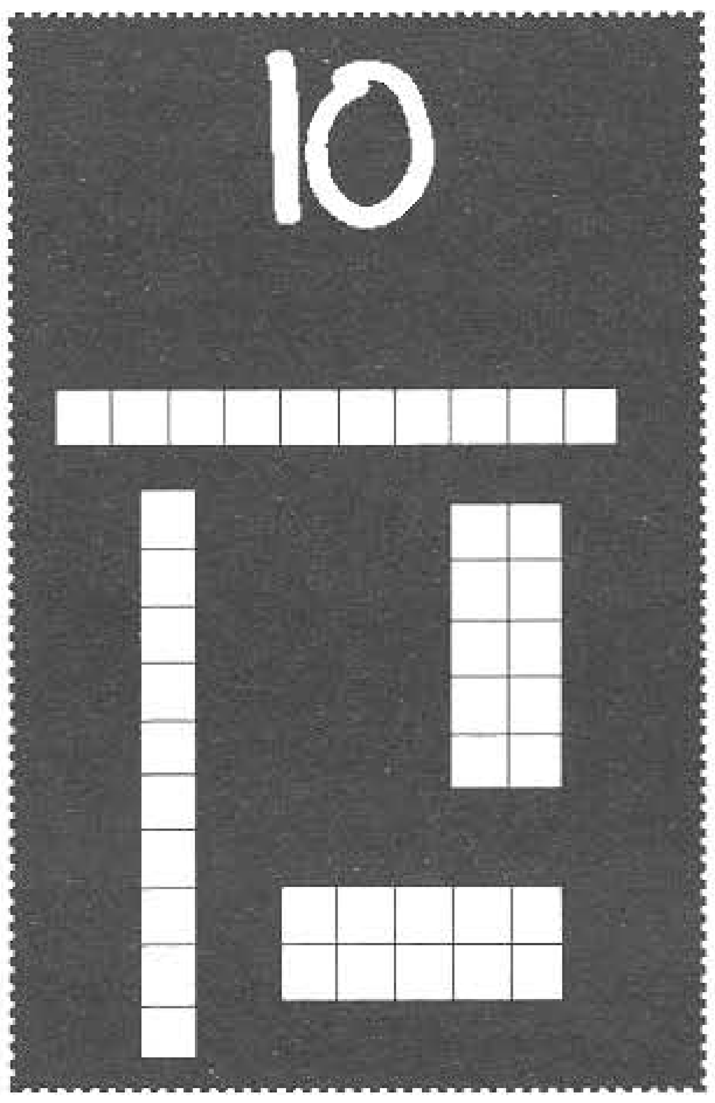
I was beginning to wonder if maybe we had become "Nimomaniacs" and it all might just be too much of a good thing. My concern was addressed on the day of our field trip to the Museum of Natural History when the buses were delayed. I hadn't really planned anything for that day because of the trip. Since we had time, one of the students asked, "Please can we play Nim?" As they got out the unifix cubes for markers, one of the parents, who was coming along as a chaperone, said to me, "My son can't get enough of this game. He loves playing it and figuring out the strategies." I had my answer.

Arrays, Composite Numbers, Square Numbers, and Primes

Mickey Jo Sobierajski

I teach at Red Creek Central School District, a rural district located at the northeastern end of Wayne County, one of the poorest counties in the state of New York.The district has about 1300 students. This unit was presented to a heterogeneous 5th-grade inclusion class of 24 students, seated in cooperative groups of four.

On the first day of this unit each student received one or two 12×18 pieces of black construction paper and centimeter graph paper. Each piece of black paper had one numeral (ranging from 1 to 30) written on the top with chalk. The construction paper was handed out randomly, but if a student received a low number, then I would try to give him/her another number so everyone would finish about the same time.



The task was to outline rectangular arrays on the centimeter paper which contained the number of squares shown on their black paper. They were to make as many as they could, then cut out the rectangles, glue them on the black paper, and label the dimensions of the rectangles. The papers were then hung around the room in numerical order, and the students discussed in their groups what they noticed about the numbers.

We regrouped and discussed their observations as a class. Students noticed that: some numbers (2, 3, 5, 7, 11, 13, 17, 19, 23, and 29) had two rectangles, but all the others had more. 24 had a lot of rectangles; some numbers (1, 4, 9, 16, and 25) had an odd number of rectangles, and they were the same numbers that had squares under them!

On the second day of this unit each student received 25 color tiles. They repeated the previous day's activity, using blocks instead of centimeter paper, but this time instead of finding all rectangles for a single number, they were to find all rectangles for each of the numbers from 1 to 25, recording their findings on a chart.

The third day was for discussion of the previous two days' work. This discussion got very lively; in fact, it already became lively by the time we got to the number 1. They could accept that 1 was a square number, since it fell on the diagonal on the multiplication table with the other square numbers. No problem. The problem arose when we discussed whether 1 was *prime* or not. This took two class meetings to discuss! The verdict was that 1 is square, but not prime. (In fact, this is how mathematicians classify the number 1.)

I found that this unit helped students understand that square numbers are special composite numbers, that prime numbers form exactly two rectangles, and that 1 is a square number, but not prime. Putting the black construction papers with the rectangles on them around the room is, visually, a very powerful activity. Numbers and their rectangles jump right out at the students.

Discussion is the most important part of this unit. Students need to have the opportunity for math talk with each other, as well as with the teacher. They need to make their own conjectures and listen to others' conjectures, then modify or continue with their conjecture. This unit gave them the opportunity to do all of that.

Leadership Program in Discrete Mathematics
Crash Course for High School Teachers

The Leadership Program in Discrete Mathematics will offer a three-day crash course in discrete mathematics for high school teachers at Rutgers University on August 15-17, 2000. The content will include paths and circuits in graphs, patterns in numbers and geometry (fractals), voting methods, and codes. The anticipated cost of the program is $225 (including lodging, double-occupancy, on August 15 and 16, and breakfasts and lunches).

For information, call Bonnie Katz, 732-445-4065, email her at bonnie@dimacs.rutgers.edu, download the materials from http://dimacs.rutgers.edu/lp/crash-course/, or write to: Leadership Program, P.O. Box 10867, New Brunswick, NJ 08906.

Credits...

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Joseph G. Rosenstein is Director of the LP and Founding Editor of this Newsletter.
joer@dimacs.rutgers.edu

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Just send your name, address, phone number and school to:
vocelaj@dimacs.rutgers.edu

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Editors...

Janice Kowalczyk is Assistant Director of the Leadership Program, and is our Discrete Reviewer.
kowlacjn@ride.ri.net

Robert Hochberg writes educational software, and is general editor.
hochberg@dimacs.rutgers.edu

Answers from Page 1: "yes" & "no."

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P.O. Box 10867
New Brunswick, NJ 08906
Phone: (732) 445-4065
Fax: (733) 445-3477
email: vocelaj@dimacs.rutgers.edu

Authors...

Elizabeth Kelsey Hicks (LP '97) has taught 6th, 7th, and 8th grade math at Algonquin Middle School in Averill Park School District over the past four years. She also taught Math 8 for three years at Stewart Middle School in Tacoma, WA.

Gina Scanlon (LP '98) teaches 8th grade math and algebra at Central Middle School in Dover, DE. She has her master's degree in special education and became retrained to teach math 13 years ago.

Laurie Sleep (LP '98) teaches math and writing to 4th and 5th graders at Camino Pablo School in Moraga, CA. She is also the math mentor for her district.

Mickey Jo Sobierajski (LP '98) is a Title I Math teacher for grades K-4 at M.W. Cuyler Elementary School in Red Creek, NY. She is the math specialist for her building. She is also a math consultant for Creative Publications.

Patricia Thelander (LP '98) is a 3rd-grade teacher at Grandview School, North Caldwell, NJ.

Discrete Mathematics Summer Institutes for K-8 teachers

The Leadership Program in Discrete Mathematics (LP) will feature one two-week residential program at Rutgers University, New Brunswick NJ, during June 26 to July 7, and four twelve-day commuter programs at Greenville NC (7/6-7/21), Auburn AL (7/17-8/1), Houston TX (7/20-8/4), and Worcester MA (7/31-8/17).

Participants will be expected to attend follow-up sessions during the school year and a one-week follow-up institute during the summer of 2001. Graduate credit will be available. Teams of teachers in schools or districts are welcome to apply. Funding by the National Science Foundation provides for all costs of the institutes. Participants will be expected to assume leadership roles in bringing discrete mathematics to their classrooms and schools, and in introducing their colleagues to these topics. For information, call Bonnie Katz, 732-445-4065, e-mail her at bonnie@dimacs.rutgers.edu, download the materials from http://dimacs.rutgers.edu/lp/institutes, or write to: Leadership Program, P.O. Box 10867, New Brunswick NJ 08906.

Bring the Leadership Program (LP) to Your District or Region
Institutes in Discrete Mathematics for K-8 Teachers

A series of 6-20 full-day workshops during the school year or summer that together would provide a broad experience in discrete mathematics similar to that provided by the LP summer institute For further information, contact Assistant Director Janice Kowalczyk at 401-841-5583 or kowalcjn@ride.ri.net.

Shortest Paths and Probability — A Game to get Students Started

Gina Scanlon

After attending the Leadership Program in the summer of 1998, I was searching for a way to integrate shortest path problems into our algebra curriculum. After many attempts an idea began to form. I would design a game board in which the object of the game was to get from one side of the board to the other, with the students choosing their own paths. My reasoning was that in the context of a game, students would try to win and would try to find the shortest path.

I knew, however, that there had to be more to the game than that. Years ago I had attended a workshop that focused on using games in math. They emphasized that an element of chance often helps to make it more interesting, so I decided to introduce rolling dice into the game. Eureka! I could include probability in the game and bring two important discrete math topics into the same activity.

The design and production of the game took a long time. It was not as polished as I would have liked, but I decided to give it a trial run in my classroom. I planned several activities using the game board, but wanted to begin with just a three-day lesson.

The game board included several paths from the bottom left corner of the board to the top right corner. Some of the paths were gray (light rectangles in the figure below) and some of the paths were brown (dark in the figure.) At first I had planned to use use a theme with cute, clever pictures at each vertex, but for the time being, I used letters instead and intended to replace them once I found a theme that I wanted to use.

Day 1

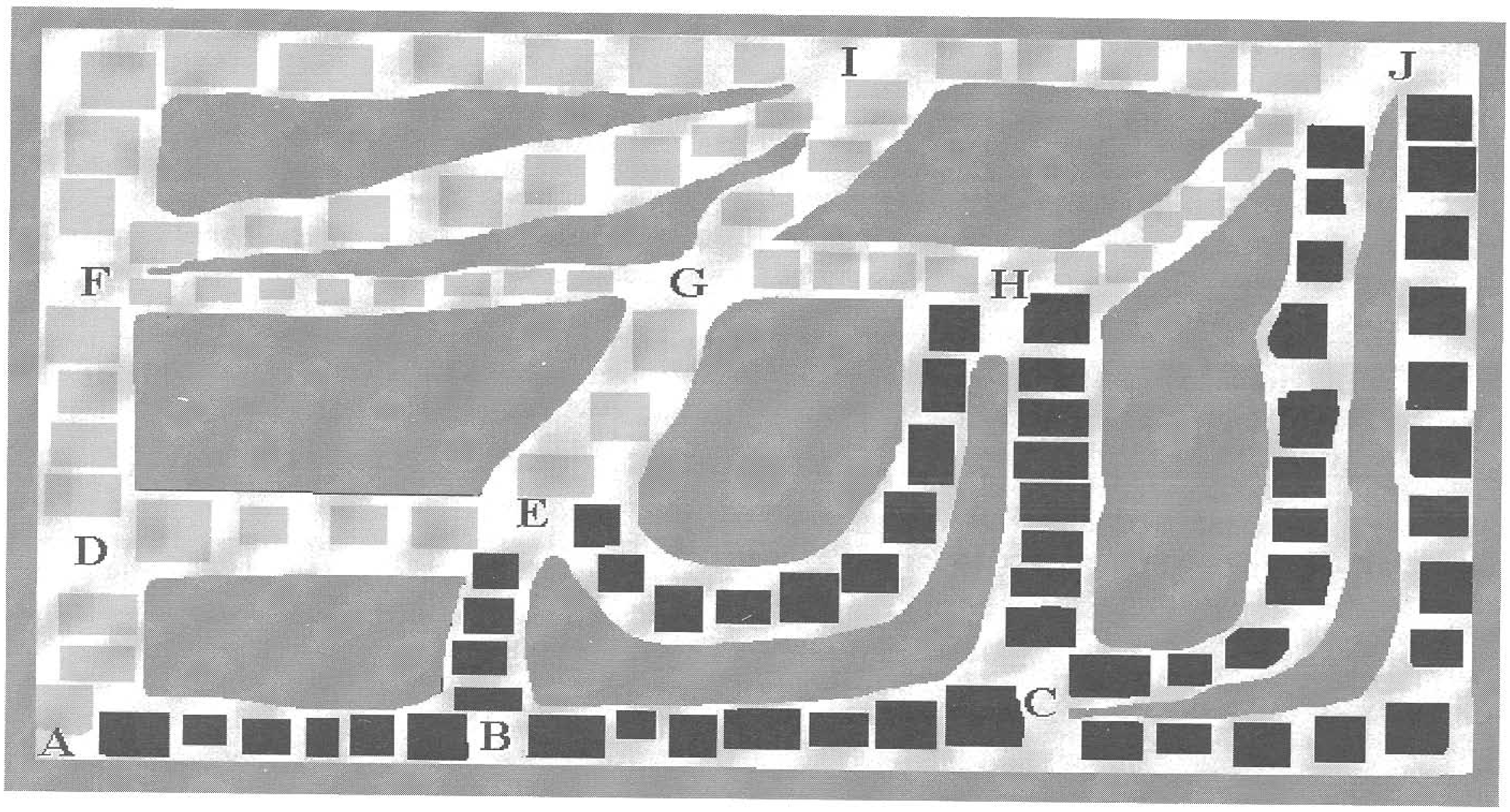
The game board was 11 by 17 and two students competed against each other. They started at point A and had to go to point J. They took turns rolling two dice; if the sum was even, they moved two spaces, if the sum was odd, they moved three spaces. We called this game *The Sum Game*. I set up this first game to deliberately bias the students towards making certain choices that would not be good choices in the second game.

Students played this game twice and I watched their strategies. Many of them didn't think about the shortest path. A minority of them did seem to be thinking about it, and there was some improvement when they played again.

After they finished two games, I demonstrated on the overhead how to translate the game board into a graph. I drew the game board with the vertices and edges on the overhead projector and counted the spaces from point A to B. I had the students draw their own graphs, label the length of the edges, and then determine the shortest path from A to J (the ending point). We discussed game strategy and what path they would take if they were to play the game again. For homework, the students worked two shortest path problems I had created.

Day 2

The next day we played a new game called *The Product Game*. On the game board some of the paths were now gray



(light-colored in the figure above) and some of the paths were brown (dark in the figure). The shortest path was gray, and had length 18, as students knew from our work the previous day; the brown path had length 24. Students played in pairs, with one using the grey route and the other using the brown route. The person using the gray route could move only if he rolled an odd product, and he would move two spaces. The person using the brown route would move only if he rolled an even product, and he would also move two spaces.

The students started playing , and pretty soon I was hearing cries of "That's not fair!" from the players on the gray paths. They had considered themselves very clever to get the gray routes, and then they would end up losing. I asked them to keep track of who won — the person on the gray route or the person on the brown route. The person on the brown route almost always won. After they played the game, we did some analysis.

SUM	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

We made two tables of the outcomes of the roll of the dice — a sum table and a product table. (In the tables shown here, the even outcomes are shaded.) We looked at the probability of getting an even or odd sum versus the probability of getting an even or odd product, but had to leave the rest of the analysis for the next day. For homework the second day the students had to do a shortest path problem on a larger graph (to reinforce the first day's activity) and they had to write a paragraph explaining why the person on the brown path almost always won. The top math section also had to write a paragraph explaining how they would change the rules of *The Product Game* to make it fairer, but keeping the rule that the person on the gray path could only move on odd products and the person on the brown path could only move on even products.

PROD	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

The Five Dollar Problem

You and your brother get $5 a week for running errands for your grandmother. Your brother has a suggestion: Instead of splitting the money, why not roll a pair of dice and multiply to see who gets it? The lowest possible product is 1, the largest is 36, and their average is 18.5. So, if the product is 19 or more, he keeps the money, and if the product is 18 or less, you get the money. He's older than you, and very good at math, so you're inclined to trust his idea. Should you? Explain your answer.

Day 3

On this day we completed our analysis of the theoretical number of rolls needed to win *The Product Game* on each of the two routes.

The gray route was 18 spaces, so it took 9 moves to complete it. The probability of getting an odd product was 9/36 (as shown in the product chart) or 1/4, and so, after x rolls, we expect $(1/4)\cdot x$ of them to be odd. To figure out how large x should be in order to make 9 moves, we set up the equation $(1/4)\cdot x = 9$. Solving for x, we determined that it would take 36 rolls to win, theoretically.

We did the same for the brown route and used the equation $(3/4)\cdot x = 12$. We determined that it would take 16 rolls to win on the brown route, theoretically. This is why the brown route was so much more likely to win! [*Ed. note: It turns out that gray will win only 1 game for every 80 games that brown wins (approximately).*]

Next, we discussed how changes to the rules could change how fair, or unfair, the game was. For example, if gray moves three spaces at a time instead of two, the equation becomes $(1/4)\cdot x = 6$, giving $x = 24$. This means that gray will take 24 rolls on average, instead of 36, to complete the game. That's a little fairer! [*Ed. question: How many spaces would gray have to move at a time in order for the game to be fair?*]"

As an extension of this lesson I selected some of the paragraphs that the students had written on making the game fairer. The students worked in small groups. Each group was assigned a paragraph to analyze. They had to present their conclusions to the class. It took about two class periods for the groups to do their reports and for the class to discuss them. In addition to the group work, each student had to pick two or three of the paragraphs to analyze by himself and everyone had to solve the *Five Dollar Problem* (see insert).

The technique of having groups present solutions was one we used in the Rutgers workshop last summer. I have used it several times this year and find it a very useful classroom strategy. Next time I teach this lesson, I would probably begin Day 3 with a review of probability, and allow additional time on other days for presentations.

"Noel is infamous for trying to pull cons and doing anything humanly possible to get out of assignments. I handed him back his report without a grade. You see, the grammar and syntax were impeccable and Noel's forte is definitely not writing! As anticipated he asked me why I hadn't put a grade on his paper. I told him I didn't believe it was his, whereupon he pulls out a report, written by his cousin (several years his elder) and retorts, "Of course it's mine; here's the report my cousin gave me!" Needless to say, it was difficult keeping a straight face at that point."

– Sylvia Nomikos, LP '99

Making Mazes

Robert Hochberg

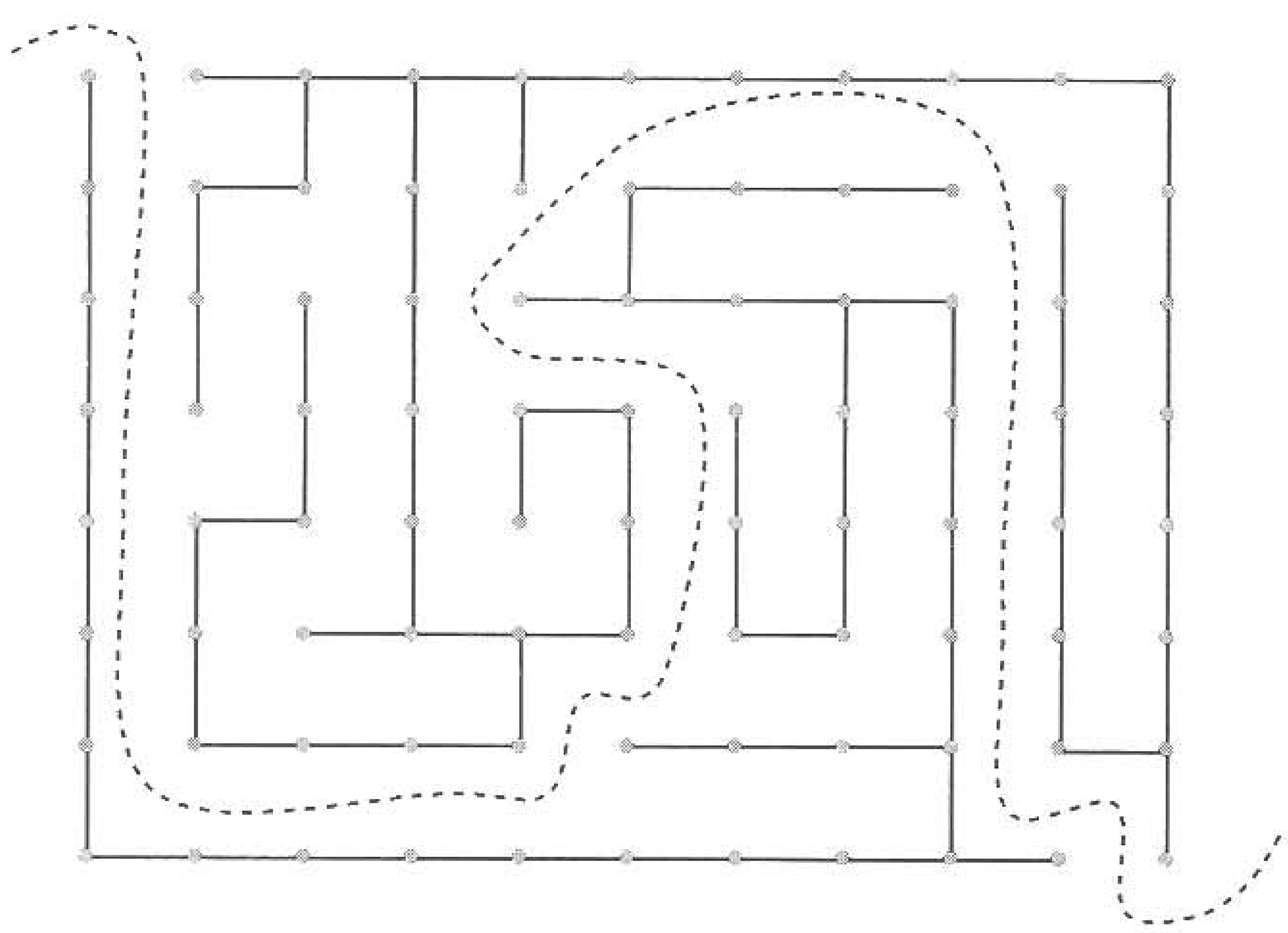
There is a lot of recreational mathematics involving mazes. Solving mazes is part of the fun, but drawing them can also be enjoyable, and poses some interesting questions. Let's look at mazes and how they work.

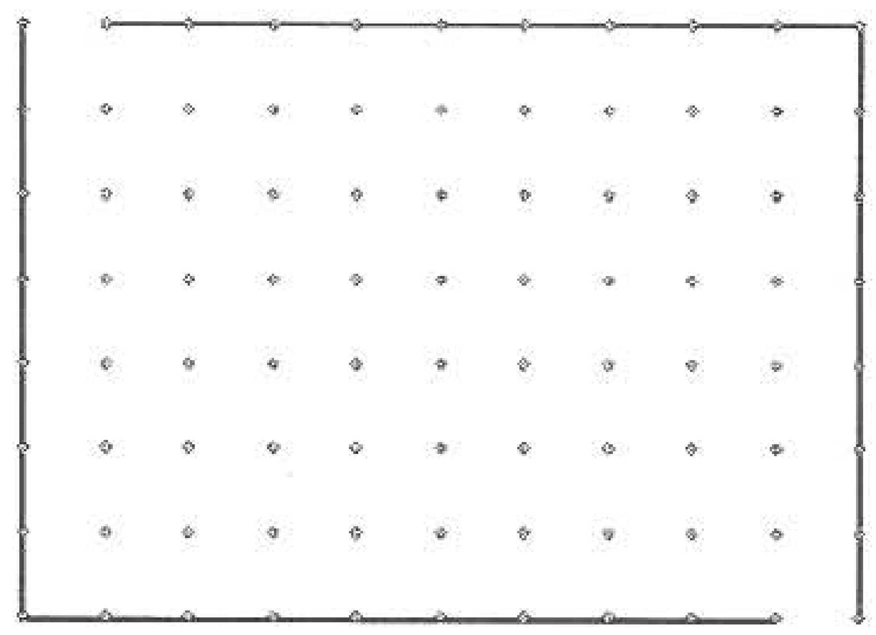
A maze has an entrance and an exit, and if it is a good maze, it will also have a way through, from the entrance to the exit! So if we imagine a person walking through a life-sized maze, trailing a red string (or a dotted line, as in the figure below) behind him from the entrance to the exit, we discover that a maze consists of two parts which don't touch one another at all; for the red string separates them. And each of these two parts is a tree, that is, a graph with no cycles.

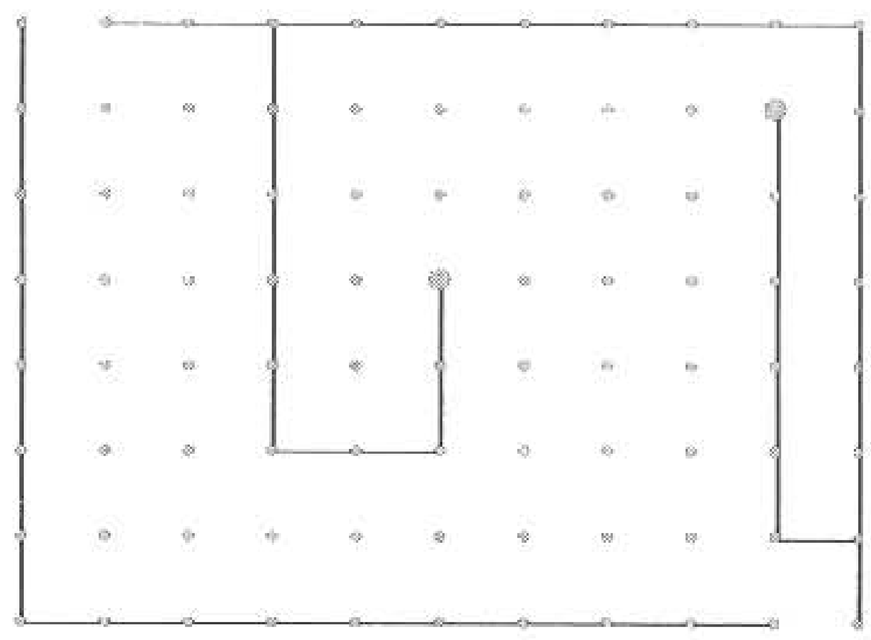
Thus, to build a maze, one simply draws two trees which don't touch one another, and specifies an entrance and an exit. And to solve the maze, one simply walks between the two parts. But to make the maze challenging, the parts should have lots of corridors to confuse the maze-solver.

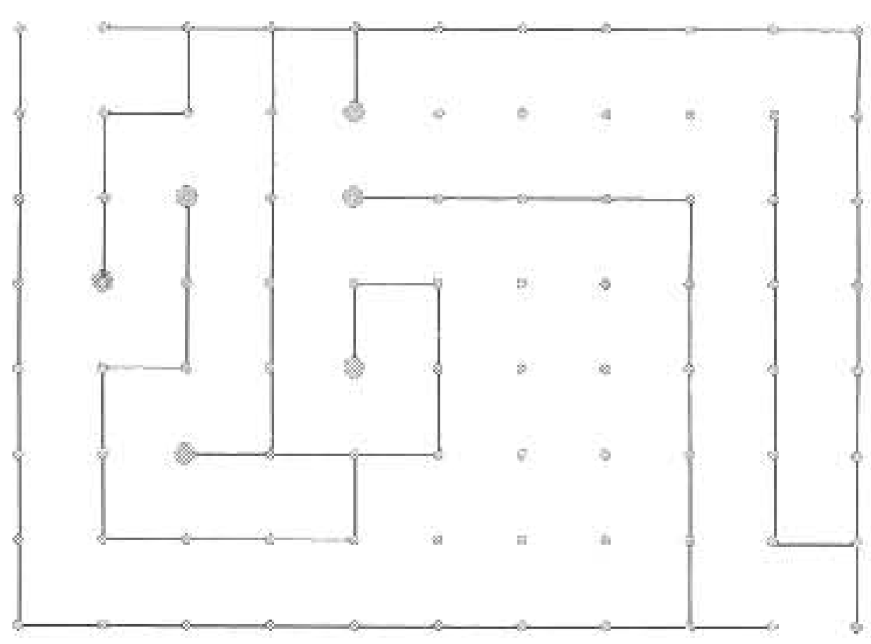
So, here's one way to build good mazes: Start with a rectangle, drawn on square dot paper, but with two "doors" cut into it (top figure). Then grow trees! A tree is grown by selecting an unused vertex within the rectangle (large dots in the middle figure) and walking horizontally and/or vertically from vertex to vertex, until a used vertex is reached. The middle figure shows two branches grown. Note that all the vertices on the two trees are now considered "used" vertices, and so other branches can grow onto them. (The bottom figure shows 6 branches so far; two of them are just single edges which grew onto another branch.) We continue growing branches, always starting at an unused vertex and walking to a used vertex, and then stopping there. When all the vertices inside the rectangle are used, we have a maze, as shown below.

Note that the entrance and exit can be anywhere on the rectangle, and you'll still get a maze. In fact, you can take a maze, close the exit, and erase a border edge to create a new exit somewhere else, and you get a new maze!







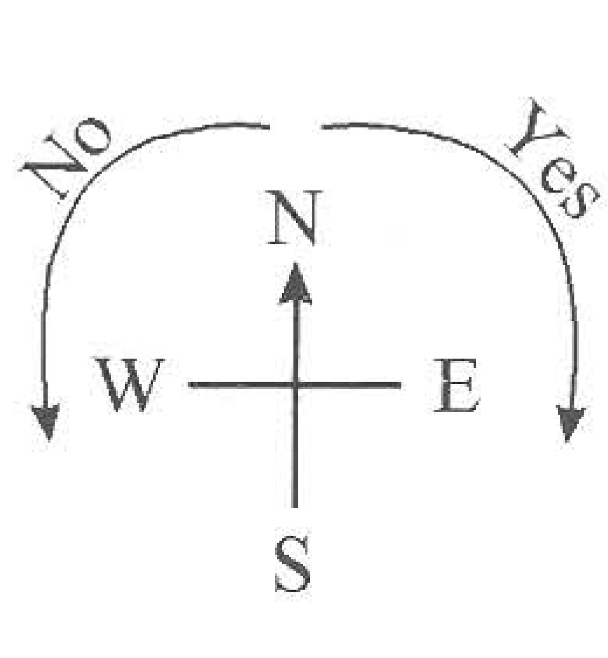


If the maze has a dots along the left side and b dots along the top, then the student will have to draw a total of $(a\text{-}2)(b\text{-}2)$ inside edges in a complete maze. Thus the maze on the left has (8-2)(11-2) = 54 edges inside the rectangle.

One classic technique for solving a maze is to imagine walking through the maze, always keeping your right hand on the wall. This method will always work on the mazes described here. (Try it out on the maze to the left!) Sometimes it will lead you to a dead end, but it will also lead you out of that dead end, never to return.

But what about blindfolded? Suppose your students have drawn a maze on the blackboard, placed a marker somewhere in the maze to mark your position, and then challenged you to get out of the maze. (We assume that they block off the entrance so you don't accidentally walk out that way!) All you can do is call directions (north, south, east or west) and the class responds "no" if a wall is blocking you in that direction and "yes" if there is an opening in that direction, in which case they move the marker. How do you get out? Tammy Wopnford was kind enough to let me try this with her 5th graders at Sonoran Sky Elementary School in Scottsdale, and we had a lot of fun.

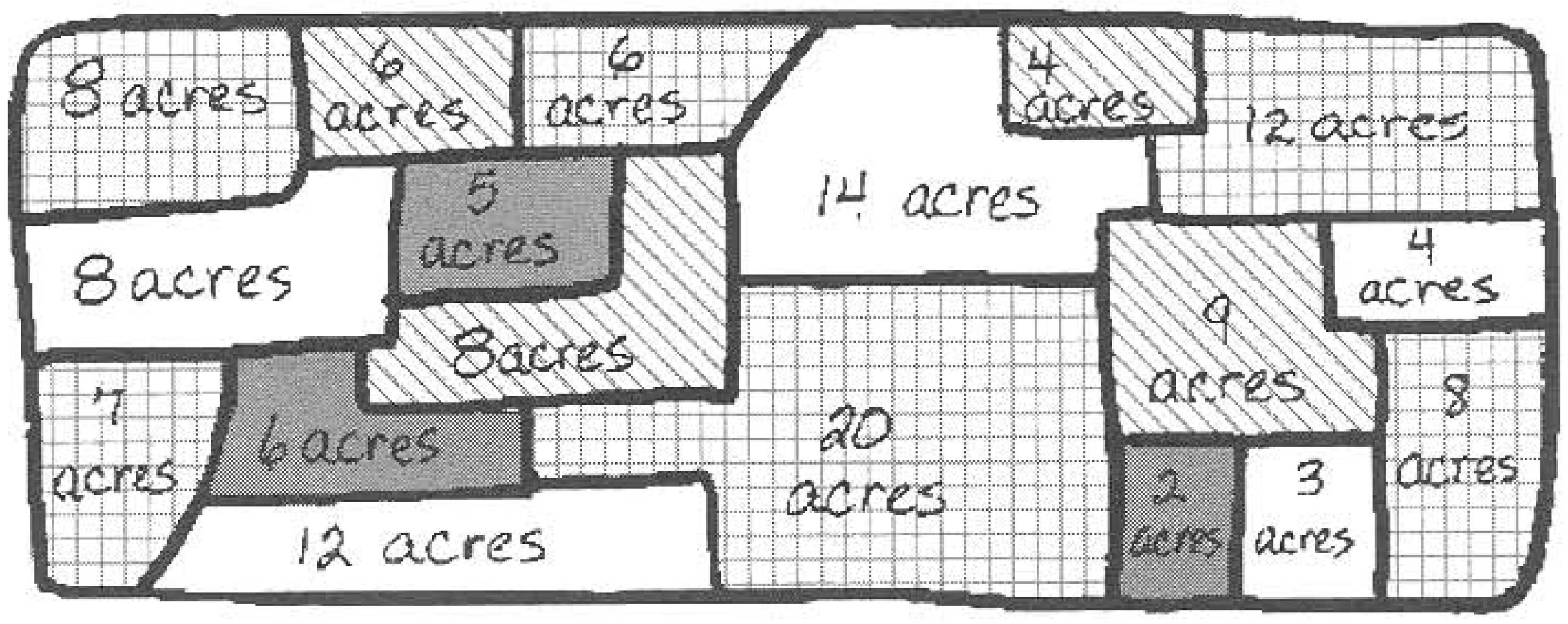
Here is a nice algorithm (look at the compass below as you follow this algorithm): Say "North" first. If the students say "yes," then turn clockwise around the compass to "East," and make that your next guess. But if they say "No" to your "North," then turn counter-clockwise and make "West" your next guess. In general, whenever they say "Yes," turn clockwise for your next guess, and whenever they say "No," turn counter-clockwise for your next guess. This will get you out of the maze, guaranteed, just as if you were keeping your right hand on the wall. And the kids will never guess your trick!



Solving A Discrete Challenge: The Farmer's Daughter Problem

Laurie Sleep

In the last issue of *In Discrete Mathematics, The Farmer's Daughter,* an interesting map coloring problem written by Jill Dunlap was presented. In this problem, a farmer had divided his small farm into 18 different fields for each of which he wished to purchase livestock. Although the farmer enjoyed many different animals, he wanted to limit his animals to as few types as possible. He also knew that he could not have like animals in pastures that share a common border because the animals would try to tear down the fences to be with their friends in the adjoining fields.



The farmer presented his situation to his problem-solving daughter with the following guidelines:

1. He wanted as many acres of cows as possible
2. Then as many acres of sheep as possible
3. Then as many acres of horses as possible
4. Then as many acres of goats as possible
5. Finally, as many acres of pigs as possible.

He decided that each acre could hold either 8 cows, 20 sheep, 6 horses, 12 goats, or 18 pigs. It was possible that he could actually end up with more sheep than cows. This was fine, as long as the cows occupied the most acreage. He handed his daughter a diagram of the field (shown below, but without the coloring) and set off with his sons to build fences while she devised a plan for placing the livestock.

A Discrete Challenge was offered to readers and their students to submit their best solutions to this problem. The response was fantastic. We received 51 solutions from students ranging from grade 5 to the college level.

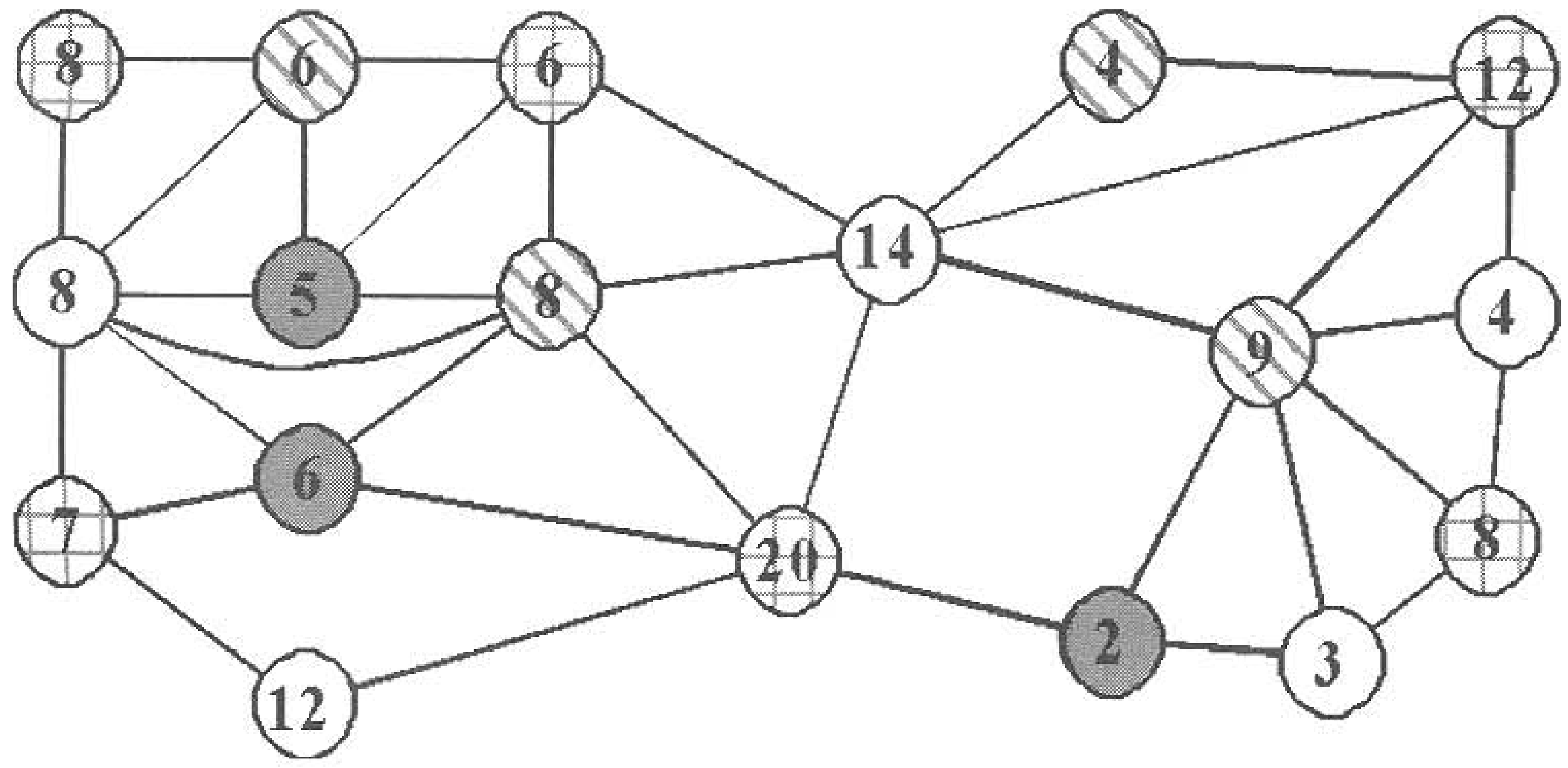
The best solution for the farmer's problem showed that he should place 488 cows in 61 acres, 820 sheep in 41 acres, 162 horses in 27 acres, and 156 goats in 13 acres. He should not buy any pigs. The coloring shown here was submitted by Priya Parayanthal, a 5th-grade student in New Jersey: The cross-hatched areas represent cows, the diagonal regions represent horses, the shaded regions represent goats, and the blank regions represent sheep.

Most students explained that they approached this problem using a combination of guess and check and logical thinking. Students began by placing cows in the 20 acre field because this was the area with the greatest acreage. They then proceeded to place cows in the next largest fields that were not adjacent to the 20 acre sector. This procedure was repeated for the sheep, horses, and goats. After solving the problem in this manner, many students confirmed their solution by trying to place the cows without utilizing the 20 acre field. They found that this resulted in a lower acreage of cows.

Other discrete math techniques were also used to solve this problem. Some students used map coloring strategies to first find that four types of animals would be needed. Students with knowledge of the Four Color Theorem might have realized that the farmer would not need to buy pigs because, no matter how the farm was partitioned, at most four types of animals would be required to ensure that like animals were not in adjacent fields. It is impossible to use only three animals because there are sectors of the map that are surrounded by an odd number of fields, forcing the use of four animals. A few students modeled the diagram of the farm using a graph, with the vertices representing the different fields and the edges indicating fields that share a

Sixteen students submitted correct solutions to this problem. Unfortunately, due to this large number, we are unable to print the work of all of these students. We would like to congratulate the following people for correctly solving *The Farmer's Daughter*:

Christine Chynoweth	*(Mrs. Burnett, Patrick McGaheran School, Lebanon, NJ)*
Yong S. Cohen	*(New York, NY)*
Allie DeGeorge	*(Mrs. Burnett, Patrick McGaheran School, Lebanon, NJ)*
David Dreifus	*(Mrs. Burnett, Patrick McGaheran School, Lebanon, NJ)*
George Flannery	*(Mrs. Burnett, Patrick McGaheran School, Lebanon, NJ)*
Jesse Gormley	*(Ms. Deihl, Stroudsburg Middle School, Stroudsburg, PA)*
Harry Harkins	*(Calamus-Wheatland School, Wheatland, IA)*
Noelle Helmstetter	*(Mrs. Burnett, Patrick McGaheran School, Lebanon, NJ)*
Addie Hill	*(Mrs. Burnett, Patrick McGaheran School, Lebanon, NJ)*
Dan Huebner	*(Ms. Deihl, Stroudsburg Middle School, Stroudsburg, PA)*
Maggie Johnson	*(Ms. Deihl, Stroudsburg Middle School, Stroudsburg, PA)*
David Mathewson	*(Mrs. Burnett, Patrick McGaheran School, Lebanon, NJ)*
John Michels	*(Mrs. Burnett, Patrick McGaheran School, Lebanon, NJ)*
Laura Munch	*(Ms. Deihl, Stroudsburg Middle School, Stroudsburg, PA)*
Priya Parayanthal	*(Mrs. Burnett, Patrick McGaheran School, Lebanon, NJ)*
Kevin Tomlin	*(Ms. Deihl, Stroudsburg Middle School, Stroudsburg, PA)*



Animal	Acres	# Animals
Cows	61	488
Sheep	41	820
Horses	27	162
Goats	13	156
Pigs	0	0

common border. Vertex coloring was then used to find the solution. Yong S. Cohen used this technique; the graph and chart are shown above.

Teachers' responses indicated that their students found *The Farmer's Daughter* to be an engaging and thought-provoking problem. It encouraged mathematical dialogue in their classrooms and served as a meaningful way to promote logical, systematic thinking, as well as incorporate basic skills such as multiplication. Based on the range of the respondents' ages, this problem demonstrates that discrete mathematics is accessible to learners of all levels. We encourage all of you to continue solving challenging math problems, and thank everyone for their participation.

We would also like to acknowledge the following students for submitting solutions. Keep up the mathematical thinking! *Michael Baxter PA, Michelle Barberi NJ, Carly Becker NJ, Sarah Ben-Asher NJ, Nicholas Chen NJ, B.J. Conklyn PA, Jonathan Costa NJ, Natha Davids PA, Jenn Deluca PA, Nicole Dietze PA, Bruce Eaton PA, Danielle Egbert NJ, Marian Fahmy NJ, Jason Fell NJ, Michael Franklin PA, Ryan Giblin PA, Ted Gonzalez PA, Caitlin Hoffman NJ, Jackie Kang NJ, Ryan Kuchinskas PA, David Lertola NJ, Reese Levale PA, Scott Mastroianni NJ, Yaroslav Naumenko PA, Kristina Newcombe PA, Phillip Ngu NJ, Paul Orlando PA, Aba Osseo-Asare PA, Matt Rivers NJ, Ronan Slater PA, Angela Wary PA, Mrs. Watson's 9th-grade honor's geometry students MA, George Vallone NJ, Emily Yama NJ, and Callie Yosh PA.*

Workshops in Your District

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BY WHOM? These workshops are presented by experienced workshop leaders who have participated in the Rutgers Leadership Program in Discrete Mathematics and have used the workshop materials in their own classrooms.

FOR FURTHER INFORMATION, INCLUDING FEES: Contact Program Coordinator Debby Toti by email at toti@dimacs.rutgers.edu or by phone at 732/445-4065, or check http://dimacs.rutgers.edu/lp/workshops/