

What Do Mathematicians Do? (See cartoon on page 12)

If one asks the “person on the street” what plumbers, electricians, chemists, or geologists do, they are likely to give you a reasonable answer. Put in more dramatic terms, when home-owners see water cascading through the ceiling, they do not call a carpenter or a mathematician. But few people on the street know when to call a mathematician.

One thing that we can do about mathematics’ image problem is to discuss how mathematics affects people’s lives, even if we cannot always do proper justice to the mathematics involved. For example, we can say that mathematicians (not chemists or plumbers) study waiting lines, and show that this can be applied at banks, airports, and in computers. Or, we can say that mathematicians find shortest paths and networks, and show that this can be applied to travel arrangements and telephone connections.

The above paragraphs are adapted from “Mathematics’ Image Problem” by Joseph Malkevitch (see address on page 11). The cartoon on page 12 was drawn by Joe Pipari one of whose colleagues at Thomas McKean High School (Wilmington, Delaware) participated in a discussion with Malkevitch at the 1990 Leadership Program in Discrete Mathematics. The situation presented in the cartoon is discussed below. ■

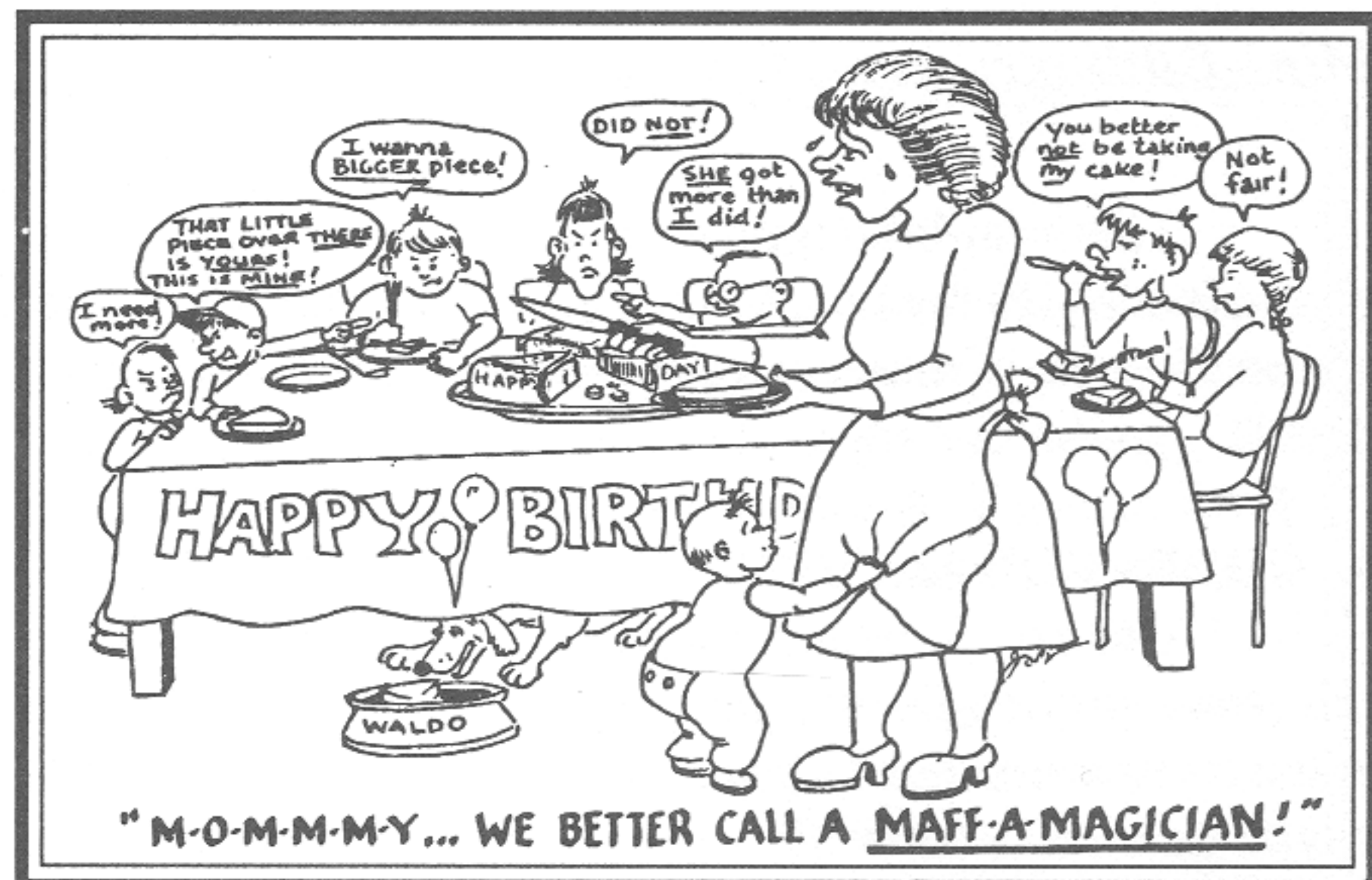
Calling that mathematician... by Ethel Breuche
(see cartoon on page 12)

The situation depicted in the cartoon points out vividly the problem of how to divide a cake fairly. (The standard answer “just divide it into thirds” would work only if we were able to do that perfectly; in real life, it is hard to divide things perfectly -- see page 2 for another example.)

In discrete mathematics, *fair division* provides a rich source of real life problems which include dividing such things as cake, candy, pizza, the air waves, property, and estates.

I have taught fair division problems in quick one day lessons, as well as an extended four day unit; this was great fun, and motivating for both the students and teacher. Cooperative learning was the teaching strategy I used most frequently. Working in small groups, my students actually performed fair division on brownies using each of the methods described in the box at the right, and several others.

Brownies are easy to bake, cheap to make, and easy to push back together for further division (especially if you don’t make them too chewy). Roles were rotated in the groups so that each person “taught” one method of fair division to the rest. Time was provided for students to discuss the fairness of the method used. Variations were added to the basic problem by introducing another player or by having certain players have particular preferences. Follow up problems for homework can be created at all levels of difficulty, and groups can be challenged to come up with their own problems.



A Sampler of Fair Division Methods

Divider Chooser Method: This method involves two players. One player divides the item (property, cake, land, etc.) into what he/she considers two equal shares. The other person chooses.

Lone Divider Method: This method involves three players (or more with variation). One player (chosen randomly) divides the item into what he/she considers to be three equal parts. Each of the other two players (the choosers) declares independently (usually by writing it down on a slip of paper) which of the pieces he/she believes to be a fair share (i.e. worth at least one-third) and therefore acceptable. More than one answer is admissible. Only one situation requires extra steps taken, when both choosers select the same one piece. In this case, the divider picks one of the other two pieces and the remaining two pieces are put together and the two choosers then use the *Divider Chooser Method*.

Lone Chooser Method: This method involves three players. One player, chosen randomly, will be the chooser of the three players. The other two players will cut the item into two pieces using the divider chooser method (i.e. one will cut the item in half and the other will choose the piece he/she will work with.) Now these two same players divide their piece into three pieces. There are six pieces altogether. The chooser picks one piece from each of the dividers and then the dividers keep the remaining two pieces.

Last Diminisher Method: This method is good with five or more players.

Round 1. Using an uncut dish of brownies (or a large batch of wrapped candy), player 1 cuts a section that he/she believes to be a fair share (one-fifth) of the whole. Player 2 now has the right to pass or to play. If player 2 thinks that player 1’s claim is a bad choice (worth less than $1/5$ th), then player 2 passes and remains in contention for a fair share of the rest of the goodies. On the other hand if player 2 thinks that player 1’s claim is a good one, then player 2 can make a claim on player 1’s claim by

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