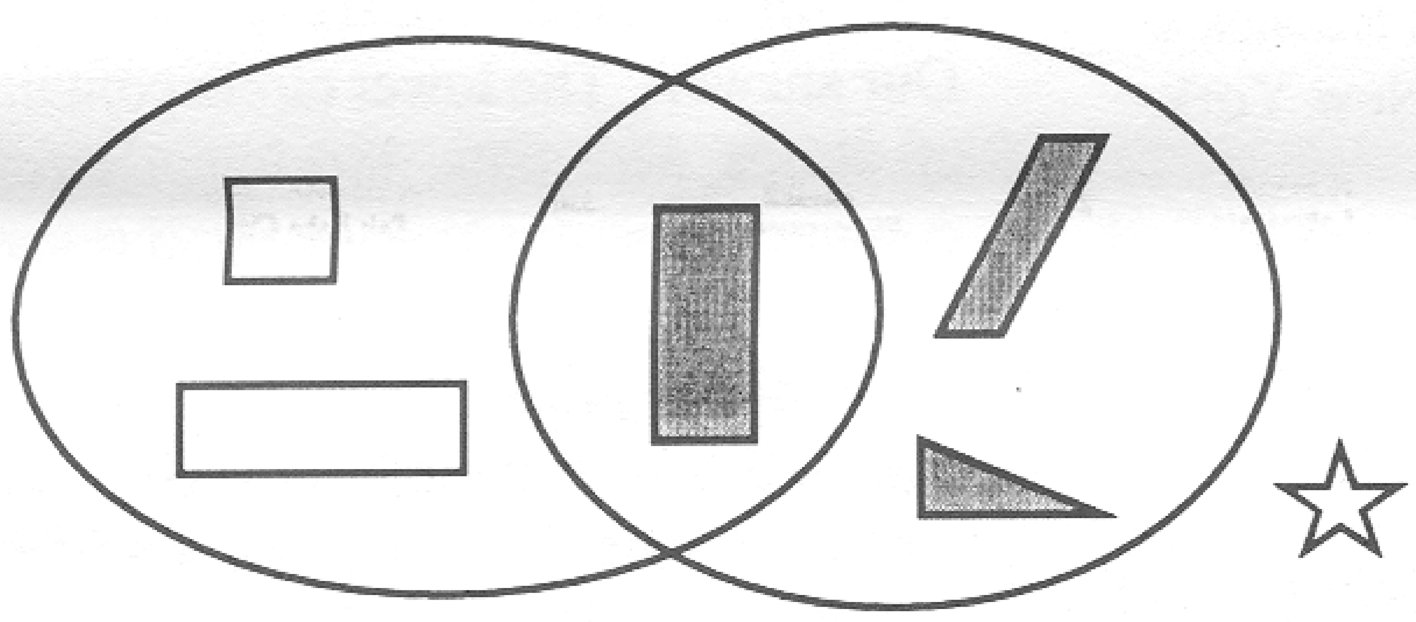
The Venn Diagram Game

by Linda Dodge

This is a popular activity that I have used to illustrate set concepts and reasoning. I have the class stand around two large circles of rope placed on the floor. I like to use two different colors of rope, let's say yellow and blue. To be sure that everyone is familiar with the idea of a Venn diagram (see figure), I will say something like, "Everyone who is wearing a white shirt please stand in the yellow circle." I allow time for them to sort themselves out, then I say, "Everyone wearing black sneakers should stand in the blue circle . . . Oh look—Josh is wearing black sneakers *and* a white shirt, where should he stand?" Then I make a *big* deal of the "complement," the set of people outside the two circles which "completes" the diagram. (Not only is the complement a useful concept, but I've found that otherwise younger children feel left out if they are not inside the circles.)

After we repeat this exercise a few times with different sets of rules, I make the game more interesting. I sort the



Venn Diagram

The left circle contains rectangles; the right circle contains filled-in shapes. The star (in the complement) is neither filled in nor a rectangle.

people—but I don't tell them the rules I am using. They have to make observations and use inductive reasoning to guess the rules. This part of the dialogue might sound something like this:

> Mary, you stand in the yellow circle but not in the intersection. Raj, you belong in the blue circle. Consuelo, you have both traits—go stand in the intersection. Jed, you are part of the complement. . . .
>
> Does anybody think they know the rule yet? If you do, raise your hand and tell me where you think you belong, but don't say the rule out loud.

After each student tells me where they think they belong, I verify whether they are right or wrong; if they're right, they can go stand in the correct part of the diagram, if not they wait where they are.

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What's So Special About 6174?

by Ronald S. Tiberio

I am always intrigued by good number patterns; here's one that that I've used with my students which requires only basic arithmetic. For reasons that I'll explain below, I'll call this "Kaprekar's game."

1. Choose any four-digit number where not all the digits are the same (not divisible by 1111). (The number can start with 0.)
2. Rearrange the digits from largest to smallest and then from smallest to largest.
3. Subtract the smaller rearrangement from the larger.
4. Take your answer from step 3 and go back to step 2.

For example, if you start with 4997, this is what happens:

9974	7551	9954	5553	9981	8820	8532	7641
-4799	-1557	-4599	-3555	-1899	-0288	-2358	-1467
5175	5994	5355	1998	8082	8532	6174	6174

Now ask your students to try to figure out the maximum number of steps you can play the game before repeating a number. The amazing answer is that if you play Kaprekar's game on *any* four-digit number (other than multiples of 1111) you will reach 6174 in at most seven steps! (A sketch of a proof using only elementary algebra appears at the end of this article, on page 9.)

I first saw this property of 6174 as an elementary problem in the *American Mathematical Monthly* [1]. The original source was a short note in an obscure journal by D.R. Kaprekar of Devlali, India [2]; since that time 6174 has been referred to as *Kaprekar's constant.*

Of course there's no need to stop with four digits. What happens if you perform this operation with a two-, three- or five-digit number? Is there always a fixed point? Can there be more than one fixed point (not counting 0, which is always a fixed point)? Such questions are the basis for the concepts of *iteration* and *dynamical systems*, and can be introduced to students with very little background. ❖

References

[1] Elementary problem E2222, *American Mathematical Monthly*, March 1970, p. 307.

[2] D. R. Kaprekar, "Another Solitaire Game", *Scripta Mathematica*, September 1949, p. 244-5.

Editor's notes

(1) This article first appeared as a posting to the Leadership Program electronic mailing list, where teachers share information and discuss issues related to teaching

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