Teaching briefs... Digraphs and Relations

by Ruth Ann Krayesky

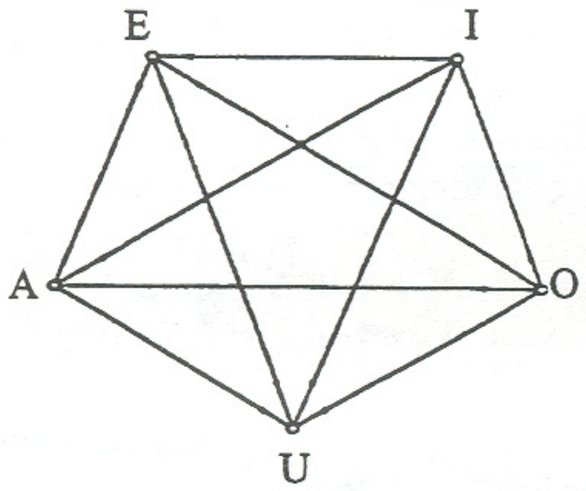
Have you ever considered using a *digraph*, or directed graph -- with arrows instead of edges, to explain relations and their properties? Fehr, Fey and Hill's *Unified Mathematics* presents a model that has been useful in helping my students to understand relations.

Starting with a finite set and a relation, arrange the elements of the set in a circular pattern. These elements become the vertices of our graph. Next draw an arrow from each element to every element of the set to which it is related. These arrows become the directed edges of our digraph.

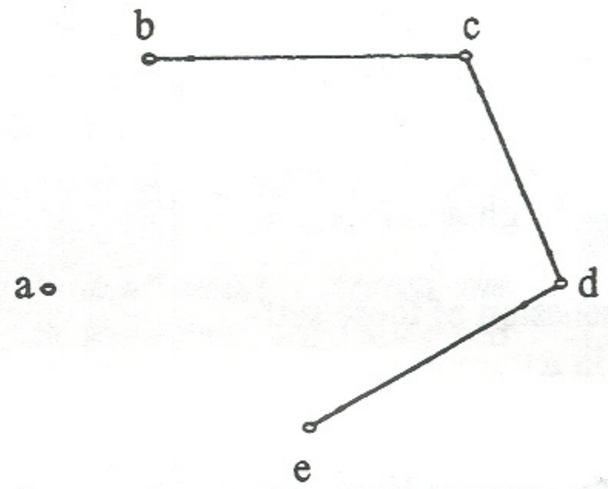
For example, the relation "X is older than Y" among five students in the class might result in Graph I.

If every element has an arrow to itself, then the relation is reflexive. If whenever there is an arrow in one direction between two elements, there is also an arrow between the two elements in the other direction, then the relation is symmetric. And if whenever there is a directed path between vertices made up of two edges there is also an arrow from the initial vertex to the terminal vertex, then the relation is transitive. (Note: In this case, there is an edge connecting the vertices whenever there is a directed path - of any number of edges -- between the vertices.)

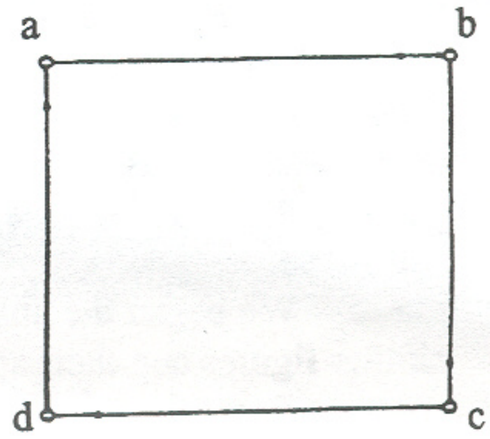
For example, the relation depicted in Graph II is symmetric and reflexive, but not transitive. Can you find the smallest transitive relation containing the relation in Graph III?



Graph I



Graph II

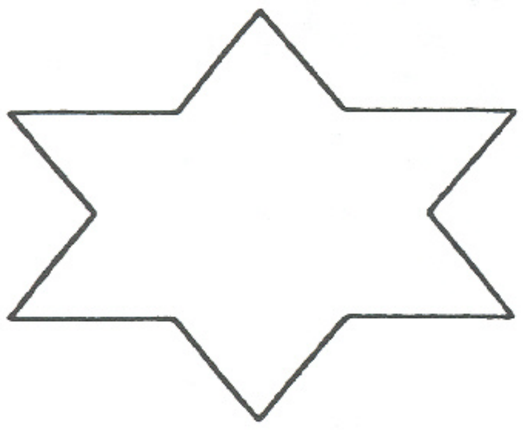


Graph III

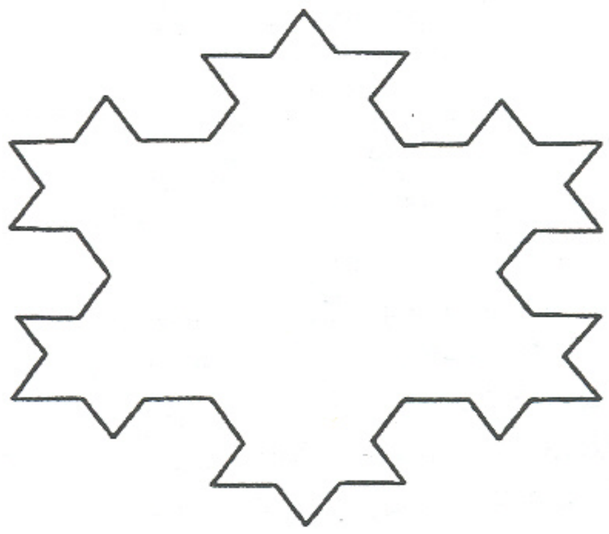
Illustrations... Koch snowflake

The Koch snowflake (referred to on page 3, column 1) is obtained by starting with an equilateral triangle and replacing each side by the pattern --

-- obtaining a six-pointed star.



Each of the twelve line segments of the resulting star is replaced by a similar pattern to obtain the third generation picture --



-- which begins to resemble the Koch snowflake obtained by repeated iterations of this "replacement procedure". To generate this snowflake, your students should first draw an equilateral triangle whose sides are all 27 centimeters. (When you try to draw it yourself, you'll see why it helps to start with a power of 3.

Worksheets for carrying out iterations for other fractals can be found in *Fractals for the Classroom: Strategic Activities Volume I*, by Heinz-Otto Peitgen et al., Springer-Verlag and NCTM, 1991.

Announcement...

To obtain a copy of a *Bibliography on Fractal Geometry and Chaos*, write to Hubert J. Ludwig, Department of Mathematical Sciences, Ball State University, Muncie, Indiana 47306.