Topics... Two Problems Involving Graphs

by Joseph G. Rosenstein

Here are two problems which can be stated very simply; the first may be familiar to you but the second you have probably not seen before. What do they have in common, and how are they connected with discrete mathematics?

PROBLEM 1: Can you use 31 1x2 dominos to cover the 62 squares of an 8x8 chessboard obtained by deleting two diagonally opposite corners?

PROBLEM 2: A mouse eats her way through a 3x3x3 cube of cheese by tunnelling through all of the 27 1x1x1 minicubes. If she starts at one corner of the cube and always moves to an adjacent uneaten mini-cube, can she finish at the center of the cube?

You may want to think about the first problem for a few minutes, in which case you should probably stop reading this article for the time being. You may even want to find a chessboard and play with the problem a bit.

If you solved the problem, you realized first that the two unused squares have the same color, and then noticed that the two squares covered by any domino have opposite colors. (Unless you are really good at "visualization", you would probably not have discovered the first fact without using a real chessboard.) Thus 31 non-overlapping dominos must cover 31 squares of one color and 31 squares of the other color, leaving uncovered one square of each color! So diagonally opposite corners cannot both be uncovered.

If your students have learned a little about graphs, they should probably be able to discover the graph that underlies this problem. It is a graph with 64 vertices, namely the squares of the chessboard; two vertices are adjacent (in the graph theory sense) if the corresponding squares are adjacent (in the physical sense).

The normal coloring of the chessboard -- involving red and black squares -- provides what is called a coloring of the graph. In a coloring of a general graph, adjacent vertices must have different colors. Graph colorings have many applications, to problems as diverse as map colorings, scheduling committee meetings (see article to the right and box on page 10), traffic lights, and radio frequency assignments. If you are interested in learning more about colorings of graphs, one source is *The Mathematician's Coloring Book*, by Richard L. Francis, in the HiMAP Module Series published by The Consortium for Mathematics and Its Applications (COMAP).

Graphs which can be colored using two colors are often called bipartite graphs, since the vertices can be separated into two sets with adjacency occurring only between vertices in different sets. The chessboard graph is bipartite

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Spreading the Word... Introducing Teachers to Discrete Mathematics

by L. Charles Biehl

On March 21-22, the Maryland State Department of Education held the third annual Dwight D. Eisenhower Mathematics and Science Conference in Baltimore, Maryland. The focus for mathematics teachers was the NCTM Standards, and I was fortunate to be selected to give a presentation in discrete mathematics, entitled *From Final Exams to Traffic Jams: Using Graphs to Resolve Conflicts*.

The audience consisted of more than forty math teachers from all over Maryland, only a few of whom were familiar with graphs. The purpose of this presentation was to give these teachers enough exposure to feel comfortable with the basic ideas of graphs and graph coloring, to provide them with access to additional materials for further study, and to enable them to teach a one or two day lesson on the topic in their own classes.

After laying the foundation for the topic, I showed that conflicts that arise in a variety of situations can be modeled with graphs; examples include scheduling meetings for people who had multiple responsibilities (see *Illustration* on bottom of page 10), assigning frequencies to mobile radio telephone relay stations, scheduling final examinations at a small college, and sequencing green lights at an intersection. To resolve the conflicts in each situation, we had to define what the conflict was and assign the minimum number of "colors" (or meeting times, or relay towers or traffic light changes) to ensure that no one had to be two places at the same time, no radio frequencies interfered with each other, no cars collided, etc.

All the actual problem solving was done by the participants; I encouraged them to work in pairs or groups. Once the ice was broken and the first problem had been solved (in more than one way, I should add) the workshop continued as a lively problem-solving session, with participants convincing themselves of the correctness of their solutions by comparing notes with neighbors. The final ten minutes of the hour was spent discussing the underlying concepts, their applications to other and more diverse situations, and a plethora of potential classroom activities.

This presentation was a big step for me. It was one thing to present this material to a small group of colleagues in my district; but to present to a group twice the size, and to strangers no less, filled me with apprehension and a fear of being asked questions I could not attempt to answer. This must be the feeling that we all had when we first entered the profession. After the first five minutes I was completely at ease, and I felt that the participants were too.

Discrete mathematics topics work extremely well in this type of environment, since they can be learned and appreciated at many levels. The same is true in the classroom,

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