Finding the Fractal Complexity of a Coastline

by William L. Bowdish

Veteran teachers are fully aware that 14 year olds know everything that there is to know about mathematics! However, when I gave this lesson on fractal dimension (complexity) to my accelerated 9th-grade honors-algebra class, they were truly amazed. I got the idea for this lesson from Terry Perciante, who taught an excellent one-week course in the Leadership Program in Discrete Mathematics at Rutgers during the summer of 1993. To prepare this three-day unit, I needed only a few hours of preparation and reading [1, 2].

I decided to ask my class to compute the complexity of the coastline of Martha's Vineyard, a gorgeous island off the coast of Massachusetts, and focus on using the "compass method" to calculate its fractal dimension. Each student received a photocopy of a hypothetical irregular coastline (see below), a map of Martha's Vineyard, and a (drawing) compass. I used transparencies to explain the calculation.

I drew a 4-inch straight line segment on a transparency, and showed, by "walking" a compass along the line, that the line is 8 units long if the units are 1/2 inch (the distance between the compass legs is 1/2 inch), and 16 units long if the units are 1/4 inch: i.e.,

$$4 = 8 \times 1/2 = 16 \times 1/4.$$

This helped motivate a procedure to compute the dimension of any curve by walking a compass along the curve for different lengths. We let X be the number of units counted for compass opening 1/2 inch, and Y the number of units for compass opening 1/4 inch. (Note: 1/2 and 1/4 are chosen for convenience, other pairs like 1/8 and 1/16 would also work in theory--but no one would have the patience in practice!)

If the dimension were one (like the straight line), we'd expect $Y = X \times 2^1$ (above, $16 = 8 \times 2^1$). By analogy, letting D be the dimension, we set

$$Y = X \times 2^D, \text{ or } 2^D = Y/X.$$

To approximate D, we need only compute X and Y by counting compass "steps." For example, if X is 10.8 and Y is 23.2, we get $2^D \cong 2.148$, or $D \cong 1.1$. The main drawback with this method is that it can be a struggle to compute X and Y using compasses.

I gave each student a copy of an imaginary coastline similar to that below (but larger), put students in groups of 2 or 3, and asked them to find the dimension.

The students found X and Y, then using a guess-check-and-revise strategy, the students solved for D. Then, with their calculators, they checked the answer using the formula

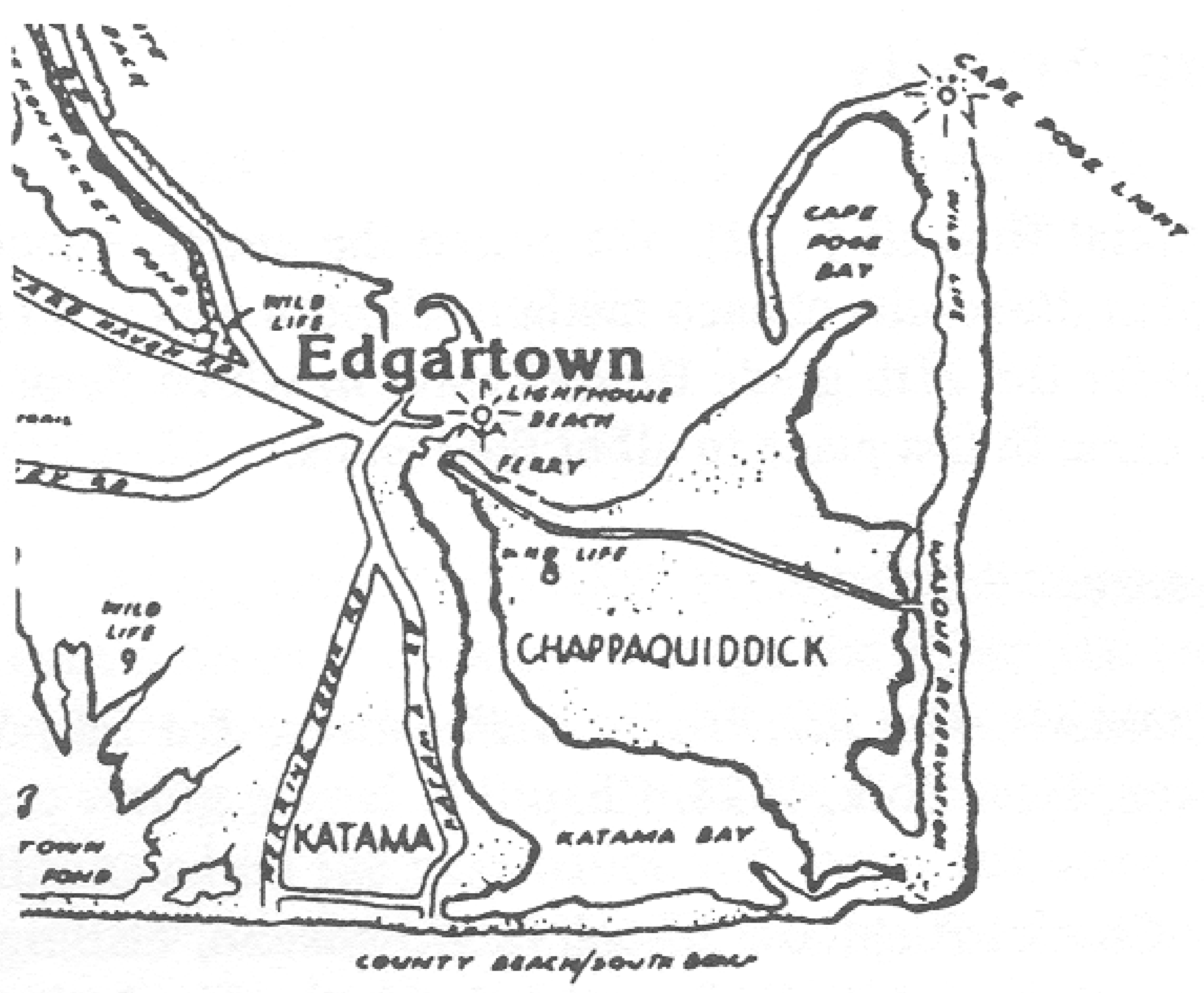
$$D = \log(Y/X)/\log 2 \quad \text{(i.e., log base 2 of Y/X).}$$

Lastly, I passed out the maps of Martha's Vineyard, and directed the groups to calculate its coastline dimension. According to my class, D is approximately 1.44.

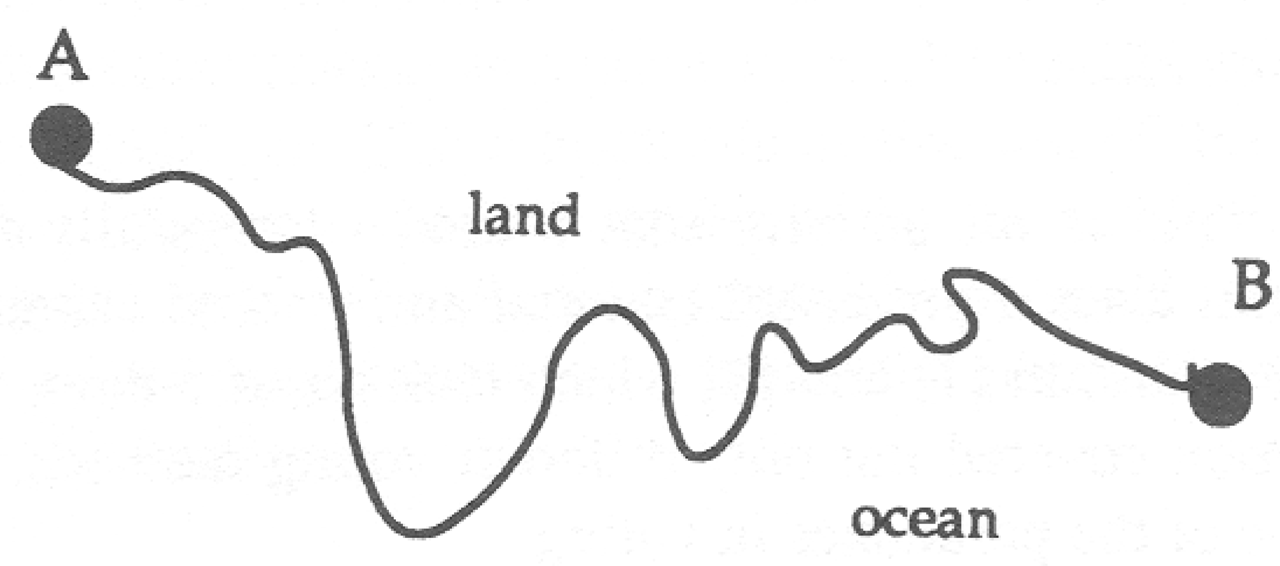
References

1. Peitgen, H., Jurgens, H., Saupe, D., Maletsky, E., Perciante, T.,Yunker, L., *Fractals for the Classroom*, Volume One. New York: Springer-Verlag, 1991.

2. Senk, S., Thompson, D., Viktora, S.. Rubenstein, R., Halvorson, J., Flanders, J., Jakucyn, N., Pillsbury, G.. & Usiskin, Z., "Dimensions and Space," in *Advanced Algebra* (pp. 857-860). Scott, Foresman and Company, Glenview, IL: 1990.



SouthEast Corner of Martha's Vineyard



Imaginary Coastline