A Classroom Dilemma

by Reuben Settergren

In the summer of 1994, I taught a course called Applications of Contemporary Mathematics (ACOM) at the Johns Hopkins University Center for Talented Youth (CTY), at its Los Angeles site. A new course, ACOM was aimed at the CTY's youngest students (who were about 12 years old). I used *For All Practical Purposes* [1] as a text, and included a unit on game theory.

My favorite activity was a game inspired by Douglas Hofstadter's article "Dilemmas for Superrational Thinkers" [2]. The purpose of the game would be defeated by cooperation, or even communication, between the players, so each of my students received a private letter from me telling them that they were selected to compete in a game, with rewards to be paid in real money. They were told the number of players, but not who the players were. Here is how the game works:

> (1) each player secretly writes down a single "move": either a "C" (cooperate) or "D" (defect);
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> (2) to determine the payoff for each player: the moves of each pair of players are compared, and their winnings are augmented according to the payoff schedule below.
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> C vs. C: each gets 5 cents
> D vs. D: each gets 1 cent
> C vs. D: D gets 9 cents, and C gets nothing.

Notice that the total payoffs in this game vary according to the number of players. For example, consider a game with 16 players. If everybody cooperates, everybody gets 5 cents, so everybody gets 15 x 5 = 75 cents. However, if everybody defects, everybody gets only 15 x 1 = 15 cents. With 8 cooperators and 8 defectors, cooperators each receive 7 x 5 + 8 x 0 = 35 cents, and defectors each receive 8 x 9 + 7 x 1 = 79 cents. And—the holy grail of defectors—if only one player defects, he or she gets 15 x 9 = 135 cents, almost twice as much as the payoff for the cooperators: 14 x 5 = 70 cents each.

If you try this with your students, you can expect that well under a fourth of your students will choose to cooperate, thus limiting the expenditure of precious school (or personal) funds. The defectors will be disappointed with their classmates' greed, and some might be ashamed of their own!

Experienced game theorists will recognize that this game is simply a many-way "Prisoner's Dilemma". For any set of moves, the defectors will always earn more than the cooperators, and any one player will always earn more against any set of opponents' moves by defecting. The "dilemma", however, is that if everybody is greedy and defects, everybody loses. My hope is that the students will discover this and begin to think about what the best strategy is in playing the game.

Now, for the game to work properly, it is essential that the students not communicate. I suggest that you use the letter in Hofstadter's article (modified, of course, to fit your class size and budget): it does an excellent job of explaining rational decision-making, and the possible rewards or hazards of the different moves. I distributed my game letters very secretively, outside of class, and required students to give me their responses personally (along with an explanation and a complete chart of payoffs, to make sure they understood the game). After the results were in, I handed out the money, and we all discussed our reactions to the game and its results.

We then read Hofstadter's article [2], and discussed possible applications. This game is in fact very similar to many everyday situations: traffic ("I can slow down to rubberneck at this accident"), art ("I'd really like to touch this Van Gogh; it's a good thing nobody else ever would"), pollution ("If I alone disregard the polluting effects of my company, I can get an edge on my competitors"), etc. We also speculated what the students' moves would be in the various other games described in the postscript to Hofstadter's article, or how their strategy would change if the game had payoffs in dollars, or millions of dollars, instead of just cents (an excellent demonstration of the non-linearity of the utility or value of money).

My kids spent two hours on a Friday afternoon exploring these issues, and even forgot to rush out the door to their sunny Los Angeles weekends at the end of class! That might be more time than you can afford to spend in your class, but perhaps the less time you spend, the more your students will come away intrigued and inquisitive.

References:

[1] COMAP, *For All Practical Purposes*, 3rd. Ed., W.H. Freeman, New York, 1994, Chap. 15.

[2] Hofstadter, Douglas R., *Metamagical Themas*, Basic Books, New York 1985.

"Dilemmas for Superrational Thinkers" (Chapter 30 of this book) originally appeared as a "Metamagical Themas" article in *Scientific American*, June, 1983. "The Tale of Happiton" (Chapter 32) is a story about cooperation that would make a nice reading assignment. See also Chapter 29, "The Prisoner's Dilemma Computer Tournaments", which explores the success of different player strategies using computer simulations.