Graph Coloring . . . Starting the Year off Right

by Richard Adkisson, Susan Howell, Steven Kepnes, and Lisa Soden-Winer

In the summer of 1994, we all attended the high-school institute of the Leadership Program in Discrete Mathematics. On the first day of the program, the instructor (and program director), Joe Rosenstein, introduced us to graph coloring. Since this worked so well for us as an introduction to discrete mathematics, we each decided to try it in our own classes in the fall. We wanted similar outcomes: a break from the monotony of "book work" or drilling fraction addition, more students confident in their own abilities, and a "proof" that math can be both useful and fun. We wanted students to get used to working in groups and to get to know one another. We found that graph coloring is a great topic for achieving all of these.

Richard, who teaches at a rural/suburban school in New Jersey, was teaching a remedial course for 17 ninth graders who had failed the state's 8th-grade "Early Warning Test". Susan and Lisa were each teaching sections of a course called "Algebra I, Part I", at a rural high school in New Jersey, each with 17-18 students. Their course enrolls students who have either done very poorly in mathematics or have special needs, and covers the standard Algebra curriculum at a much slower pace. Lisa's section had mostly ninth graders, while Susan's had mainly tenth and eleventh graders. Susan and Lisa each work with a second teacher, who has experience with special-needs students, who assists them and works with students individually during activities. Steve Kepnes was teaching a mainstream eighth-grade mathematics course.

Each of us followed a similar outline, although the students in the remedial courses needed more time to complete the activities (Steve used two days, while Lisa needed two weeks). We began by handing out maps with markers or crayons, and asking students to find a good way to color the maps so that no two regions sharing a border had the same color. We then introduced the problem of finding the minimum number of colors needed; some of us motivated the problem by observing that in map-making, each new color costs extra.

After the students had colored maps successfully, we showed them how to represent the problem as coloring the vertices of a graph, and defined the chromatic number of a graph (the minimum number of colors needed). Through experimentation, students observed that a graph with a triangle required 3 colors, and several conjectured the 4-color theorem for maps.

Finally, we introduced conflict problems which can be solved by graph coloring, such as the problem of scheduling committees with overlapping membership (see [1]), or that of assigning frequencies to mobile radio telephones ([2], p. 108, problem 8), as shown in the sidebar on page 10.

Steve asked students to present their solutions to the map-coloring problems to the class, letting them discuss whether the solutions were correct, and if better solutions (with fewer colors) were possible. He says that "if my students had the same enthusiasm every day as they had that day in wanting to present their solutions on the overhead projector, I probably wouldn't have to show up—the students could conduct the class themselves!"

Lisa found an interesting way to color a graph. She placed a graph (which the students had created for homework, whose vertices represented the townships of Monmouth County) on an overhead projector. She started by coloring one vertex blue, then putting pennies on the vertices attached to it by edges. She repeated the process, coloring a penniless vertex blue, and putting pennies on the vertices attached to it, continuing until all vertices were either colored blue or covered with a penny. She then removed the pennies and started over, coloring a penniless vertex red, continuing (adding new colors when necessary) until all the vertices were colored. At the end, she placed a transparency of the map on the graph so that the students could see that the coloring of the vertices truly represented a coloring of the townships.

Susan had her students present their solutions to the conflict problems that they were given for homework, and was impressed by the care the students took in their work

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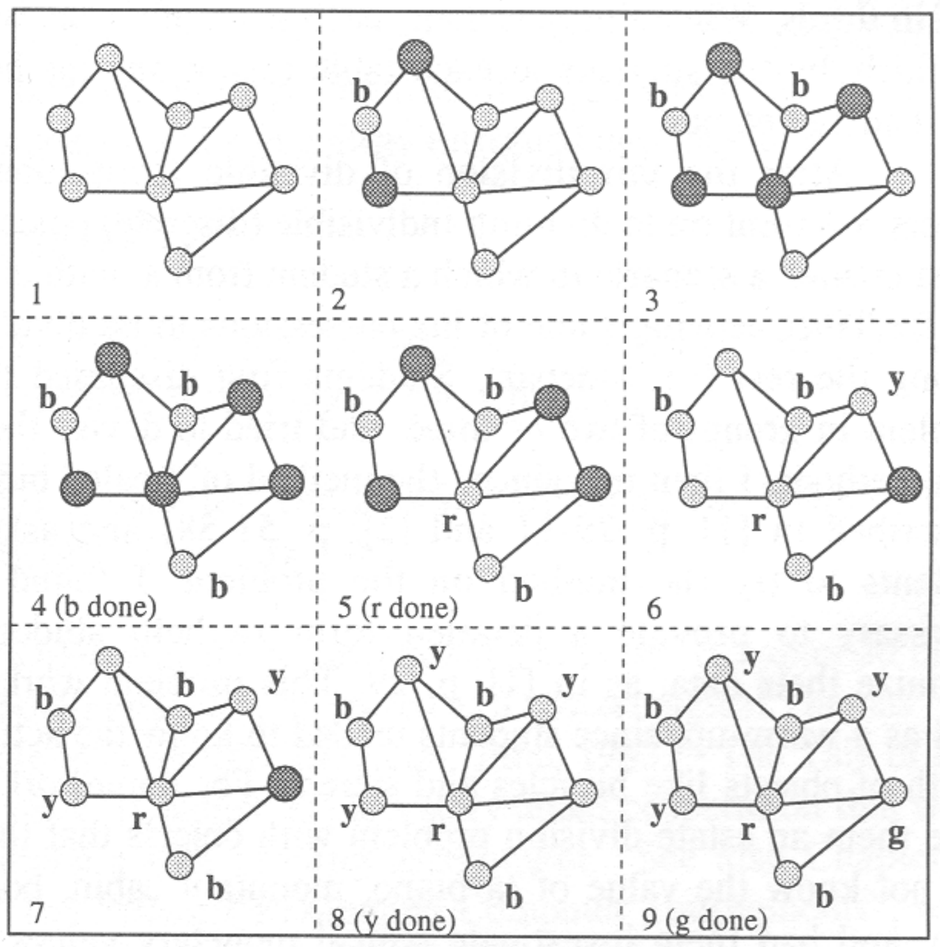

Lisa's graph-coloring method