Game Theory *(Continued from page 1)*

Recently, interest in game theory has been reawakened by the awarding of the 1994 Nobel Prize in Economics to John Harsany, John Nash (a mathematician), and Reinhard Selten [2, 3, 4] for work involving game theory. Moreover, new applications are emerging; for example, biologists have begun with some success to try predicting and explaining animal behavior using game models [5, 6].

An emerging subarea of game theory is the theory of voting. One important problem in this area is that of upholding the concept of "one person-one vote" within legislatures in which representatives represent parties with unequal population. One solution to this problem is to use what is called "weighted voting" in which the votes of different representatives have different weights (see [9, 10, 11]). Often, however, naive methods of assigning weights give unintuitive results (see sidebar), and mathematical analysis is needed. Examples in which weighted voting is used include the Electoral College, stockholder meetings, and the United Nations Security Council (see [11]). An example which has been in the news recently [7, 8], concerns the changes in voting within the legislative bodies of the European Union, necessitated by the addition of new countries with very different populations, size, and economic strength [7] (see also [10], p. 370-71).

Game theory has its limitations, of course: if (in trying to reflect reality) the game models become too complex, they are difficult to analyze; if the models are too simplified, the resulting solution may not address the original question adequately. However, as is apparent in the weighted-voting example, game theory can provide a framework in which relatively simple mathematical reasoning can be used to gain insight into real political situations.

References

(For further reading, see also the Game Theory Bibliography on page 9.)

[1] J. Von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton Univ. Press, 1944 (3rd Ed., 1953).

Classic, pioneering work on game theory and its application to economic and social problems.

[2] P. Passell, "Game theory captures a Nobel", *NY Times*, v. 144, Oct. 12, 1994, p. C1.

A description of the winners of the 1994 Nobel Prize in Economics and some of the work that led to the award.

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A Paradox in Weighted Voting

Imagine a county legislature having one member for each of four towns, which at some time in the past had approximately equal populations. Now, thirty years later, the populations of the towns are:

Town A: 70,343 (41%) Town C: 29,857 (18%)
Town B: 60,123 (35%) Town D: 10,099 (6%)

Or, pictorially:

A B C D

Rather than redistrict or add more legislators, the town inhabitants agree to allow each member of the legislature to have a vote that has "weight" larger than one, if necessary. What is a fair way of assigning the weights? Given the data above (in which each population is near a multiple of 10,000), a natural assignment might be to add one to the "weight" of each legislator's vote for each group of 10,000 people in the district they represent (rounding to the nearest integer). If this is done the total weights assigned would be 7, 6, 3, and 1, respectively.

There is a total weight of 17; assuming that a majority is needed, there must be a weight of 9 to pass a bill. For example, if B and C both vote "yes", then the bill will pass. One can think of this as a game in which A, B, C, and D are players, who can only make one of two choices.

An analysis of the "power" that each player has in this game shows that this method of assigning weights is less fair than it seems. If you list the possible ways that the votes can be distributed, you will see that it doesn't matter what D does; the final outcome will be decided solely by the votes of A, B and C. In other words, D has positive weight but no power! (In fact, any assignment in which the weights are in proportion to the populations will have this feature.) In a slightly more complex setting, this situation actually occurred on the Nassau County Board of Supervisors in New York, and led to several lawsuits (see [9], and [10], p. 359-60, 372-73).

Is there a way to assign weights so that each representative ("player") has a fair fraction of the total "power"? A number of methods have been devised to try to answer this question, based on special "power indices" which are used to try to measure the power of players in a weighted-voting game [9, 10, 11].