

Project title: Extremal Properties of  $k$ -Partite Graphs of Large Girth,  $k = 2, 3$

Project Description: Extremal graph theory is a branch of graph theory that is interested in studying the following scenario: Suppose  $\mathcal{G}$  is a family of graphs,  $P$  a property and  $\mu(G)$  an invariant for each  $G \in \mathcal{G}$ . We wish to determine a value  $m$  such that whenever  $\mu(G) > m$  for  $G \in \mathcal{G}$ , then  $G$  has property  $P$ . Those graphs  $G \in \mathcal{G}$  for which  $\mu(G) = m$  and  $G$  does **not** have property  $P$  are said to be the extremal graphs with respect to property  $P$  and invariant  $\mu(G)$ . For example, suppose  $\mathcal{G}$  is the family of simple graphs on  $n$  vertices,  $n$  a positive integer. Moreover, suppose  $P$  is the property that “ $G$  contains a cycle,”  $G \in \mathcal{G}$  and the invariant  $\mu(G)$  is the number of edges of  $G$ ,  $\mu(G) = e(G)$ . Then  $m = n - 1$  (the number of edges a graph on  $n$  vertices can possess and still not contain a cycle) and the extremal graphs are trees on  $n$  vertices.

Consider all  $k$ -partite simple graphs ( $k = 2, 3$ ) on a finite set of vertices. The objective of this project is to explore bounds on the number of edges that  $k$ -partite graphs can have and still be of large girth. That is, what is the maximum number of edges a bipartite or tripartite graph can have and avoid cycles of length  $n$  ( $n = 3, 4, 5, \dots$ )?