Fast Discretized Geometric Algorithms for Union and Envelope Computations

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Arrangement Problems

- **Arrangements**
  - Decomposition of space into connected open cells
  - Fundamental problem in computational geometry and related areas
- **Underlying structure in many geometric applications**
  - Swept Volumes
  - Minkowski Sums
  - CSG or Boolean operations
  - Many more…….

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Basic Computational Pipeline

- Enumerate a set S of primitives that contribute to the final surface
- Compute the arrangement A(S) by performing intersection and trimming computations
- Traverse the arrangement and extract a substructure $\delta A(S)$

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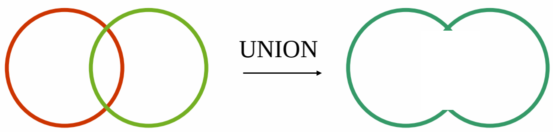
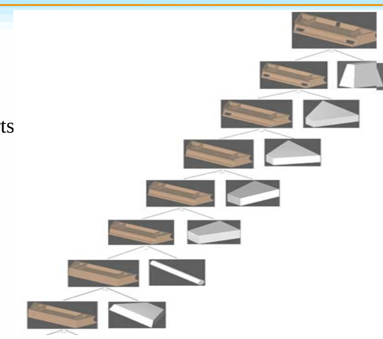
Example: CSG Union Operation

Boundary = outer envelope in the arrangement of the primitives



CSG Operations

Design of complex parts



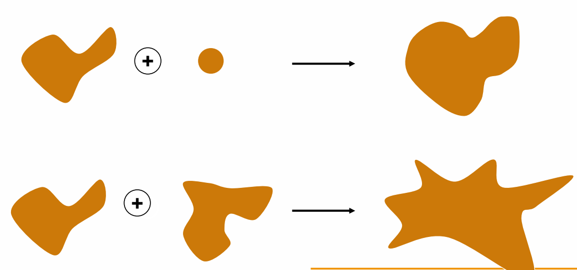
Boundary Evaluation of Complex CSG Models

Bradley Fighting Vehicle
1200+ solids
8,000+ CSG operations

Minkowski Sum

$A \oplus B = \{ a+b \mid a \in A, b \in B \}$

OFFSET



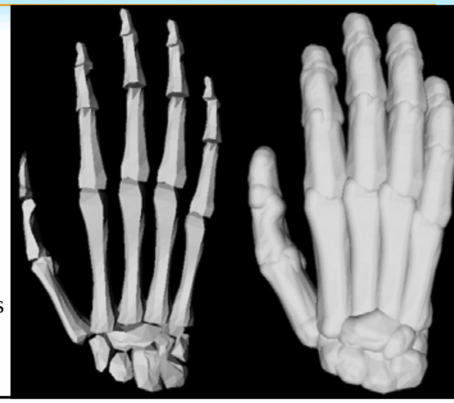
Minkowski Sums: Motivation

- Configuration space computation
- Offsets
- Morphing
- Packing and layout
- Friction model

Offset Computation

Offset: Minkowski sum with a sphere

Input: 2982 triangles



Minkowski Computation

- Decompose A and B into convex pieces
- Compute pairwise convex Minkowski sums
- Compute their union

<u>**Issues:**</u>

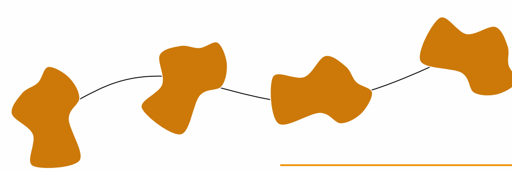
–High combinatorial complexity = $O(n^6)$

–Exact computation almost impractical

Swept Volume (SV)

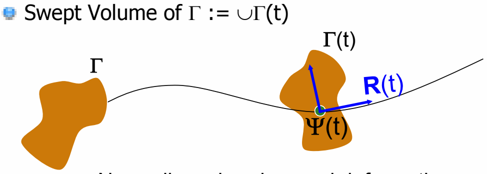
- Volume generated by sweeping an object in space along a trajectory

- **Goal:** Compute a boundary representation of SV



Sweep Equation

- $\Gamma(t) = \Psi(t) + R(t)\,\Gamma$, $0 \le t \le 1$
  - Γ **:** Generator (polyhedron)
  - Ψ**(t)** : Smooth vector in R^3 (sweeping path)
  - R**(t) :** Local orientation
- Swept Volume of $\Gamma := \cup\Gamma(t)$

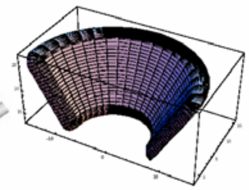


• No scaling, shearing, and deformation

Swept Volume: Applications

Numerically Controlled Machine Verification

Tool and workpiece



Material removal

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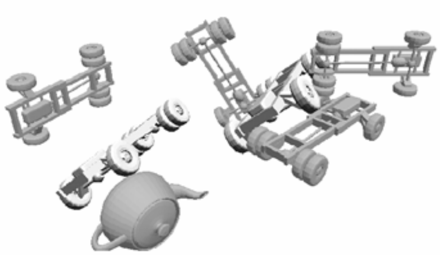
Swept Volume: Applications

Collision detection between discrete instances



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Swept Volume Computation

- Enumerate ruled and developable surfaces
- Boundary of SV = outer envelope of the arrangement

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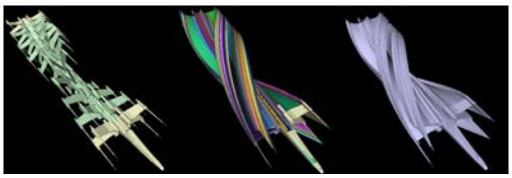
Swept Volume Computation

- X-Wing Model
  - 2496 triangles
  - 3931 ruled and developable surfaces
  - Intersection curves of degree as high as nine



Sweep Trajectory   Arrangement   Boundary of SV

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Computation of Swept Volumes

- Generate ruled and developable surfaces
- Compute their arrangement
- Traverse the arrangement and extract the *outermost* boundary (outer envelope computation)

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Complexity of Arrangements

- High computational and combinatorial complexity
  - Super-quadratic in number of surfaces
- Accuracy and robustness problems
- No good practical implementations are available

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Approximation Pipeline

- Enumerate surface primitives
- Compute distance fields on a voxel grid
- Perform filtering operations on distance fields
- Use improved reconstruction algorithms

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Approximation Pipeline

- Enumerate surface primitives
- Compute distance fields on a voxel grid
- Perform filtering operations on distance fields
- Use improved reconstruction algorithms
  - Max-norm computations for reliable voxelization
  - Recover all connected components
  - Faithfully reconstruct sharp features

Organization

- Fast distance field computation
- Max-norm based voxelization
- Boundary reconstruction
- Analysis
- Applications
  - Boundary evaluation
  - Swept volume computation
  - Medial axis computation
  - Minkowski sums

Organization

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Distance Fields

- **Distance Function**

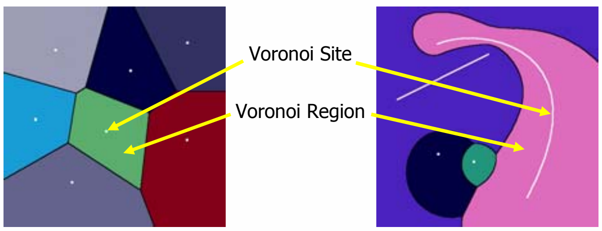
  For a site a scalar function $f:R^n \to R$ representing the distance from a point $P \varepsilon R^n$ to the site

- **Distance Field**

  For a set of sites, the minima of all distance functions representing the distance from a point $P \varepsilon R^n$ to closest site

Voronoi Diagrams

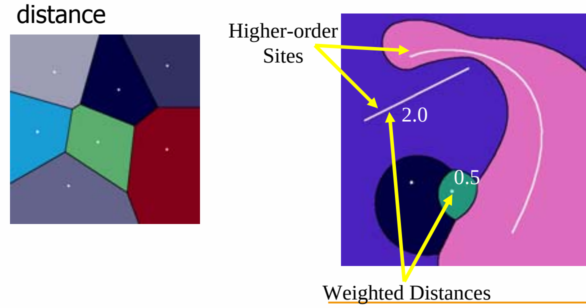
Given a collection of geometric primitives, it is a subdivision of space into cells such that all points in a cell are *closer* to one primitive than to any other



Ordinary Generalized

Ordinary
- Point sites
- Nearest Euclidean distance

Generalized
- Higher-order site geometry
- Varying distance metrics



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Voronoi Diagram & Distance Fields

- Minimization diagram of distance functions generates a Voronoi Diagram
- ***Projection of lower envelope*** of distance functions

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Distance Fields: Applications

Collision Detection
Surface Reconstruction
Robot Motion Planning
Non-Photorealistic Rendering
Surface Simplification
Mesh Generation
Shape Analysis

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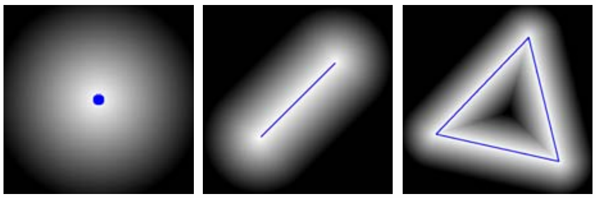
GPU Based Computation

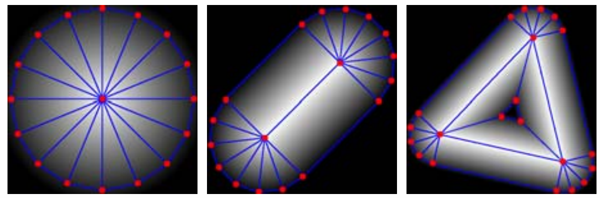
- **HAVOC2D, HAVOC3D [Hoff et al. 99,01]**
  - Evaluate distance at each pixel for all sites
  - Evaluate the distance function using graphics hardware



Point Line Triangle

Approximating the Distance Function

- Avoid per-pixel distance evaluation
- Point-sample the distance function
- Reconstruct by rendering polygonal mesh

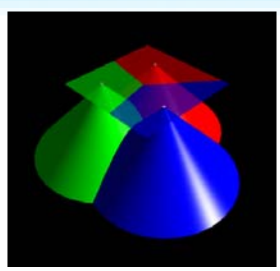


Point Line Triangle

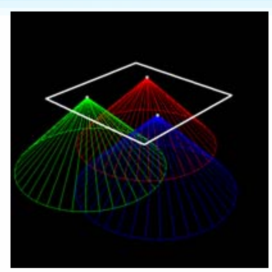
GPU Based Computation

- Triangular mesh approximation of distance functions
- Render distance meshes using graphics hardware

Meshing the Distance Function



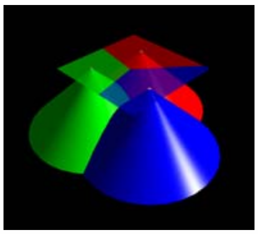
Shape of distance function for a 2D point is a cone



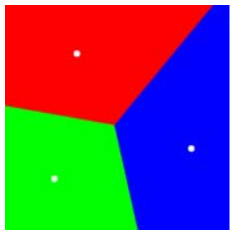
Need a bounded-error tessellation of the cone

Graphics Hardware Acceleration

- Rasterization to reconstruct distance values
- Depth test to perform minimum operator



Perspective, 3/4 view



Parallel, top view

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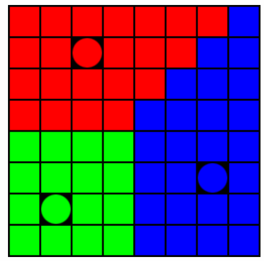
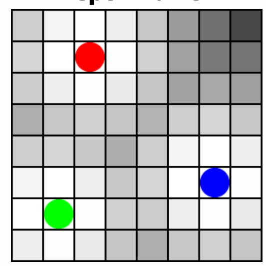
Results in the Frame Buffer

Color Buffer



Voronoi Regions

Depth Buffer



Distance Field

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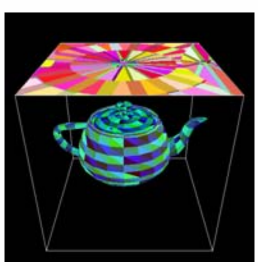
3D Voronoi Diagrams

- Graphics hardware can generate one 2D slice at a time
- Sweep along 3rd dimension (Z-axis) computing 1 slice at a time



Distance Field of the Teapot Model

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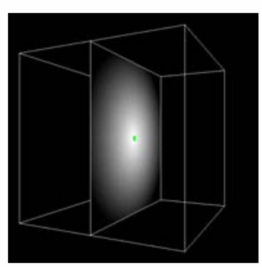
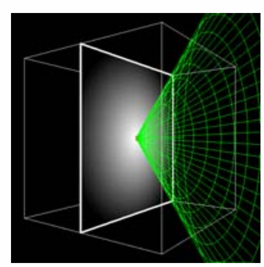
Shape of 3D Distance Functions



Slices of the distance function for a 3D point site



Distance meshes used to approximate slices

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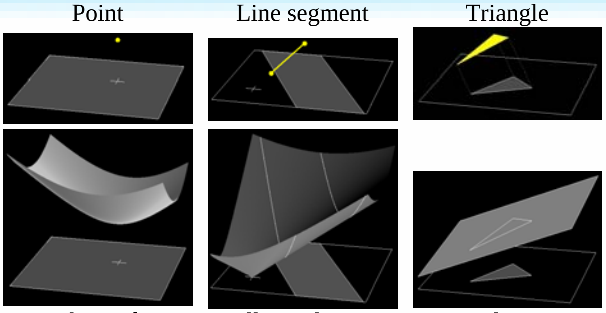
Shape of 3D Distance Functions

Point | Line segment | Triangle



1 sheet of a hyperboloid | Elliptical cone | Plane

Bottlenecks in HAVOC3D

- **Rasterization:**
  - Distance mesh can fill entire slice
  - Complexity for *n* sites and *k* slices = O(*kn*)
  - Lot of Fill !
- **Readback:**
  - Stalls the graphics pipeline
  - Not suitable for interactive applications

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Improved Distance Field Computation (DiFi)

- Use graphics hardware
- Exploit spatial coherence between slices
- Use the programmable hardware to perform computations

[Sud and Manocha 2003]

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Improved Distance Field Computation (DiFi)

Reduce fill: Cull using estimated voronoi region bounds

- **Along Z:** Cull sites whose voronoi regions don't intersect with current slice
- **In XY plane:** Restrict fill per site using planar bounds of the voronoi region

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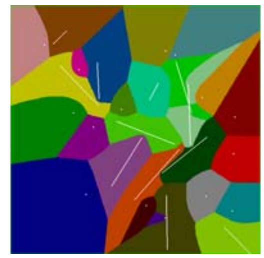
Voronoi Diagram Properties

- Within a bounded region, all voronoi regions have a bounded volume

9 Sites, 2D

Voronoi Diagram Properties

- Within a bounded region, all voronoi regions have a bounded volume
- As site density increases, average spatial bounds decrease



27 Sites, 2D

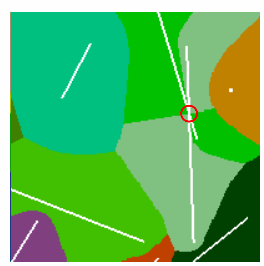
Voronoi Diagram Properties

Voronoi regions are connected

- Valid for l_2 , l_{inf} etc. norms

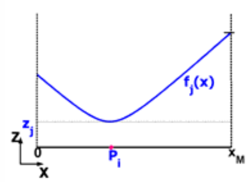
Voronoi Diagram Properties

- **Voronoi regions are connected**
  - Valid for l_2 , l_{inf} norms
- **Special cases:** Overlapping features



Voronoi Diagram Properties

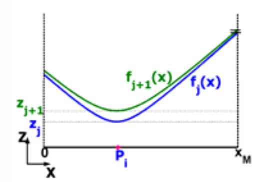
- High distance field coherence between adjacent slices
- Change in distance function between adjacent slices is bounded



Distance functions for a point site P_i to slice Z_j

Voronoi Diagram Properties

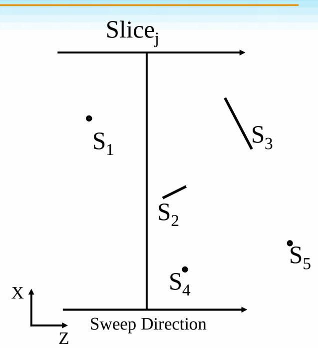
- High distance field coherence between adjacent slices
- Change in distance function between adjacent slices is bounded



Distance functions for a point site P_i to slice Z_{j+1}

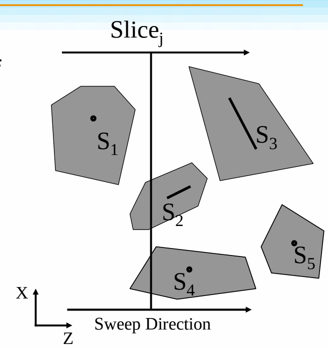
Site Culling: Classification

For each slice partition the set of sites



Site Culling: Classification

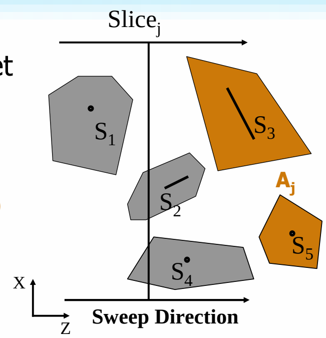
For each slice partition the set of sites using voronoi region bounds



Site Culling: Classification

For each slice partition the set of sites, using voronoi region bounds:

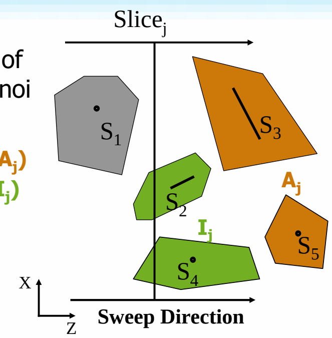
- Approaching (A_j)



Site Culling: Classification

For each slice partition the set of sites, using voronoi region bounds:

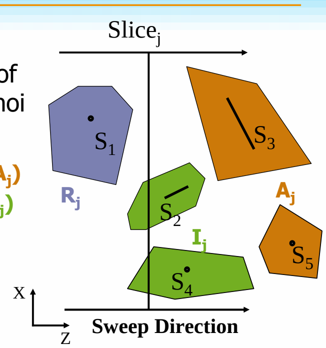
- Approaching (A_j)
- Intersecting (I_j)



Site Culling: Classification

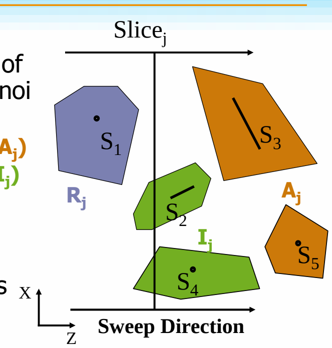
For each slice partition the set of sites, using voronoi region bounds:

- Approaching (A_j)
- Intersecting (I_j)
- Receding (R_j)



Site Culling: Classification

- For each slice partition the set of sites, using voronoi region bounds:
  - Approaching (A_j)
  - Intersecting (I_j)
  - Receding (R_j)
- Render distance functions for Intersecting sites only

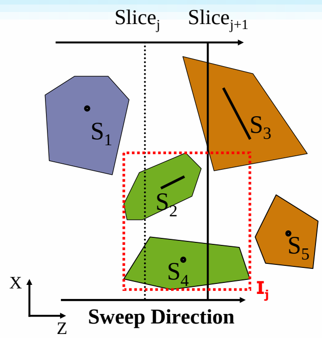


Coherence: Adjacent Slices

Updating $\mathbf{I_j}$

$\mathbf{I_{j+1} = I_j \ldots}$

Previously intersecting



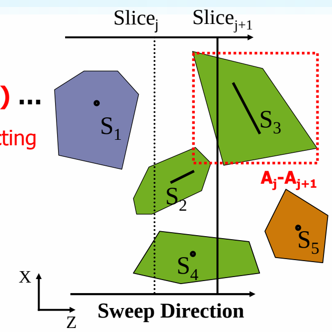
Coherence: Adjacent Slices

- **Updating $\mathbf{I_j}$**

$\mathbf{I_{j+1} = I_j}$

$\mathbf{+ (A_j - A_{j+1}) \ldots}$

Approaching → Intersecting



Coherence

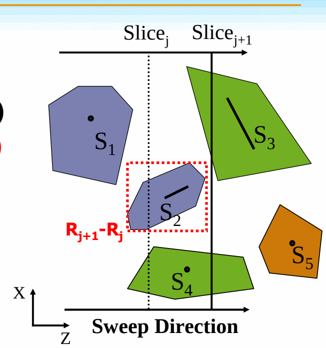
- **Updating $\mathbf{I_j}$**

$\mathbf{I_{j+1} = I_j}$

$\mathbf{+ (A_j - A_{j+1})}$

$\mathbf{- (R_{j+1} - R_j)}$

Intersecting → Receding



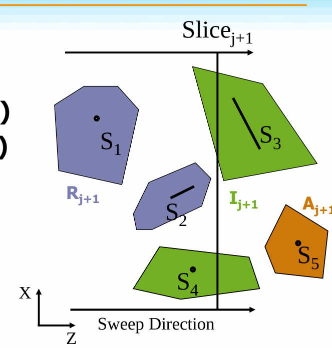
Coherence

- **Updating $\mathbf{I_j}$**

$\mathbf{I_{j+1} = I_j}$

$\mathbf{+ (A_j - A_{j+1})}$

$\mathbf{- (R_{j+1} - R_j)}$

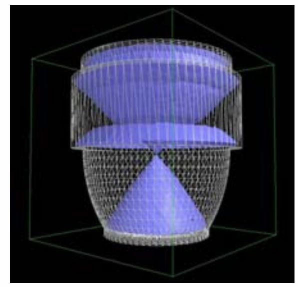


Estimate Potentially Intersecting Set (PIS)

- Computing exact intersection set = Exact voronoi computation
- **Conservative Solution**:
  - Use hardware based occlusion queries
  - Determine number of visible fragments
  - Computes *potentially intersecting* sites (PIS) $\hat{\mathbf{I}}$

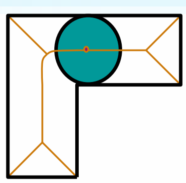
$$\hat{\mathbf{I}}_j \supseteq \mathbf{I}_j$$

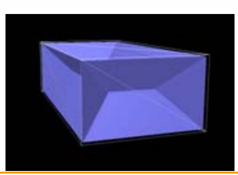
Application to Medial Axis Computation



Blum Medial Axis

- Locus of centers of maximal contained balls
- Well-understood medial representation
- **Applications**
  - Shape analysis
  - Mesh generation
  - Motion planning
- Exact computation is hard





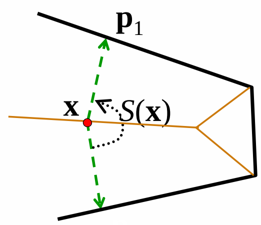
θ-Simplified Medial Axis M_θ

- A subset of the full medial axis M
- Relies on *separation angle* from points on the medial axis to the boundary
- More stable than Blum medial axis

[Foskey, Lin and Manocha 2002]

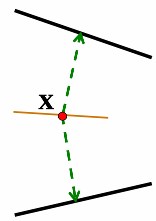
Separation Angle

- Angle separating the vectors from x to nearest neighbors
- If more than 2 nearest neighbors, maximum angle is used



Large Separation Angle

Point is roughly *between* its nearest neighbor points



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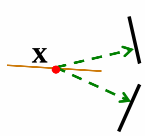
Small Separation Angle

Point is *off to one side* of its nearest neighbor points



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Simplified Medial Axis

$$\mathrm{M}_\theta = \{\mathbf{x} \in M \mid S(\mathbf{x}) > \theta\}$$

- Start with medial axis M
- Eliminate portions with $S(\mathrm{x}) \leq \theta$

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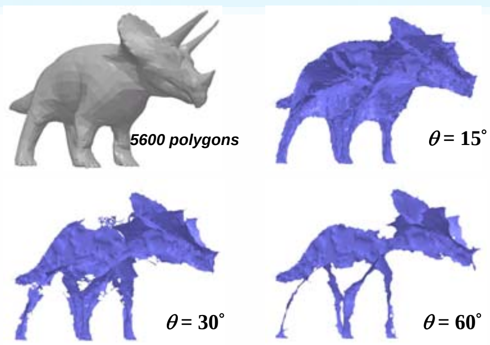
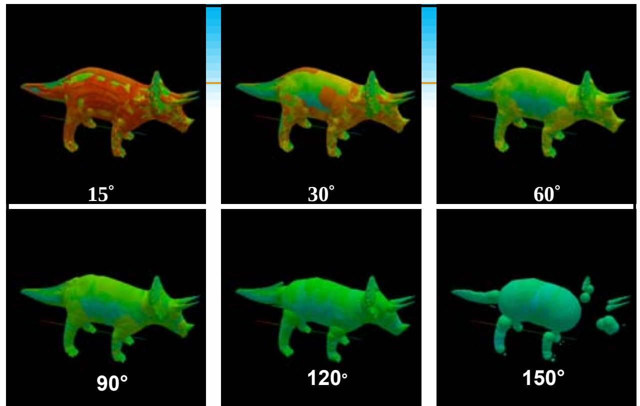
3D Example: Triceratops





Shape Simplification using Simplified Medial Axis

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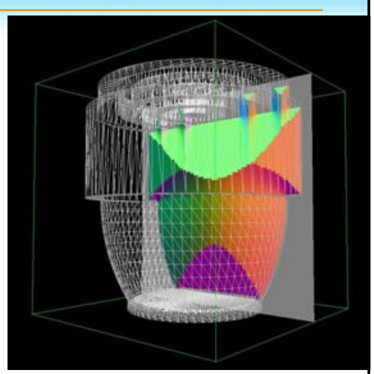
Direction Field

- Gradient of Distance Field
- Direction image rendered for each slice (constant z)
- Direction vectors encoded as RGB triples
- Length encoded in depth buffer



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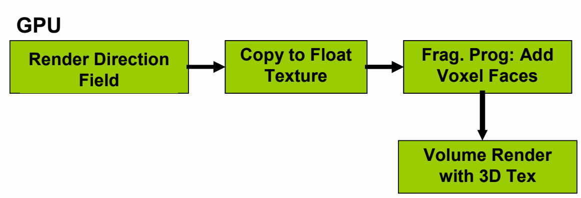
Simplified MAT Computation using GPUs

Computation using **DiFi** [Sud et al. 2003]



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Simplified MAT Computation using Graphics Hardware

Real-time Capture from a Dell Laptop with NVIDIA GeForce4 To Go graphics card

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Simplified MAT Computation using Graphics Hardware

Real-time Capture from a Dell Laptop with NVIDIA GeForce4 To Go graphics card

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Direction Field Computation

- **4 -20 times speedup over HAVOC3D**

Model	Polys	Resolution	HAVOC (s)	DiFi (s)
Shell Charge	4460	128x126x126	31.69	3.38
Head	21764	79x106x128	52.47	13.60
Bunny	69451	128x126x100	212.71	36.21
Cassini	90879	94x128x96	1102.01	47.90

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Surface Reconstruction

- **2 - 75 times speedup**

Model	Resolution	CPU (s)	GPU (s)
Shell Charge	128x126x126	3.50	0.14
Head	79x106x128	0.18	0.08
Bunny	128x126x100	0.68	0.13
Cassini	94x128x96	7.59	0.1

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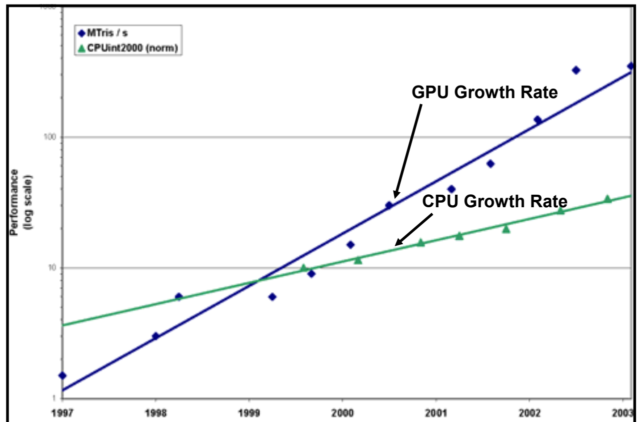
Reconstruction: CPU vs. GPU

- **Depends on grid size**
- **2 - 75 times speedup via GPUs**

Model	Resolution	T(CPU) (s)	T(GPU) (s)
Shell Charge	128x126x126	3.50	0.14
Head	79x106x128	0.18	0.08
Bunny	128x126x100	0.68	0.13
Cassini	94x128x96	7.59	0.1



Benefits of using GPUs

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Organization

- Fast distance field computation
- Max-norm based voxelization
- Boundary reconstruction
- Analysis
- Applications
  - Boundary evaluation
  - Swept volume computation
  - Medial axis computation
  - Minkowski sums

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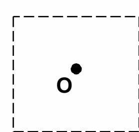
Max-Norm (l_∞) Computation

- Max-Norm
  - Natural metric for axis-aligned voxels

$$|| \mathbf{p} ||_\infty = \max (|x|, |y|, |z|)$$



Iso-distance ball
$|| \mathbf{x} ||_\infty = c$
is a cube

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Applications of Max-Norm Computation

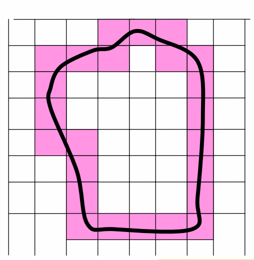
- Markov decision processes [Tsitsiklis et al. 96, Guestrin et al. 2001]
- Discrete objects in supercover model [Andres et al. 96]
- Image analysis [Lindquist 99]
- Volume graphics [Wang & Kaufman 94, Sramek & Kaufman 99]

Goal

- Efficiently compute max-norm distance between a point and a wide class of geometric primitives
- Motivation
  - Voxelization

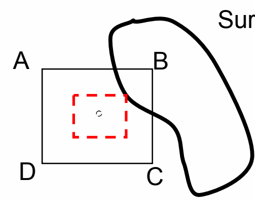
Voxelization

- Represent a scene by a discrete set of voxels



Voxelization

- Reduce to max-norm distance computation

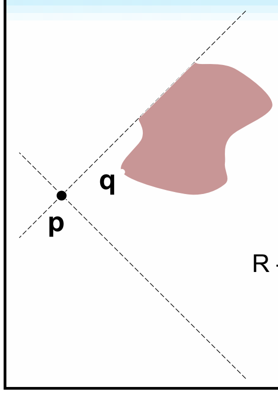


Surface intersects voxel ABCD if l_∞ iso-distance cube is smaller than the voxel

Outline

- l_∞ Distance Computation
  - Optimization Framework
  - Specialized Algorithms
- Complex Models
  - Bounding Volume Hierarchy
  - Graphics Hardware Approach

Optimization Framework



Minimize x

subject to

q lies on the primitive

q lies within R

Non-linear optimization

R – x+ dominating region

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Convex Primitives

- Non-linear optimization reduces to convex optimization
- Simpler solution when the query point is inside the primitive

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Outline

- l_∞ Distance Computation
  - Optimization Framework
  - Specialized Algorithms
    - Convex Primitives
    - Algebraic Primitives
    - Triangulated Models
- Complex Models

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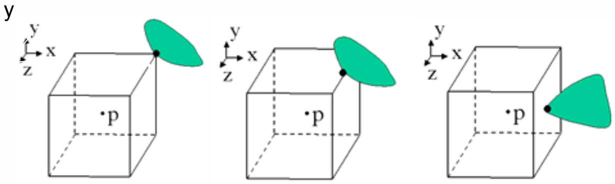
Algebraic Primitives

- Equation solving approach
- Applicable to convex and non-convex primitives
- Solve for the closest point, x



Vertex Edge Face

Equation Solving

- Solve above equations for each vertex, edge and face
- Solution set is finite in general
- Obtain a set X of feasible values for the closest point
- Calculate min { $||x-p||_\infty$ | x ε X}

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Equation Solving

- **Quadrics**
  - Quadratic Equation
- **Torus**
  - Symmetry
  - Degree 8 polynomial

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Outline

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Distance Computation for a Triangle

- Optimization framework applied to the special case of a triangle
- Split the triangle with respect to the partitioning triangles

Bounding Volume Hierarchy

- Large polyhedral model
- Naïve algorithm
  - Minimum over distance to each triangle
- Speed it up using a precomputed bounding volume hierarchy

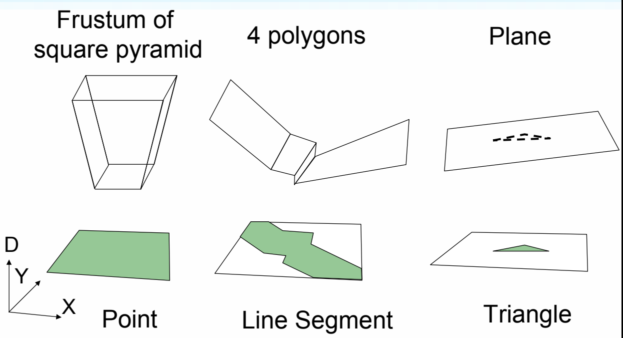
Outline

- l_∞ Distance Computation
  - Optimization Framework
  - Specialized Algorithms
    - Convex Primitives
    - Algebraic Primitives
    - Triangulated Models
- Complex Models
  - Bounding Volume Hierarchy
  - Graphics Hardware Approach

Graphics Hardware Approach

- Approach similar to [Hoff et al. 1999]
- Render distance function for each primitive
- Z-buffer holds the distance field

Linear Distance Functions for l_∞ Computations



Organization

- Fast distance field computation
- Max-norm based voxelization
- **Boundary reconstruction**
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Surface Reconstruction

- Objective – obtain a triangular mesh representation
- To extract the surface
  - Compute the zero-set { p | D(p) = 0 }

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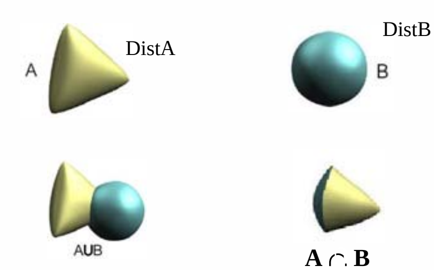
Boolean Operations



Min { DistA, Dist B} == 0   Max { DistA, Dist B} == 0

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Isosurface Extraction

- Marching Cubes [Lorensen & Cline 87]
- Extended Marching Cubes [Kobbelt et al. 01]
- Dual Contouring [Ju et al. 02]
- Extended Dual Contouring [Varadhan et al. 03]

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Marching Cubes

- Given the distance field grid,
  - Reconstruct the surface within each grid cell
- Once done with one cell (cube), march to the next

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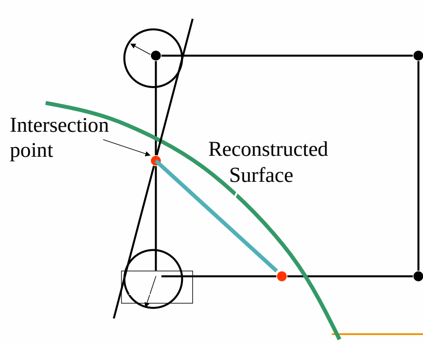
Marching Cubes



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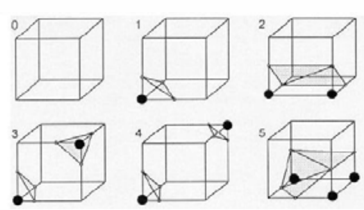
Marching Cubes



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Marching Cubes

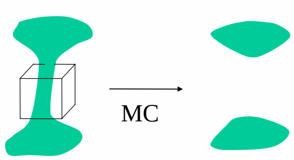
- Handle each cell independently
- Because intersection points along grid edges are consistent between adjacent cells
  - Reconstructed surface matches at cell boundaries and doesn't leave holes

Our Approach

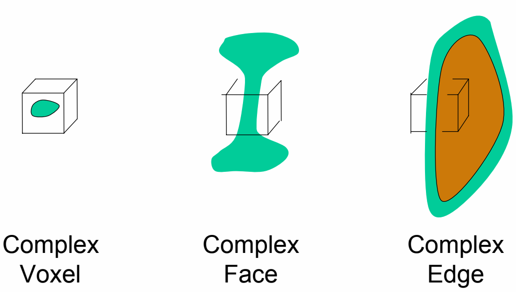
1. Generate distance field D for the union
2. Obtain an approximation by extracting an
   - Isosurface { p | D(p) = 0 }

Issues

- Accuracy of the algorithm dependent on resolution of the underlying grid
  - Insufficient resolution can result in unwanted handles or disconnected components



Complex Cells



Complex Cells

- How do you detect them?
  - Solution: Max-Norm Distance Computation

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Complex Cells

- Express voxel, face and edge intersection tests in terms of 3D, 2D and 1D max-norm distance respectively.
- A voxel, face, or edge is *complex* if it is intersecting but does not exhibit a sign change (i.e., a different in the outside/inside status)

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Complex cells

- Once detected, how do you handle them?
  - Subdivide them

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Issues

- Many cells in the grid do not contain a part of the final surface
  - Cull them away
- For each grid cell, first perform the voxel intersection test
- If the test fails, do not consider the voxel any further
- Makes the algorithm *output-sensitive*

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Issues

- Large number of primitives
- Each distance and outside/inside query defined in terms of all the primitives

Local Queries

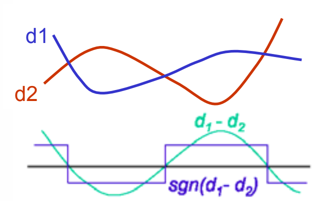
- Perform a local query within each cell by considering only the primitives intersecting the cell
  - Preserves correctness of the query
  - Drastically improves performance

Sharp Features

- Surface-surface intersection causes many sharp features on the boundary of the final surface
- When do two surfaces S1 and S2 intersect each other?
  - Track the bisector surface d1-d2, where d1, d2 are the distance functions for the two surfaces [Varadhan et al. 03]

Bisector Surface

- Bisector surface (d1-d2) contains the intersection curve
- It changes sign at intersection
  - Track sgn(d1-d2)



Grid Generation

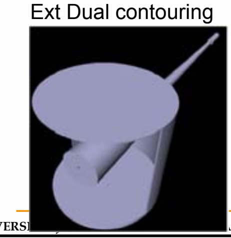
- Can reconstruct atmost one sharp feature per voxel
- Subdivide voxels with more than one sharp feature

Reconstruction algorithm

- Extended dual contouring algorithm [Varadhan et al. 03]
  - can reconstruct arbitrary thin features without creating handles

Dual contouring

Ext Dual contouring



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Organization

- Fast distance field computation
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  - Medial axis computation
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Bounds on Approximation

Let **S**: exact answer of the union or envelope computation
B(**S**): boundary of S

.
Our approximation algorithm takes as input $\varepsilon > 0$, and generates an approximation $A(\varepsilon)$

$B(A(\varepsilon))$: denote the boundary of the approximation

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Bounds on Approximation

Theorem 1: Given any $\varepsilon > 0$, our algorithm computes an approximation $B(A(\varepsilon))$ such that

2-Hausdorff($B(A(\varepsilon))$, $B(S)$) < ε,

where 2-Hausdorff is the two sided Hausdorff distance

Bounds on Approximation

Theorem 2:Given any $\varepsilon > 0$, our algorithm computes an approximation $A(\varepsilon)$ to the exact union or envelope **S** such that $A(\varepsilon)$ has the same number of connected components as **S**

Corollary: S is connected if and only if $A(\varepsilon)$ is connected

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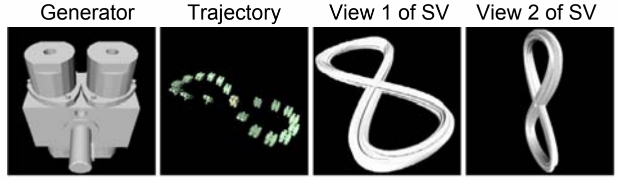
Swept Volume Computation

Generator | Trajectory | View 1 of SV | View 2 of SV



2,280 triangles

1,152 surfaces

Time = 12 secs

[Kim et al. 2003]
http://gamma.cs.unc.edu/SV

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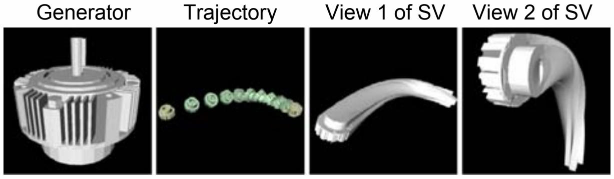
Results: Swept Volume

Input Clutch Model

Generator | Trajectory | View 1 of SV | View 2 of SV



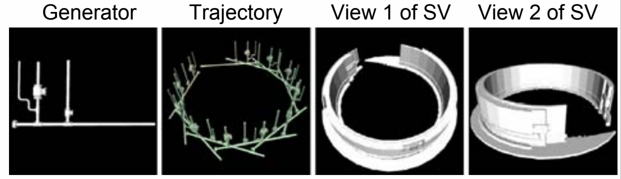
2,116 triangles | 1,175 surfaces | Time = 21 secs

[Kim et al. 2003]
http://gamma.cs.unc.edu/SV

Results

Pipe Model

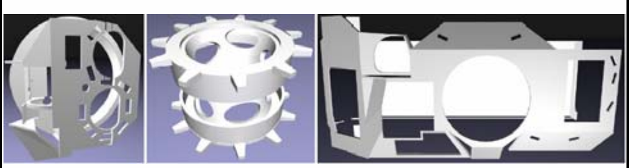
Generator | Trajectory | View 1 of SV | View 2 of SV



10,352 triangles | 15,554 surfaces | Time = 67 secs

Boundary Evaluation of Complex CSG Models

30-40 solids defined using 2-7 Boolean operations
8-13 secs per solid



Turret | Drivewheel | Hull

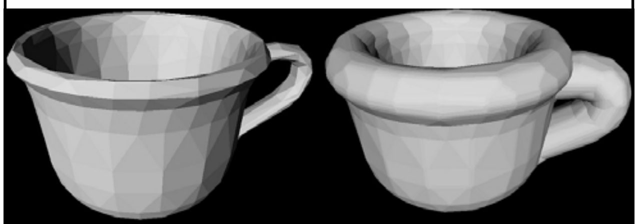
[Varadhan et al. 2003]
http://gamma.cs.unc.edu/recons

Boundary Evaluation of Complex CSG Models

Bradley Fighting Vehicle
1200 solids
8,000 CSG operations
Took 2 hours

[Varadhan et al. 2003]
http://gamma.cs.unc.edu/recons

Cup: Offset Computation



1000 triangles:
338 convex pieces

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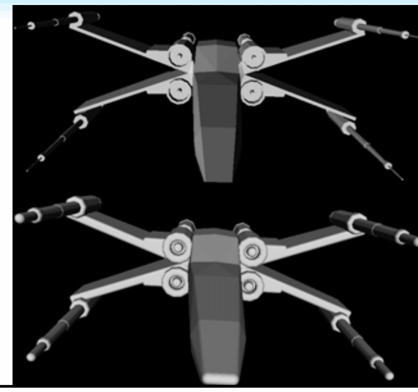
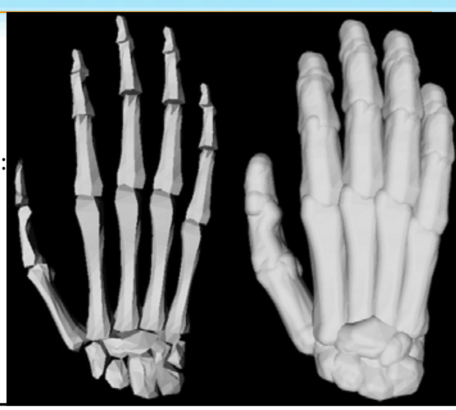
Xwing: Offset Computation

2496 triangles:
1294 convex pieces

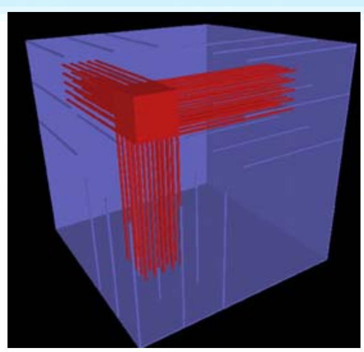


Hand: Offset Computation

2982 triangles:
910 convex pieces



Minkowski Computation



Non-convex polyhedra

Red polyhedra: 1134 polygons
Blue polyhedra: 444 polygons

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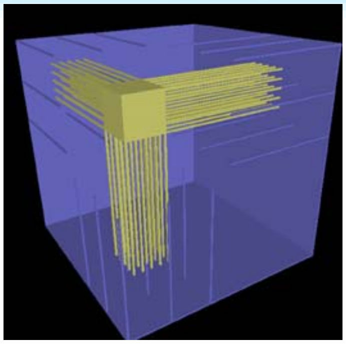
Minkowski Computation



Non-convex polyhedra

Yellow polyhedra: 1134 polygons

Blue polyhedra: 444 polygons

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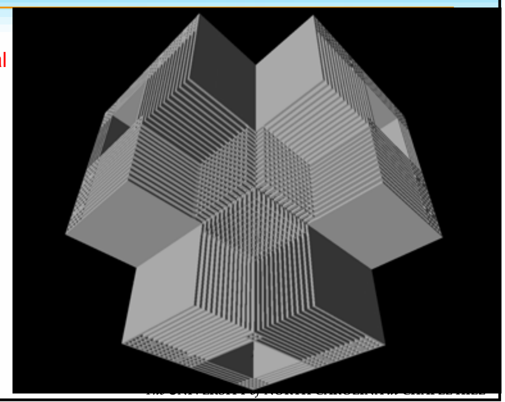
Worst Case Minkowski Sum

$O(n^6)$ Combinatorial complexity



Application to Continuous Collision Detection

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Conclusions

- Discretized geometric computations
- Union and envelope comptuations
- Fast distance field computation
- Max-norm computation algorithms
- Application to medial, swept volume, Minkowski and CSG computations
- Use of GPUs for geometric computations
- Benefits
  - Improved performance
  - Robust implementations

Acknowledgements

- Army Research Office
- Intel
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The End

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