Optimization Models for Container Inspection

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Problem

- Finding ways to intercept illicit nuclear materials and weapons destined for the U.S. via the maritime transportation system is an exceedingly difficult task. Today, only a small percentage of containers arriving to U.S. ports are inspected.
- Inspection involves checking paperwork, using various imaging sensors, and manual inspection.
- Objectives involve maximizing detection rate, minimizing unit cost of inspection, rate of false positives, time delays, etc.
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Detection rate: 60%

Inspection cost: $0.4C_{CHK} + C_a$
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Container Inspection
A small example involving two sensors

Sensor a
- Good
- Bad
- Sensor reading $t_a$

Sensor b
- Good
- Bad
- Sensor reading $t_b$

Detection rate
- Sensor a: 60%
- Sensor b: 64%

Inspection cost
- $0.4C_{CHK} + C_a$
- $0.1C_{CHK} + C_a + 0.5C_b$

Tree diagram:
- Node a: OK
- Node b: OK
- Node CHK: 50% 20%
- 50% 80%
- 40% 10%
- 10% 64%
A small example involving two sensors

sensor a

sensor b

Detection rate

Inspection cost

0.4C_{CHK} + C_a
0.1C_{CHK} + C_a + 0.5C_b
A small example involving two sensors

**Detection rate**
- Sensor A: OK with 60%, CHK with 40%
- Sensor B: OK with 64%, CHK with 36%

**Inspection cost**
- Sensor A: $0.4C_{CHK} + C_a + 0.5C_b$
- Sensor B: $0.1C_{CHK} + C_a + 0.5C_b$
Mathematical Model

Maximize detection rate $\Delta(D, t)$

- over all decision trees $D$ and threshold selections $t$
- subject to budget, capacity, and delay constraints

A possible solution (Stroud and Saeger, 2003)

- Enumerate all possible (binary) decision trees and compute best possible threshold selections for each.
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  - Number of decision trees is doubly exponential!
  - Enumeration is possible only for $s \leq 4$!
  - Too expensive to analyze tradeoffs!
  - Why only 1-1 thresholds?
  - Why a single decision tree?
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Developed a polyhedral description of all possible decision trees.

Formulated a large scale LP model for optimal inspection policy; maximization of detection rate, while limiting unit cost of inspection, rate of false positives, and time delays, etc.

Off the shelf LP packages can find optimal inspection strategies up to 6-8 sensors.

Detection rate and inspection cost ROC curve can be tabulated.

Effects of capacity and time delay limitations can be analyzed.

Robustness of new sensor technologies can be evaluated.
Large Scale LP Formulation

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Experiments with 4 sensors (Stroud and Saeger, 2003)

- Detection rate $\geq 81.5\%$
- *Threshold-optimized* pure strategy found by Stroud and Saeger (2003)
- *Non-optimized* threshold grid; savings of $\approx 10\%$
Experiments with 4 sensors (Stroud and Saeger, 2003)

- 7 non-optimized thresholds per sensor
- 1 non-optimized thresholds per sensor

<table>
<thead>
<tr>
<th>Cost</th>
<th>Detection Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10</td>
<td>75%</td>
</tr>
<tr>
<td>$20</td>
<td>80%</td>
</tr>
<tr>
<td>$30</td>
<td>85%</td>
</tr>
<tr>
<td>$40</td>
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</tr>
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</tr>
<tr>
<td>$60</td>
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