In Search of Various Oh’s

Find $a$ and $b$ such that: $f(n) = an^b - R(n)$

- $R(n) \geq 0$, but hopefully small, for all $n > n_0$
- $R(n) \leq 0$, but hopefully small, for all $n > n_0$
- $|R(n)|$ as small as possible, for all $n > n_0$
First things a statistician will probably want to talk about:

Where are the physical sources of variation?
• problem-to-problem for the same $n$ ...
• computer-to-computer for the same problem ...
• execution-to-execution for the same computer ...

Where are the structural uncertainties that cannot be avoided?
• functional form of $R$ ...
• possibility that $a$ isn’t really constant, even if $O(a) = 1$ ...
• possible “granular” response to discrete $n$ (e.g. discontinuous $R$) ...
“Find $a_L$, $b_L$, $a_U$, and $a_U$ such that

$$a_L n^{b_L} < f(n) < a_U n^{b_U} \ldots$$

(what statisticians don’t do much) ... is provably true for all functions in a specified class, perhaps assuming a relationship between the observed $n$’s and $n_0$.”

(what statisticians do more of) ... is true except with some controllable and quantifiable risk∗ for functions in a perhaps richer class.”

∗ relative to the sources of variability, noise, and uncertainty previously mentioned
Standard regression methods ...

- are good for modeling the response near the data
- are generally not so good for revealing model structure

They typically produce confidence bounds that grow to asymptotic uselessness with $n$ ... this will make them of little value here.

Generally need to add information/assumptions to reflect how structure is more apparent with larger $n$ (same intuition as with PW3).
Statistical intuition toward this end: Need information concerning:

- $an^b$ (2 degrees of freedom)
- How large is $R$ relative to $a$?
- How quickly does $R$ die out with $n$?
- How simple/smooth/crazy is $R$? (...min 5 d.f. so far)

If there is also rough/“discontinuous” (in $n$) noise

- How large, relative to $a$?
- How quickly does it die out?

Sounds like you need ... well, maybe I need ... substantially more than 5 data points. (Statisticians are famous for saying things like this.)
How about this?

\[ f(n) = an^b \text{(dominant)} + a_1 n^{b_1} + a_2 n^{b_2} + ... \]
\[ = an^b [1 + \frac{a_1}{a} n^{-(b-b_1)} + \frac{a_2}{a} n^{-(b-b_2)} + ...] \]
\[ \ln(f(n)) = \ln(a) + b \times \ln(n) + \ln[''] \]
\[ \approx \ln(a) + b \times \ln(n) + \{r_1 n^{-\delta_1} + r_2 n^{-\delta_2} + ...} \]

Model \( Z(n) = \{-\} \) as a random function with:

- \( E[Z(n)] = 0 \)
- \( SD[Z(n)] = \sigma n^{-\delta} \) (size and decay rate of extra)
- \( Corr[Z(n), Z(n')] = exp(-\theta[\ln(n) - \ln(n')]^2) \) ("smoothness")

Think about lower and upper confidence limits for \( b \) ...
Relatively vague priors, design = \{2, 4, 8, ..., 1023\}, MCMC, 2.5% and 97.5% points of posterior:

<table>
<thead>
<tr>
<th>function</th>
<th>(\hat{b}_L)</th>
<th>(\hat{b}_U)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3n^2 + 100) (#2)</td>
<td>0.169</td>
<td>0.175</td>
</tr>
<tr>
<td>(3n^8 - n^2) (#6)</td>
<td>0.822</td>
<td>0.853</td>
</tr>
<tr>
<td>(3n^8 + n^6) (#8)</td>
<td>0.834</td>
<td>0.848</td>
</tr>
<tr>
<td>(3n^{1.2} - 2n^8 + n^4)</td>
<td>1.158</td>
<td>1.168</td>
</tr>
</tbody>
</table>

Excuses: In each case, \(\hat{\delta}\) was very, very small ... suggesting that the model isn’t tracking the “smaller-term decay” adequately.