The best-deterministic method for the stochastic unit commitment problem

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Joint work with Hugo P. Simao, Warren B. Powell

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The challenge of using wind energy

Wind: complex to forecast; high-dimensional process.

Generating units must balance variations from stochastic injections (load:- and wind:+) in real-time. The control relies on frequency changes and on signals sent by the system operator.

Net power injections at each bus (node) from generators and loads, including wind:

\[ f(v, \theta) = p \]

active power balance

\[ g(v, \theta) = q \]

reactive power balance

voltage magnitude and angle at each bus, assuming steady state @ 60Hz

This is 5-min data of energy injected by a wind farm.

Generating units must balance variations from stochastic injections (load:- and wind:+) in real-time. The control relies on frequency changes and on signals sent by the system operator.

\[ f(v, \theta) = p \]

active power balance

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voltage magnitude and angle at each bus, assuming steady state @ 60Hz

This is 5-min data of energy injected by a wind farm.
Steam units need time to start up and be online (spin at required frequency). They must be committed to produce power in advance.

Start-up of a gas-fired steam turbine after a 7-hour shutdown.

Initial period where the unit is committed to produce power
Aggregated cost curves say: Do not wait too long

Cost-based offer curve of dispatchable units

Units that can be started up on short notice

Units to be committed in advance

Assumptions for this graph:
No transmission constraints. No startup costs.
Not plotted: Pumped Storage, Hydro, Wind, Solar.
We are plotting curves from cost estimates, not bids.
Offer dynamics for peaker units

daily bids of a combustion turbine bidding a single price-quantity block, year 2010

Price [USD/MWh]

Quantity [MW]

Natural Gas Price [USD/MMBtu]

Temperature [F]

summer

winter

winter
Multistage stochastic unit commitment

Stochastic formulation with **startup decision time lags** $\delta_j$ (12h, 6h, 3h, 1h,...)
given

\{${W}_{tj}$\} : random process for variable energy resource $j$ in $J^{\text{VER}}$
\{${L}_t$\} : random demand process

\[
\text{minimize} \quad \mathbb{E}\{ \sum_{t=1}^{T} \sum_{j=1}^{J} c_{tj} \text{start} v_{t-\delta_j, tj} + c_{tj} p_{ttj} \} \\
\text{subject to} \quad \sum_{j=1}^{J} p_{ttj} = {L}_t \text{ a.s., for each } t
\]

energy balance (assuming NO demand-side flexibility)

constraints for dispatchable $j \in J^D$, for each $t$:

\[
\begin{align*}
&v_{t-\delta_j, tj} - w_{t-\delta_j, tj} = u_{t-\delta_j, tj} - u_{t-\delta_j-1, t-1, j} \\
&u_{t-\delta_j, tj} \geq p_{jt} \leq u_{t-\delta_j, tj} P_j \\
&-R_j^{\text{down}} \leq p_{ttj} - p_{t-1, t-1, j} \leq R_j^{\text{up}} \\
&\text{lagged startup decisions}
\end{align*}
\]

capacity constraints

ramping constraints

(simplified statement)

constraints for variable energy resources $j$ in $J^{\text{VER}}$

\[
p_{ttj} \leq u_{t-\delta_j, tj} {W}_{tj} \text{ a.s., for each } t
\]

[curtailment]

\[
u_{t-\delta_j, tj}, v_{t-\delta_j, tj}, w_{t-\delta_j, tj} \in \{0,1\}.
\]
Multistage stochastic unit commitment

updated information

D-1 12:00

samples in high-dimensional uncertainty space

D-1 18:00

D 0:00

D 1:00

D ...

recommitments and redispachting

Locked commitments for slow-start units
Two-stage stochastic unit commitment

Stochastic MILP formulation in the day-ahead paradigm:
Time lags $\delta_j$ valued in \{12h, 0h\} only (slow- and fast- start).

Minimize

$$\mathbb{E}\left\{ \sum_{t=1}^{T} \sum_{j=1}^{J} c_{tj}^{\text{start}} v_{t-\delta_j, tj} + c_{tj} p_{t-\delta_j, tj} \right\}$$

subject to

$$\sum_{j=1}^{J} p_{ttj} = L_t \quad \text{a.s., for each } t$$

Constraints for dispatchable $j \in J^D$:

- $v_{0tj} - w_{0tj} = u_{0tj} - u_{0, t-1,j}$ \quad $j$ in **slow-start units**:
- $u_{0tj} P_j \leq p_{ttj} \leq u_{0tj} \bar{P}_j$ \quad lock the day-ahead startups
- $v_{ttj} - w_{ttj} = u_{ttj} - u_{t-1, t-1,j}$ \quad $j$ in **fast-start units**:
- $u_{ttj} P_j \leq p_{ttj} \leq u_{ttj} \bar{P}_j$ \quad do not lock day-ahead startups

Constraints for variable energy resources $j$ in $J^{VER}$

$$p_{ttj} \leq u_{ttj} W_{tj} \quad \text{a.s., for each } t$$

$$u_{0tj}, v_{0tj}, w_{0tj} \ (j \text{ slow}), u_{ttj}, v_{ttj}, w_{ttj} \ (j \text{ fast}) \in \{0,1\}.$$

Each $u_{0tj} \ (j \text{ slow start})$ is implemented as a here-and-now decision.
Two-stage stochastic unit commitment

- D-1 12:00
- Samples in high-dimensional scenario space
- Updated information

Locked commitments for slow-start units

Perfect dispatch over day D (since whole day is visible)

0:00-23:55 (i.e. whole day D)
Deterministic unit commitment

$W_{tj}, L_t$ are set to forecasts $W_{0tj}, L_{0t}$.

minimize $\Sigma_{t=1}^{T} \Sigma_{j=1}^{J} c_{tj}^{\text{start}} v_{0tj} + c_{tj} p_{0tj}$

subject to $\Sigma_{j=1}^{J} p_{0tj} = L_{0t}$

$\Sigma_{j=1}^{J^D} (u_{0tj} \bar{P}_j - p_{0tj}) \geq S_{0t}$ reserve requirements

constraints for dispatchable $j \in J^D$:

$v_{0tj} - w_{0tj} = u_{0tj} - u_{0, t-1, j}$

$u_{0tj} \bar{P}_j \leq p_{0tj} \leq u_{0tj} \bar{P}_j$

$-R_j^{\text{down}} \leq p_{0tj} - p_{0, t-1, j} \leq R_j^{\text{up}}$

constraints for variable energy resources $j$ in $J^{\text{VER}}$

$p_{0tj} \leq u_{0tj} W_{0tj}$

$u_{0tj}, v_{0tj}, w_{0tj} \in \{0,1\}$.

Each $u_{0tj}$ ($j$ slow start) is implemented as here-and-now decision.
## Practical complexity of stochastic unit commitment

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1968</td>
<td>Early stochastic mixed-integer linear programming (MILP) model for unit commitment</td>
</tr>
<tr>
<td>1990</td>
<td>Parallel computing for solving stochastic programs</td>
</tr>
<tr>
<td>2006</td>
<td>Convex multistage stochastic programming is intractable (*)</td>
</tr>
<tr>
<td>2010</td>
<td>PJM completes a 6-year effort of deploying and integrating its security-constrained MILP unit commitment</td>
</tr>
</tbody>
</table>

### References


### Abstract idealized setup:

**Dream:** solve the 2-stage MILP model

1st-stage decision \( \downarrow \)

probability of scenario \( k \)

\[
\begin{align*}
\text{SP: } & \min f(x) + \sum_{k=1}^{K} \ p_k \ g(x, y_k, \xi_k) \\
& \text{s.t. } x \in X, \ y_k \in Y(x, \xi_k), \ k=1, \ldots, K.
\end{align*}
\]

2nd-stage decisions \( \uparrow \)

scenario \( k \)

\[
\text{P(}\xi\text{): } \min f(x) + g(x, y, \xi) \\
\text{s.t. } x \in X, \ y \in Y(x, \xi).
\]

**Reality:**

*We have tools to reduce to 1-2% the optimality gap of the MILP*
Best-Deterministic Approximation

• Let $v^*$, $S$ be the optimal value and first-stage solution set of the stochastic program. Let $x^* \in S$.

• Let $v(x)$ be the optimal value of the stochastic program when the first-stage decision is fixed to $x$. We have $v(x^*) = v^*$ for all $x^* \in S$. $v(x)$ can be evaluated by optimizing separately over each scenario.

• Let $S'(\xi)$ be the optimal first-stage solution set of the stochastic program with its probability distribution degenerated to $\xi$. Let $x'(\xi) \in S'(\xi)$.

• Value of the Stochastic Solution [Birge 1982]:
  \[ VSS = v(x'(\bar{\xi})) - v(x^*) \]
  where $\bar{\xi} = \sum_{k=1}^{K} p^k \xi^k$

• Value of the stochastic solution over the best-deterministic solution:
  \[ VSS^{BD} = \inf_{\xi \in \Xi} [v(x'(\xi)) - v(x^*)] \]
  for $\Xi$ : space easy to cover.

• Best-deterministic approximation:
  Try to find $\xi^* \in \text{argmin}_{\xi \in \Xi} v(x'(\bar{\xi}))$ and then implement $x'(\xi^*)$.
Pictorial representation for the VSS-BD

Near-optimal solution to two-stage stochastic MILP

Goal: minimize \( VSS^{BD} \) given search space, cpu time budget.

First-stage solution to deterministic MIP

Solutions to stochastic MILP with fixed first-stage decision (fully separable).
“Best-Deterministic” unit commitment

\( W_{tj}, L_t \) are set to planning forecasts \( \overline{W}_{0tj}, \overline{L}_{0t} \).

minimize \( \sum_{t=1}^{T} \sum_{j=1}^{J} \tilde{c}_{tj} \text{start} v_{0tj} + c_{tj} p_{0tj} \)

subject to \( \sum_{j=1}^{J} p_{0tj} = \overline{L}_{0t} \)

\( \sum_{j=1}^{J^D} (u_{0tj} \overline{P}_j - p_{0tj}) \geq \overline{S}_{0t} \)

constraints for dispatchable \( j \in J^D \):

\( v_{0tj} - w_{0tj} = u_{0tj} - u_{0, t-1, j} \)

\( u_{0tj} \overline{P}_j \leq p_{0tj} \leq u_{0tj} \overline{P}_j \)

\( -R_j^\text{down} \leq p_{0tj} - p_{0, t-1, j} \leq R_j^\text{up} \)

constraints for variable energy resources \( j \) in \( J^{\text{VER}} \)

\( p_{0tj} \leq u_{0tj} \overline{W}_{0tj} \) \text{ planning forecast}

\( u_{0tj}, v_{0tj}, w_{0tj} \in \{0, 1\} \).

Each \( u_{0tj} \) (j slow start) is implemented as here-and-now decision.
VSS-BD for unit commitment (test 1)

day-ahead start

<table>
<thead>
<tr>
<th></th>
<th>Expected Cost</th>
<th>Time [s]</th>
<th>Gap [%]</th>
<th>Loss [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stochastic MIP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>high-accuracy</td>
<td>2.70335e+07</td>
<td>285.93</td>
<td>0.10</td>
<td>0.00</td>
</tr>
<tr>
<td>Stochastic MIP</td>
<td>2.70501e+07</td>
<td>9.11</td>
<td>0.46</td>
<td>0.06</td>
</tr>
<tr>
<td>Middle scenario</td>
<td>2.78027e+07</td>
<td>2.90</td>
<td>0.48</td>
<td>2.85</td>
</tr>
<tr>
<td>Mean scenario</td>
<td>2.71157e+07</td>
<td>1.56</td>
<td>0.46</td>
<td>0.30</td>
</tr>
<tr>
<td>50-quantile</td>
<td>2.77531e+07</td>
<td>0.92</td>
<td>0.41</td>
<td>2.66</td>
</tr>
<tr>
<td><strong>60-quantile</strong></td>
<td>2.70375e+07</td>
<td>0.75</td>
<td>0.48</td>
<td><strong>0.01</strong></td>
</tr>
<tr>
<td>70-quantile</td>
<td>2.73184e+07</td>
<td>0.21</td>
<td>0.33</td>
<td>1.05</td>
</tr>
</tbody>
</table>

5 scenarios $\xi_k$ of net load [MW]

60th-percentile scenario [MW]

code: www.princeton.edu/~defourny/MIP_UC_example.m
VSS-BD for unit commitment (test 2)

Test with transmission constraints.

<table>
<thead>
<tr>
<th></th>
<th>Expected Cost</th>
<th>Time [s]</th>
<th>Gap [%]</th>
<th>Loss [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stochastic MIP</strong></td>
<td>2.00287e+07</td>
<td>19481.00</td>
<td>0.50</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Stochastic MIP</strong></td>
<td>2.00552E+07</td>
<td>1657.00</td>
<td>0.85</td>
<td>0.13</td>
</tr>
<tr>
<td><strong>Stochastic MIP</strong></td>
<td>2.04341e+07</td>
<td>31.67</td>
<td>0.50</td>
<td>2.02</td>
</tr>
<tr>
<td><strong>Stochastic MIP</strong></td>
<td>2.01821e+07</td>
<td>0.84</td>
<td>0.43</td>
<td>0.77</td>
</tr>
<tr>
<td><strong>60-60-60 quantile</strong></td>
<td>2.01821e+07</td>
<td>0.78</td>
<td>0.45</td>
<td>0.77</td>
</tr>
<tr>
<td><strong>60-60-70 quantile</strong></td>
<td>2.04505e+07</td>
<td>0.81</td>
<td>0.43</td>
<td>2.11</td>
</tr>
<tr>
<td><strong>60-70-60 quantile</strong></td>
<td>2.01514e+07</td>
<td>1.79</td>
<td>0.42</td>
<td>0.61</td>
</tr>
<tr>
<td><strong>60-70-70 quantile</strong></td>
<td>2.00866e+07</td>
<td>2.45</td>
<td>0.00</td>
<td>0.30</td>
</tr>
<tr>
<td><strong>60-70-70 quantile</strong></td>
<td>2.01514e+07</td>
<td>1.51</td>
<td>0.47</td>
<td>0.61</td>
</tr>
<tr>
<td><strong>70-70-60 quantile</strong></td>
<td>2.00866e+07</td>
<td>2.15</td>
<td>0.10</td>
<td>0.30</td>
</tr>
<tr>
<td><strong>70-70-70 quantile</strong></td>
<td>2.01514e+07</td>
<td>1.09</td>
<td>0.47</td>
<td>0.61</td>
</tr>
<tr>
<td><strong>70-70-70 quantile</strong></td>
<td>2.06064e+07</td>
<td>3.24</td>
<td>0.48</td>
<td>2.88</td>
</tr>
</tbody>
</table>

code:  www.princeton.edu/~defourny/MIP_UC_3node.m
Guiding the search

Rather than finding a best-deterministic solution by direct search, we could compute a priori a single scenario (by stochastic programming).

- **Stochastic optimization of wind forecasts and reserve requirements**

Optimization of the wind that can be scheduled in day-ahead, along with various reserves for hedging against wind being lower than expected, using a very simplified expression of the costs and constraints.

<table>
<thead>
<tr>
<th>wind energy [MW]</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>cumulative distribution function (cdf) of wind energy</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
</tr>
</tbody>
</table>

- **Curtail excess wind**
  - quantile level $\beta$

- **Call spinning reserve**
  - quantile level $\alpha$

- **Start up fast units**
  - unhedged
  - reserve

B. Defourny (Princeton) The best-deterministic method for the stochastic unit commitment problem DIMACS 2/21/2013 17/28
Optimality of quantile solutions

Let us recall a textbook result:

The newsvendor problem

\[
\text{Max } -c x + \mathbb{E}\{ p \min[x, D]\}
\]

where 0 < c < p, and D is a r.v. with cdf \( G \) (demand)
admits the optimal solution \( x = G^{-1}(\alpha), \:\alpha = (p - c)/p \).

\( x = G^{-1}(\alpha) \) is a quantile of the distribution of \( \xi \).

The same problem can also be written as

\[
\begin{align*}
\text{Min } & \mathbb{E}\{ (c-p) D + c [ x - D ]^+ + (p-c) [ D - x]^+ \}. \\
\text{exogenous} & \text{ overage cost} & \text{underage cost}
\end{align*}
\]
Extension to multiple quantiles

Let \( 0 \leq c_1 < c_2 < c_3 < d_2 < d_1 \). Let \( w \) (wind) be a positive, \( \text{abs. cont. r.v., with cdf } G. \)
Let \( L > 0 \) (fixed load; dedicated reserve assumed to be in place.)

**Proposition:**

The stochastic program

\[
\begin{align*}
\text{minimize} & \quad c_1 x_1 + c_2 x_2 + c_3 x_3 + E\{d_1 y_1 + d_2 y_2\} \\
\text{subject to} & \quad x_1 + x_2 + x_3 = L, \quad x_3 \geq 0 \quad \text{(day-ahead schedule meets load)} \\
& \quad w + y_1 + y_2 \geq x_1 + x_2 \quad \text{a.s.} \quad \text{(compensation of missing wind)} \\
& \quad 0 \leq y_1 \leq x_1, \quad 0 \leq y_2 \leq x_2 \quad \text{a.s.} \quad \text{(consequence of reserve choices)}
\end{align*}
\]

admits an optimal solution based on quantiles as long as \( x_3 \geq 0 \).

\( x_1 + x_2 \) : total wind energy to be “scheduled” day-ahead.
\( x_3 \) : energy from dispatchable units committed in day-ahead (rarely < 0.)
Recursive algorithm

Function \( (x_1, \ldots, x_n) = \text{SOLVE}(c_1, \ldots, c_n, d_1, \ldots, d_{n-1}, L; G) \)

Step 1. Define \( \alpha_i = (c_{i+1} - c_i)/(d_i - d_{i+1}) \), \( i = 1, \ldots, n-1 \),
where \( d_n = 0 \).

Step 2. If \( J = \{ i : \alpha_i < \alpha_{i-1} \} \) is empty, go to Step 3.
Otherwise: select \( j = \inf J \).

Set \( x_j = 0 \).

Set \((x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n) = \text{SOLVE}(c_1, \ldots, c_{j-1}, c_{j+1}, \ldots, c_n, d_1, \ldots, d_{j-1}, d_{j+1}, \ldots, d_{n-1}, L; G)\).

Return \((x_1, \ldots, x_n)\).

Step 3. Set \( x_1 = G^{-1}(\alpha_1) \), \( x_i = G^{-1}(\alpha_i) - G^{-1}(\alpha_{i-1}) \),
\( x_n = L - (x_1 + \ldots + x_{n-1}) \).

Return \((x_1, \ldots, x_n)\).

Shows that the optimal solution is formed of zeros and differences of quantiles.
Quantile levels as a function of wind speed mean and standard deviation

optimized quantile levels of scheduled wind

Power curve of Vestas V90
Learning algorithm

1. Start with some parameters for setting the wind forecast \( \bar{Y}_1, \ldots, \bar{Y}_T \).
2. Solve the UC problem given the forecast.
3. Given simulations of forecast errors and adjustment costs, estimate average overage & underage costs \( C_1^+, \ldots, C_T^+; \ C_1^-, \ldots, C_T^- \).
4. Update the parameters and go back to Step 2.

Forecast parameter update

\[ q_t = \frac{c_t^-}{c_t^+ + c_t^-} \]

If forecasting too much wind is relatively expensive, the quantile level will decrease.

\[ y_t = F_t^{-1}(q_t) \]
Goal: explaining the successive quantile levels by other processes, such as the load. Let $X_t$ be that process. Let $\bar{X}_t$ be its forecast.

Justification: the cost of adjustments is influenced by the state of the grid (load, congestions, ...)

- quantile level
  \[ q_t = \frac{c_t^-}{c_t^+ + c_t^-} \]

- quantile level function
  \[ \rho(\bar{X}_t) \approx q_t \]

- regression model
  \[ \rho(\bar{X}_t) = \frac{1}{1+e^{-(\alpha + \beta \cdot \bar{X}_t)}} \]

- forecast parameters
  \[ \alpha, \beta \]

- wind power

- time

\[ y_t = F_t^{-1}[\rho(\bar{X}_t)] \]
Numerical test: Stochastic processes

1. Sample N times uniformly in [0,T]

2. Use the N values of the function with uniform time increments

3. Add “vertical” noise

Processes with random time shifts and random magnitude shifts
Direct quantile search

Sample paths over 24 hours

- Load
- Net load (100% wind)
- Wind

Time (hours)

marginal cost

$/\text{MWh}

0 1 2 3 4 5 6

Total generation

Load

schedule

more wind

Day-ahead prices

× 10 GW

Expected cost for some fixed quantile $q$ at each hour

Optimum @ $q=0.4$

empirical distributions from sample paths

$E_{\text{cost}}(q)$

36720

$\sigma: 8.1$

36412

$\sigma: 6.7$

36412

$\sigma: 6.7$

$q$

$q=0.4$
Learned time-varying quantiles

Wind forecast @ iter 10

Wind forecast @ iter 10
Summary of the talk

- Value of the stochastic solution over the best-deterministic solution.
- Best-deterministic approximation presented as a particular algorithmic approach to two-stage stochastic unit commitment.
- Search space based on quantiles: the motivation is that quantile solutions can be optimal for wind and reserve scheduling without capacity constraints.

codes:  
www.princeton.edu/~defourny/MIP_UC_example.m  
www.princeton.edu/~defourny/MIP_UC_3nodes.m

Thank you!

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