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Sonar Placement in Ports and Waterways
Protect against terrorist attacks

- Divers
- AUV’s
- Hull mounted objects
Environment

- Infeasibility of electromagnetic sensors
- A type of sensor called SONAR (SOund NAvisation and Ranging) is used
- Sonars work based on sound waves
Sonar’s advantage over radar

- Electromagnetic waves get stuck in sea water
- Sound waves can travel in sea water even for tens of miles
Sonar Types

- **Active**

- **Passive**
A Risk Minimization problem, with integer (binary) decision variables

With:
- Multiple coverage
- Detection probability reduces by distance from the sonar
- Various properties of sonars
- Different sonar types

Are considered in the model.

Discretization
Characteristic Values
Assessing $a_{ij}$ Values

```
0 0 1 1 0 1
3 3 3 5 4 3
2 2 5 6 3 2
4 3 3 4 3 2
3 3 4 4 1 2
```
A Simple Model!
Notation

\[ a_{ij} = \text{characteristic value of cell } i, j \]

\[ p = \text{detection probability of a sonar} \]

\[ c = \text{budget for placing sonars} \]

\[ n = \text{number of cells a sonar can cover} \]

\[ NC_{ij} = \text{set of neighboring cells of } i, j \text{ that a sonar positioned at cell } i, j \text{ can cover including } i, j \text{ itself} \]

\[ x_{ij} = \begin{cases} 
1 & \text{if a sonar is placed at cell } i, j \\
0 & \text{otherwise} 
\end{cases} \]

\[ y_{ij} = \begin{cases} 
1 & \text{if cell } i, j \text{ is covered by a sonar} \\
0 & \text{otherwise} 
\end{cases} \]
Formulation

\[ \text{Min} \sum_i \sum_j a_{ij} (1 - p \cdot y_{ij}) \]

\[ n \cdot x_{ij} \leq \sum_{k,l \in NC_{ij}} y_{kl} \quad (1) \]

\[ y_{ij} \leq \sum_{k,l \in NC_{ij}} x_{kl} \quad (2) \]

\[ c \sum_i \sum_j x_{ij} \leq b \quad (3) \]

\[ x_{ij}, y_{ij} \in \{0, 1\} \]
Explanation

\[ \text{Min } \sum_{i} \sum_{j} a_{ij} (1 - p_{ij}) \]

This objective function minimizes a risk-like measure according to cell coverage and also the importance of cells (\(a_{ij}\) values)

\[ R = E[C] = E[C | \text{Successful Attack}] \cdot P(\text{Successful Attack}) = \\
E[C | \text{Successful Attack}] \cdot P(\text{Successful Attack} | \text{Attack Happens}) \cdot P(\text{Attack Happens}) = C \cdot V \cdot T \]
Main Model

- Featuring
  - Multiple detection of sonars
  - Range dependent detection probability
  - Various types of sonars
Optimization Model

\[
\begin{align*}
\text{Min } & \sum_i \sum_j a_{ij} \{1 - [((1-t_{ij}) \cdot \sum_n dp_n \cdot y_{ijn}) + t_{ij} \cdot dp_{\text{max}}]\} \\
\text{St:} & \quad d_{mn} \cdot x_{ijm} \leq \sum_{(k,l)\in N_{ijmn}} y_{klm} \quad \forall i, j, m, n \\
& \quad y_{ijn} \leq \sum_m \sum_{(k,l)\in N_{ijmn}} x_{klm} \quad \forall i, j, n \\
& \quad \sum_i \sum_j \sum_m c_m \cdot x_{ijm} \leq b \\
& \quad \sum_n y_{ijn} - 1 \leq M \cdot t_{ij} \quad \forall i, j \\
& \quad M (1-t_{ij}) + \sum_n y_{ijn} \geq 2 \quad \forall i, j \\
x_{ijm}, y_{ijn}, t_{ij} \in \{0, 1\}
\end{align*}
\]

Placement Constraints
Cost Constraint
Multiple Coverage Constraints
Decision Variables
Decision Variables

\[ x_{ijm} = \begin{cases} 
1 & \text{if a sonar of type } m \text{ is placed in cell } (i, j) \\
0 & \text{otherwise} 
\end{cases} \]

\[ y_{ijn} = \begin{cases} 
1 & \text{if cell } (i, j) \text{ is covered by coverage type } n \\
0 & \text{otherwise} 
\end{cases} \]

\[ t_{ij} = \begin{cases} 
1 & \text{if cell } (i, j) \text{ is covered by more than one sonar} \\
0 & \text{otherwise} 
\end{cases} \]
Test Case
New York Harbor
(as an example)
Terminals
Grid
Defining Criticality level of Cells
A Sonar Placement Scheme
A Sonar Placement Scheme
Thank you!