Strategic Analysis for Prediction Markets

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Joint work with
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Prediction Market Games

- Agent has
  - Beliefs (about uncertain proposition, other agents)
  - Preferences (for money, other outcomes)
  - Opportunity to trade in a prediction market

- Defines a game
  - Severely incomplete and imperfect info, dynamic revelation, huge strategy space, …

- What will or should it do?
Empirical Evidence

- Observations
  - Data from prediction markets “in the field”: IEM, historical PE markets, FX, HSX, NewsFutures, Costa Rican bookies, …
- Experimentation
  - Controlled laboratory settings: CalTech, PSU, HP, Humboldt, Frankfurt, Maastricht, GMU…

Theory as a Guide

- Assume (e.g.): common prior, common knowledge of rationality
- Q: How should a rational agent trade on asymmetric information?
- A: Don’t (Milgrom & Stokey, 1982)
- To get trading (in theory), need some irrationality, reasons beyond info asymmetry, subsidies, …
Irrational Model #1

- Agents ignore information signal in prices
  - Also competitive (price takers)
  - E.g.: Manski, Wolfers & Zitzewitz, Pennock & Wellman
- Price aggregation depends on form of utility function
  - GLU: (weighted) arithmetic mean (LinOP)
  - CARA: (weighted) geometric mean (LogOP)

Learning from prices

$\text{Pr}_1(E_1), \text{Pr}_1(E_2), \ldots$

$\text{Pr}_2(E_1), \text{Pr}_2(E_2), \ldots$

Supra Bayesian
Learning from prices

\[ \text{Pr}_1(E_1), \text{Pr}_1(E_2), \ldots \]

Supra Bayesian

Bernoulli trials model

\[ \frac{s}{n} \text{ successes in } n \text{ trials} \]

\[ \frac{s'}{n'} \text{ successes in } n' \text{ trials} \]

\[ \text{Pr}(E|p^{<E>}) = w\text{Pr}(E) + (1-w)p^{<E>} \]

where \( w = \frac{n}{n+n'} \)

"Market"
Equilibrium with Learning

- Weighted average update
  \[ \Pr(E|p^{<E>}) = w\Pr(E) + (1-w)p^{<E>} \]
  and agents with GLU
  \( \Rightarrow \) still LinOP prices
  confidence-based wts

- Geometric average update
  \[ \Pr(E|p^{<E>}) \propto \Pr(E)^{w}(p^{<E>})^{(1-w)} \]
  and agents with CARA
  \( \Rightarrow \) still LogOP prices

Market Adaptation

- Single security
- Multiperiod market
- Agents with GLU
- Fixed beliefs

\begin{itemize}
  \item\hspace{1cm} Beta(1,1)
  \item\hspace{1cm} Beta(1,2)
  \item\hspace{1cm} Beta(2,2)
  \item\hspace{1cm} Beta(3,2)
  \item\hspace{1cm} Beta(3,3)
  \item\hspace{1cm} Beta(4,3)
  \item\hspace{1cm} Beta(6,6)
  \item\hspace{1cm} Beta(11,11)
  \item\hspace{1cm} Beta(18,14)
  \item\hspace{1cm} Beta(25,17)
  \item\hspace{1cm} Beta(31,21)
\end{itemize}
Agent-Based Simulations

- Tell us what happens when agents follow particular policy in particular environment
- Still does not tell us what agents will or should do
- Sometimes augmented with evolutionary dynamics to search for stable populations

Alternative: Empirical Game-Theoretic Analysis

- Explore restricted set of strategies
- Estimate empirical game from simulated profiles
- Analyze using standard game-theoretic concepts and tools
Example: Simultaneous Ascending Auctions (SAAs)

- Ubiquitous
  - Explicit design: FCC, others…
  - Implicit due to independence of operation
- Large gap in strategic understanding
- Simulation studies using portfolio of old and new strategy proposals

Example Payoff Matrix (2 strategies)
TAC/SCM Three-Strategy Game

Profile Space

4.2 million
### Experiments on U(5,5)

- Evaluated 4916 strategy profiles
- 200K–200M samples each (avg. 10M)

**Key Result**
- Profile with all 5 PP($F^{SC}$) is pure-strategy NE
- Payoff = 4.51
- Can verify by examining only 53 profiles (this one plus 52 deviations).

### Other Equilibria?

- Not among 4915 other pure profiles tested
  - Can rule out all at $\epsilon > 0.13$
  - Can rule out 4913 at $\epsilon > 0.25$

- Not within *cliques*
  - **Clique**: strategy set for which we evaluated all profile combinations
  - PP($F^{SC}$): only strategy surviving iterated elimination of dominated strategies in clique subgames

- Not among 46 pairs
Searching for Walverine…

- Michigan’s TAC Travel-Shopping agent
- Parametrized strategy space
  - Flight delay parameters
  - Entertainment trading policy
  - Hotel bid shading…
- Restrict attention to a discrete set of $S$ strategies (parameter settings).

Profile Space

49 million

$\binom{N+S-1}{N}$
Reduced Games

- Let each “player” control two TAC agents
- Transformed to 4-player game
  - Less fidelity
  - More tractable
  - \((S = 31, \text{ only } 46,376 \text{ profiles})\)
- 2-player: 496 profiles
- 1-player: 31 profiles

Why Trust Reduced-Game Results?

- Claim: Equilibria in reduced game likely to be relatively stable in full game
- Evidence:
  - Random instances of local-effect games
  - 2-strategy
  - 8-player

![Bar chart showing comparison between 1-player, 2-player, 4-player, and global opt.](chart.png)
Searching $N$-Player TAC Classic ($S=31$)

<table>
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<th>$N$</th>
<th>Profiles</th>
<th>Explored</th>
<th>Expl %</th>
<th>samples/profile</th>
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<td>4</td>
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<td>1452</td>
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<td>354</td>
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<td>98.5</td>
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</table>

Analyzing (Partial) Reduced Games

- $N=1$ (31 profiles)
  - Identified unique pure-strategy NE (PSNE)
- $N=2$ (344)
  - “Confirmed” 1 PSNE, refuted 340 ($\varepsilon > 10$)
  - 3 confirmed eq. mixture pairs
  - Refuted 304 candidate mixture pairs ($\varepsilon > 10$); 292 ($\varepsilon > 20$)
- $N=4$ (1429)
  - Refuted 1423 candidate PSNE ($\varepsilon > 10$); 1421 ($\varepsilon > 20$)
  - Est. 114 candidate mixture pairs
    - Confirmed 1 ($\varepsilon < 1$)
    - refuted 99 ($\varepsilon > 10$); 83 ($\varepsilon > 20$)
Conclusions

- Prediction markets pose interesting open problems in trading strategy
- Empirical game analysis
  - Bridges simulation and game theory
  - Can provide conclusions given partial data
  - Provides insights about SAA, TAC games
- Any bets on what it might show for prediction markets?