HYBRID ARQ IN WIRELESS NETWORKS

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AUTOMATIC REPEAT REQUEST

• The receiving end detects frame errors and requests retransmissions.

• $P_e$ is the frame error rate, the average number of transmissions is

$$1 \cdot (1 - P_e) + \cdots + n \cdot P_e^{n-1}(1 - P_e) + \cdots = \frac{1}{1 - P_e}$$
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• Hybrid ARQ uses a code that can correct some frame errors.
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• Hybrid ARQ uses a code that can correct some frame errors.

• In HARQ schemes
  – the average number of transmissions is reduced, but
  – each transmission carries redundant information.
THROUGHPUT IN HYBRID ARQ
BPSK, AWGN, BCH Coded

$E_s/N_0$ [dB]
THROUGHPUT IN HYBRID ARQ
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![Graph showing throughput versus $E_s/N_0$ in dB for uncoded and BCH coded transmissions.]
THROUGHPUT IN HYBRID ARQ
BPSK, AWGN, BCH Coded

![Graph showing throughput vs. E_s/N_0 in dB for different codes.](image)
THROUGHPUT IN HYBRID ARQ
BPSK, AWGN, BCH Coded

\[ E_s/N_0 \text{ [dB]} \]
THROUGHPUT IN HYBRID ARQ
BPSK, AWGN, BCH Coded

![Graph showing throughput as a function of $E_s/N_0$ [dB]. The graph includes curves for different BCH codes: [255,247,3], [255,215,11], [255,177,23], and the uncoded case. The cutoff rate is also shown.]
THROUGHPUT IN HYBRID ARQ
BPSK, AWGN, BCH Coded

![Graph showing throughput in hybrid ARQ with BPSK, AWGN, and BCH-coded signals.](image)
TYPE II HYBRID ARQ
Incremental Redundancy

• Information bits are encoded by a (low rate) mother code.

• Information and a selected number of parity bits are transmitted.
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Incremental Redundancy

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• Information and a selected number of parity bits are transmitted.

• If a retransmission is not successful:
  – transmitter sends additional selected parity bits
  – receiver puts together the new bits and those previously received.

• Each retransmission produces a codeword of a stronger code.

• Family of codes obtained by puncturing of the mother code.
INCREMENTAL REDUNDANCY

A Rate 1/5 Mother Code

... at the transmitter
INCREMENTAL REDUNDANCY

A Rate 1/5 Mother Code

at the transmitter

transmission # 1

at the receiver
INCREMENTAL REDUNDANCY
A Rate $1/5$ Mother Code

at the transmitter
transmission # 1
transmission # 2

at the receiver
INCREMENTAL REDUNDANCY

A Rate 1/5 Mother Code

at the transmitter

transmission # 1

transmission # 2

transmission # 3

at the receiver
INCREMENTAL REDUNDANCY
A Rate 1/5 Mother Code

at the transmitter

transmission # 1

transmission # 2

transmission # 3

transmission # 4

at the receiver
THROUGHPUT IN HYBRID ARQ

HARQ Scheme based on Turbo codes in AWGN Channel

- Throughput of new puncturing scheme
- Throughput of standard
- BPSK Capacity
- Cutoff Rate

Throughput vs. Es/N0 (dB) graph showing the performance of different ARQ schemes.
RANDOMLY PUNCTURED CODES

• The mother code is an \((n, k)\) rate \(R\) turbo code.

• Each bit is punctured independently with probability \(\lambda\).
Randomly punctured codes

- The mother code is an \((n, k)\) rate \(R\) turbo code.
- Each bit is punctured independently with probability \(\lambda\).
- The expected rate of the punctured code is \(R/(1 - \lambda)\).
- For large \(n\) we have

\[
\begin{array}{c}
\text{TURBO CODE} \\
\text{PUNCTURING DEVICE} \\
\end{array}
\]

- \(k\) bits \(\rightarrow\) \(n\) bits \(\rightarrow\) \((1 - \lambda)n\) bits
A FAMILY OF RANDOMLY PUNCTURED CODES

Rate Compatible Puncturing

- The mother code is an \((n, k)\) rate \(R\) turbo code.

- \(\lambda_j\) for \(j = 1, 2, \ldots, m\) are puncturing rates, \(\lambda_j > \lambda_k\) for \(j < k\).

- If the \(i\)-th bit is punctured in the \(k\)-th code and \(j < k\), then it was punctured in the \(j\)-th code.
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- If the \(i\)-th bit is punctured in the \(k\)-th code and \(j < k\), then it was punctured in the \(j\)-th code.

- \(\theta_i\) for \(i = 1, 2, \ldots, n\) are uniformly distributed over \([0, 1]\).

- If \(\theta_i < \lambda_l\), then the \(i\)-th bit is punctured in the \(l\)-th code.
MEMORYLESS CHANNEL MODEL

- Binary input alphabet \( \{0, 1\} \) and output alphabet \( \mathcal{Y} \).

- Constant in time with transition probabilities \( W(b|0) \) and \( W(b|1) \), \( b \in \mathcal{Y} \).
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• Binary input alphabet \(\{0, 1\}\) and output alphabet \(\mathcal{Y}\).

• Constant in time with transition probabilities \(W(b|0)\) and \(W(b|1)\), \(b \in \mathcal{Y}\).

• Time varying with transition probabilities at time \(i\) \(W_i(b|0)\) and \(W_i(b|1)\), \(b \in \mathcal{Y}\).

• \(W_i(\cdot|0)\) and \(W_i(\cdot|1)\) are known at the receiver.
PERFORMANCE MEASURE

Time Invariant Channel

• Sequence $x \in C \subseteq \{0, 1\}^n$ is transmitted, and $x'$ decoded.

• Sequences $x$ and $x'$ are at Hamming distance $d$. 
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• Sequences $x$ and $x'$ are at Hamming distance $d$.

• The probability of error $P_e(x, x')$ can be bounded as

$$P_e(x, x') \leq \gamma^d = \exp\{-d\alpha\},$$

where $\gamma$ is the Bhattacharyya noise parameter:

$$\gamma = \sum_{b \in \mathcal{Y}} \sqrt{W(b|x = 0)W(b|x = 1)}$$
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where \( \gamma \) is the Bhattacharyya noise parameter:

\[
\gamma = \sum_{b \in \mathcal{Y}} \sqrt{W(b|x = 0)W(b|x = 1)}
\]

and \( \alpha = -\log \gamma \) is the Bhattacharyya distance.
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- The union-Bhattacharyya bound on word error probability:

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P^C_W \leq \sum_{d=1}^{n} A_d e^{-\alpha d}.
\]
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- Weight distribution \(A_d\) for a turbo code?
PERFORMANCE MEASURE

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• The union-Bhattacharyya bound on word error probability:

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P_{W}^{C} \leq \sum_{d=1}^{n} A_d e^{-\alpha d}.
\]

• Weight distribution \(A_d\) for a turbo code?

• Consider a set of codes \([C]\) corresponding to all interleavers.

• Use the average \(\overline{A}[C](n)\) instead of \(A_d\) for large \(n\).
There is an ensemble distance parameter $c_0^C$ s.t. for large $n$

$$A_d^{[C]}(n) \leq \exp(d c_0^{[C]})$$

for large enough $d$. 
Theorem by Jin and McEliece

- There is an ensemble distance parameter $c_0^C$ s.t. for large $n$

\[ A_d^{[C]}(n) \leq \exp(d c_0^C) \] for large enough $d$.

- For a channel whose Bhattacharyya distance $\alpha > c_0^C$, we have

\[ P_W^{[C]}(n) = O(n^{-\beta}). \]

- $c_0^C$ is the ensemble noise threshold.
PUncTURED TuRBO CODE ENSEMBLES
ITW, April 2003

- $c_0^{[CP]}$ is the punctured ensemble noise threshold:

$$A_d^{[CP]}(n) \leq \exp(dc_0^{[CP]})$$

for large enough $n$ and $d$. 
$c_0^{[C_P]}$ is the punctured ensemble noise threshold:

$$A_d^{[C_P]}(n) \leq \exp(d c_0^{[C_P]}) \text{ for large enough } n \text{ and } d.$$ 

- If $\log \lambda < -c_0^{[C]}$, 

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\]

• If $\log \lambda < -c_0^{[C]}$,

\[
c_0^{[C_P]} \leq \log \left[ \frac{1 - \lambda}{\exp (-c_0^{[C]}) - \lambda} \right].
\]
PUNCTURED TURBO CODE ENSEMBLES
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HARQ MODEL

- There are at most $m$ transmissions.
- $\mathcal{I} = \{1, \ldots, n\}$ is the set indexing the bit positions in a codeword.
- $\mathcal{I}$ is partitioned in $m$ subsets $\mathcal{I}(j)$, for $1 \leq j \leq m$.
- Bits at positions in $\mathcal{I}(j)$ are transmitted during $j$-th transmission.
HARQ MODEL

- There are at most \( m \) transmissions.

- \( \mathcal{I} = \{1, \ldots, n\} \) is the set indexing the bit positions in a codeword.

- \( \mathcal{I} \) is partitioned in \( m \) subsets \( \mathcal{I}(j) \), for \( 1 \leq j \leq m \).

- Bits at positions in \( \mathcal{I}(j) \) are transmitted during \( j \)-th transmission.

- The channel remains constant during a single transmission:

\[
\gamma_i = \gamma(j) \quad \text{for all } i \in \mathcal{I}(j).
\]
PERFORMANCE MEASURE

Time Varying Channel

- Let $W^n(y|x) = \prod_{i=1}^{n} W_i(y_i|x_i)$. 
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Time Varying Channel

• Let $W^n(y|x) = \prod_{i=1}^{n} W_i(y_i|x_i)$.

• Sequence $x \in C \subseteq \{0, 1\}^n$ is transmitted, and $x'$ decoded.

• The probability of error $P_e(x, x')$ can be bounded as

\[
P_e(x, x') \leq \sum_{y \in Y^n} \sqrt{W^n(y|x)W^n(y|x')}
= \prod_{i=1}^{n} \left( \sum_{b \in Y} \sqrt{W_i(b|x_i)W_i(b|x'_i)} \right)
\leq \prod_{i: x_i \neq x'_i} \gamma_i
\]
HARQ PERFORMANCE

- $d_j$ is the Hamming distance between $x$ and $x'$ over $I(j)$.

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\[ P_e(x, x') \leq \prod_{j=1}^{m} \gamma(j)^{d_j} \]
HARQ PERFORMANCE

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• The probability of error $P_e(x, x')$ can be bounded as

$$P_e(x, x') \leq \prod_{j=1}^{m} \gamma(j)^{d_j}$$

• $A_{d_1...d_m}$ is the number of codewords with weight $d_j$ over $I(j)$.

• The union bound on the ML decoder word error probability:

$$P \leq \sum_{d_1=1}^{|I(1)|} \cdots \sum_{d_m=1}^{|I(m)|} A_{d_1...d_m} \prod_{j=1}^{m} \gamma(j)^{d_j}$$
HARQ PERFORMANCE

Random Transmission Assignment

• A bit is assigned to transmission $j$ with probability $\alpha_j$.

• $d$ is the weight of the original codeword.

• $d_j$ is the weight of the $d$-th transmission sub-word.

• The probability that the sub-word weights are $d_1, d_2 \ldots, d_m$ is

$$
\binom{d}{d_1} \binom{d-d_1}{d_2} \cdots \binom{d-d_1 \cdots - d_{m-1}}{d_m} \alpha_1^{d_1} \alpha_2^{d_2} \cdots \alpha_m^{d_m}
$$
HARQ PERFORMANCE
Random Transmission Assignment

- The union bound on the ML decoder word error probability:

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P \leq \sum_{d_1=1}^{\lvert I(1) \rvert} \cdots \sum_{d_m=1}^{\lvert I(m) \rvert} A_{d_1 \ldots d_m} \prod_{j=1}^{m} \gamma(j)^{d_j}
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\]

• The expected value of the union bound is

\[
\sum_{d} A_d \left( \sum_{j=1}^{m} \gamma(j) \alpha_j \right)^h.
\]

• The average Bhattacharyya noise parameter:

\[
\bar{\gamma} = \sum_{j=1}^{m} \gamma(j) \alpha_j
\]
A RANDOMLY PUNCTURED TURBO CODE
An Example of Random Transmission Assignment

- The puncturing probability is $\lambda$.
- Transmission over the channel with noise parameter $\gamma$. 
A RANDOMLY PUNCTURED TURBO CODE
An Example of Random Transmission Assignment

• The **puncturing** probability is \( \lambda \).

• Transmission over the **channel** with noise parameter \( \gamma \).

• Equivalent to having **two transmissions**:
  – first with assignment probability \( (1 - \lambda) \) and noise parameter \( \gamma \);
  – second with assignment probability \( \lambda \) and noise parameter \( 1 \).
A RANDOMLY PUNCTURED TURBO CODE
An Example of Random Transmission Assignment

- The puncturing probability is $\lambda$.
- Transmission over the channel with noise parameter $\gamma$.
- Equivalent to having two transmissions:
  - first with assignment probability $(1 - \lambda)$ and noise parameter $\gamma$;
  - second with assignment probability $\lambda$ and noise parameter $1$.
- The average noise parameter is $\bar{\gamma} = (1 - \lambda)\gamma + \lambda$.
- Requirement $- \log \bar{\gamma} > c_0^{[C]}$
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- The puncturing probability is $\lambda$.
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- Equivalent to having two transmissions:
  - first with assignment probability $(1 - \lambda)$ and noise parameter $\gamma$;
  - second with assignment probability $\lambda$ and noise parameter $1$.
- The average noise parameter is $\overline{\gamma} = (1 - \lambda)\gamma + \lambda$.
- Requirement $-\log \overline{\gamma} > c_0^{[c]}$ translates into

\[-\log \gamma > \log \left[ \frac{1 - \lambda}{\exp (-c_0^{[c]}) - \lambda} \right].\]
INCREMENTAL REDUNDANCY

Concluding Remarks

at the transmitter
INCREMENTAL REDUNDANCY

Concluding Remarks

at the transmitter

transmission # 1

at the receiver
INCREMENTAL REDUNDANCY

Concluding Remarks

at the transmitter

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transmission # 2

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INCREMENTAL REDUNDANCY

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at the transmitter

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transmission # 2

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at the receiver
INCREMENTAL REDUNDANCY

Concluding Remarks

- Transmission #1 at the transmitter
- Transmission #2 at the receiver
- Transmission #3 at the receiver
- Transmission #4 at the receiver