The Erdős-Rényi Phase Transition
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In their great 1960 paper On the Evolution of Random Graphs Paul Erdős and Alfred Rényi expresses a special interest in the behavior of the random graph $G(n, p)$ when $p$ was near $n^{-1}$. Today we view it through the prism of Percolation Theory. If $p = cn^{-1}$ and $c < 1$ the process is subcritical and all components are small and simple. But for $c > 1$ the process is supercritical and a complex giant component has emerged. We now understand the fine structure: the critical window is parametrized $p = n^{-1} + \lambda n^{-4/3}$, with $\lambda \rightarrow -\infty$ and $\lambda \rightarrow +\infty$ represently the barely subcritical and barely supercritical phases. We discuss the behaviors and the arguments, making particular use of the similarities to the Galton-Watson Birth Process when the expected number of children is near one.