

DIMACS Center
Rutgers University

**Special Focus on Mathematics and the Foundations of
Computer and Information Sciences**

Final Report

November 2003

Ia. Participants from the program

Participants:

Special Focus Organizers:

Michael Fredman, Rutgers University Computer Science
Janos Komlos, Rutgers University Mathematics
Fred Roberts, Rutgers University Mathematics

Workshop Organizers:

Ingrid Daubechies, Princeton University
Jonathan Farley, Massachusetts Institute of Technology
Michael Fredman, Rutgers University
Richard Gundy, Rutgers University
Melvin Janowitz, DIMACS / Rutgers University
Janos Komlos, Rutgers University
Jimmie Lawson, Louisiana State Univeristy
Joel Lebowitz, Rutgers University
Michael Mislove, Tulane University
Ricky Pollack, New York University
Jean-Francois Roy, University of Rennes
Micha Sharir, Tel Aviv University
Amin Shokrollahi, Digital Fountain
Dan Spielman, MIT
Wim Sweldens, Bell Labs
Ruediger Urbanke, EPFL
Guido Weiss, Washington University

Visitors:

Enrico Presutti, University of Roma Tor Vergata
11/29/00-12/22/00

Graduate Students:

Amr Elmasry, Rutgers University
Winter 2001/2002
David Nacin, Rutgers University
Summer 2002
Amy Stern, Rutgers University
Summer 2002

Ib. Participating Organizations

Telcordia Technologies: Facilities; Personnel Exchanges
Partner organization of DIMACS. Individuals from the organization participated in the program planning and workshops.

AT&T Labs - Research: Facilities; Personnel Exchanges
Partner organization of DIMACS. Individuals from the organization participated in the program planning and workshops.

NEC Laboratories America: Facilities; Personnel Exchanges

Partner organization of DIMACS. Individuals from the organization participated in the program planning and workshops.

Lucent Technologies, Bell Labs: Facilities; Personnel Exchanges

Partner organization of DIMACS. Individuals from the organization participated in the program planning and workshops.

Princeton University: Facilities; Personnel Exchanges

Partner organization of DIMACS. Individuals from the organization participated in the program planning and workshops.

Avaya Labs: Facilities; Personnel Exchanges

Partner organization of DIMACS. Individuals from the organization participated in the program planning and workshops.

HP Labs: Facilities; Personnel Exchanges

Partner organization of DIMACS. Individuals from the organization participated in the program planning and workshops.

IBM Research: Facilities; Personnel Exchanges

Partner organization of DIMACS. Individuals from the organization participated in the program planning and workshops.

Microsoft Research: Facilities; Personnel Exchanges

Partner organization of DIMACS. Individuals from the organization participated in the program planning and workshops.

Office of Naval Research: Facilities; Personnel Exchanges

ONR partially funded one of the workshops.

1c. Other Collaborators

The project involved scientists from numerous institutions in numerous countries. There were hundreds of attendees at our workshops, coming from a variety of types of institutions and disciplines. The resulting collaborations also involved individuals from many institutions in many countries.

II. Project Activities

The rapidly changing world of information, communications, and computation is a stimulus for the development of new models and new concepts lying at the foundation of computer and information science and their development will require ever-closer connections with many methods of mathematics. We investigated the connections between foundational issues in the computer and information sciences and different branches of mathematics in a series of five “working” workshops involving both mathematicians and computer scientists. We de-emphasized formal talks and emphasized discussion time and opportunities to explore new potential collaborations between computer scientists and

mathematicians. We followed up the workshops with opportunities for participants to meet at DIMACS to pursue the research directions identified.

During the first 30 to 40 years of the computer age, the science of computation and communication has been profoundly and broadly influenced by the models, conceptual framework, and methods developed within theoretical computer science (TCS). For example, the theory of languages and automata led to the development of high level programming languages and compilers; the design and implementation of large databases was fundamentally based on theoretical models; the notion of polynomial time tractability which developed within the field of computational complexity provided a fundamental criterion for algorithmic efficiency that is now universally applied; the field of algorithm design and analysis yielded a large library of practical algorithms, as well as basic principles for attacking new computational problems; the study of private secure communication within computational complexity revolutionized the classical field of cryptography, broadening its scope enormously to complex multi-party transactions, and providing fundamental new methods for implementing such transactions.

TCS is inherently a highly mathematical discipline. Its basic philosophy and methodology -- formulating and analyzing precise models of computational systems that abstract key aspects of their behavior -- has been fundamentally influenced by the methodology of applied mathematics. TCS draws most heavily on discrete mathematics, probability, and logic, but it has growing connections throughout mathematics. The field of TCS initially grew out of logic and the basic notions of computability and complexity. Discrete mathematics soon became a central tool in the design and analysis of algorithms, as well as in the formulation and analysis of general computational models. Probabilistic methods have become increasingly important both to algorithm design, where randomized algorithms and probabilistic analysis have become essential tools, and complexity theory, where the probabilistic viewpoint is fundamental to our understanding of computational models.

Computational geometry has become a major subfield of TCS, drawing heavily on both discrete mathematics and classical geometry. Number theory has played a key role in the design of cryptographic protocols. Computational algebra, algebraic topology, Fourier analysis, and geometry play an increasingly important role in proving lower bounds on computational complexity, as well as in the design of algorithms. Coding theory, long influenced by deep mathematical methods, particularly from algebra and algebraic geometry, presents methods for the robust representation of information and has combined with methods of computational information theory to achieve codes that are close to theoretical limits. The role of these and other areas of mathematics in the development of computer and information science, and in particular in theoretical computer science, was the major theme of this special focus.

At the same time that TCS is increasingly influenced by the methods and results of mathematics, the algorithmic methods of TCS are increasingly important in the analysis of fundamental problems of mathematics. For example, the discovery of algorithms that permit the rapid processing of vectors, polynomials, matrices, and other central mathematical concepts can be a tool in the solution of important mathematical questions. Exploring this direction of the connection between mathematics and TCS was a second theme of this special focus.

The major organized events were a series of workshops and a working group.

Workshop: Algorithmic and Quantitative Aspects of Real Algebraic Geometry in Mathematics and Computer Science

Dates: March 12 - 16, 2001

Location: DIMACS Center, CoRE Building, Rutgers University

Organizers: Ricky Pollack, New York University; Jean-Francois Roy, University of Rennes; Micha Sharir, Tel Aviv University

Attendance: 48

At this workshop we brought together researchers in algorithmic and quantitative aspects of real algebraic geometry and other mathematicians and computer scientists who are concerned with related problems in the hope of promoting more productive interaction between them.

Some topics in algorithmic or quantitative real algebraic geometry are listed below. They give a flavor of the issues we addressed at the workshop. There were 10 invited one hour lectures to survey and introduce these topics. There was also 21 contributed shorter talks by other participants as well as ample time for discussion.

- Interactions between real algebraic geometry and computational geometry.
- Efficient recent algorithms in real algebraic geometry and their implementation.
- Quantitative aspects of basic semi-algebraic sets.
- Lower bounds and real algebraic geometry.
- Quantitative aspects of Pfaffian sets.
- Combinatorial characterization real algebraic varieties.
- Combinatorial patchworking for constructing real algebraic curves and hypersurfaces.
- Real aspects of enumerative geometry.
- Realization problems in geometry.

Some very valuable outcomes of our workshop were:

- 1) An Oberwolfach workshop on positive polynomials, February 2002.
- 2) A refereed proceedings from the conference, edited by S. Basu and L. Gonzalez-Vega, has been published by the American Mathematical Society as part of the DIMACS-AMS Volume Series. See Section on Papers/Books/Internet.
- 3) The publication of the book "Algorithms in Real Algebraic Geometry" by Basu-Pollack-Roy, November, 2003.
- 4) Recent exciting papers by many of the participants. Among these are:
 - "Betti numbers for quantifier-free formulae" by Gabrielov-Vorobjov
 - "Topology of definable Hausdorff limits" by T. Zell
 - "On the Betti Numbers of Sign Conditions" by Basu-Pollack-Roy
 - "Computing the Euler-Poincare Characteristic of Sign Conditions" by Basu-Pollack-Roy

Workshop: Analysis with Wavelets, Signals and Geometry

Dates: April 4 - 6, 2001

Location: DIMACS Center, CoRE Building, Rutgers University

Organizers: Richard Gundy, Rutgers University; Ingrid Daubechies, Princeton University; Wim Sweldens, Bell Labs; Guido Weiss, Washington University

Attendance: 37

This workshop explored new directions in the fields of signal processing and wavelet analysis. It brought together mathematicians, computer scientists, and engineers for an interdisciplinary exchange of research and problems in new areas of time-scale analysis.

Special attention was devoted to digital geometry processing. This refers to a new collection of techniques for processing digital data taken from surfaces, or manifolds where the geometry is not flat. For example, data obtained from PET scanners is used to obtain a three-dimensional reconstruction of internal organs. The curvature of the "signal source" presents new quantization problems, different from those that arise in processing sound, images, or video. In each of the latter categories, the data set is taken from a flat section of Euclidean space. For sound, the space is two dimensional, the signal being represented as a function of time; for images, intensity(gray level) is plotted as a function of position in the plane and for video, a third time dimension is added. In each of these cases, the data is adapted to sampling at regular intervals and pixels. Regular sampling schemes are adapted to discrete Fourier analysis, or cascade algorithms using mirror filters. If the data arise from sources with curvature, especially non-constant curvature, new "quadrature" problems arise. These problems have been attacked by second generation wavelet techniques, and these were one of the subjects of the workshop.

Wavelet analysis is usually understood as harmonic analysis where the emphasis is on time-scale rather than time- frequency. The workshop brought together some mathematicians working on extensions of the wavelet transform arising from groups other than representations of the affine group. The notion of a "coherent state transform" has unified the decomposition/reconstruction formulas for the windowed Fourier transform and the continuous wavelet transform. The coherent state transform has recently been studied for other groups, some exotic, and others, like the integers, not so exotic. These results were discussed as part of the workshop. Analysis with wavelets has many facets. One of the participants discussed multifractals, wavelets, and models for internet traffic. Probabilistic methods in wavelet analysis, minimal characterization of wavelets are two additional topics that were discussed. The talks were scheduled to encourage maximum participation and exchange. There were research presentations, tutorial-survey talks, and organized panel discussions.

Workshop: Codes and Complexity

Dates: December 4 - 7, 2001

Location: DIMACS Center, CoRE Building, Rutgers University

Organizers: Amin Shokrollahi, Digital Fountain; Dan Spielman, MIT; Ruediger Urbanke, EPFL

Attendance: 90

Ever since Shannon's paper on information theory more than 50 years ago, the construction of codes which have efficient encoders and decoders with performance arbitrarily close to Shannon's bound has been the supreme goal of coding research. The last few years have witnessed tremendous progress towards achieving this goal. One of the most intriguing aspects of recent developments has been a cross fertilization of ideas in coding and information theory, theoretical computer science, and physics. Especially the connection to theoretical computer science ultimately led to an asymptotic analysis of many classes of codes based on graphs; conversely, coding theory has turned out to be an indispensable tool in many recent exciting developments in theoretical computer science, such as the design of probabilistically checkable proofs, or pseudo random generators.

The goal of this workshop was to bring together researchers in coding and information theory, theoretical computer science, and physics in the hope of further stimulating cross-collaboration. The workshop was preceded by a tutorial on low-density parity-check codes intended to bring graduate students and other interested researchers with little or no previous background up to speed in this important area of research. There were several invited 45 minute talks on various topics that elucidate connections of coding theory to the above mentioned areas. There were also a number of contributed talks.

Working Group Meeting: Computational Complexity, Entropy, and Statistical Physics

Dates: December 12 - 13, 2001

Location: DIMACS Center, CoRE Building, Rutgers University

Organizers: Michael Fredman, Janos Komlos and Joel Lebowitz, Rutgers University

Attendance: 33

Workshop: Computational Complexity, Entropy, and Statistical Physics

Dates: December 14, 2001

Location: DIMACS Center, CoRE Building, Rutgers University

Organizers: Michael Fredman, Janos Komlos and Joel Lebowitz, Rutgers University

Attendance: 82

A working group is an interdisciplinary group of researchers that meets to discuss a research area, has informal presentations, and defines research issues for further discussion and collaboration. The working group met just prior to a larger more formal workshop.

The working group and workshop explored some underlying connections between biology, computational complexity, discrete mathematics, dynamical systems and statistical physics. All these disciplines use in one way or another something called entropy, a word first introduced by Rudolf Clausius in 1865 when he enunciated his famous two laws:

1. The energy of the universe stays constant
2. The entropy of the universe always increases.

But just what is entropy? It is frequently said that entropy is a measure of disorder, and while this needs many qualifications and clarifications it does represent something essential about it. By comparing the uses of entropy in these very different contexts we were able to gain new insights into some universal aspects common to all of them.

The public workshop featured a variety of connections between entropy and other fields, including computer science. Jennifer Chayes, Microsoft Research, talked about the role of entropy in statistical mechanics, the field where this concept originated. Yuval Peres, UC Berkeley, described how this concept has come to play a role in discrete mathematics. David Zuckerman, University of Texas, Austin, talked about computational complexity and entropy. Yasha Sinai, Princeton University, related entropy to dynamical systems. Though we usually think of biology as studying systems with increasing organization, John Hopfield, Princeton University, explained how entropy also plays an important role in that field.

Workshop: Applications of Lattices and Ordered Sets to Computer Science

Dates: July 8 - 10, 2003

Location: DIMACS Center, CoRE Building, Rutgers University

Organizers: Jonathan Farley, Massachusetts Institute of Technology; Melvin Janowitz, DIMACS / Rutgers University, Jimmie Lawson, Louisiana State University; Michael Mislove, Tulane University

Attendance: 43

Lattices and ordered sets play an important role in many areas of computer science. These range from lattices as models for logics, which are fundamental to understanding computation, to the ordered sets as models for computation, to the role both lattices and ordered sets play in combinatorics, a fundamental aspect of computation. In addition, many applications utilize lattices and ordered sets in fundamental ways. These include such areas as knowledge representation, text categorization and data mining, where

order plays a fundamental organizing principle, to the use of lattices and ordered sets to analyze crypto-protocols in security, to inductive logic programming, where ordered sets form basic models. A number of more esoteric structures, such as Kleene algebras and quantales, also feature in recent advances to understanding computation. This workshop brought researchers from the many areas that rely on lattices and ordered sets together with those doing research in these and related structures, in order to accomplish two goals:

- 1) to survey the many areas of computation where lattices and ordered sets play a role in order to better understand the problems common across these areas, and
- 2) to enhance the interactions between researchers in the areas of lattice theory and ordered sets, and those who utilize these structures in modeling computation and in areas of application.

Researchers in lattice theory and ordered sets benefited by discovering new and interesting problems where their expertise can be applied, and those who work on applications had a chance to learn of new advances in these areas that could be applied to problems where lattice theory and ordered sets traditionally have played an important role. In addition to these goals, the intention also was to present talks which are accessible to the broader public that will help those working in related areas to understand the role lattice theory and ordered sets play in theoretical computation and its applications.

The workshop featured several plenary lectures, whose goal was to give a broad background to the workshop. These lectures were complemented by several shorter talks that focused on particular areas of application, as well as on recent advances in lattice theory and ordered sets. The talks by leaders in lattice theory and ordered sets focused on applications to computer science, while the talks by leading researchers in these areas described how techniques from lattice theory and ordered sets have recently led to results and applications to problems in computation.

III. Project Findings

Summary of some of the research results reported to us by special focus participants

Results on Betti Numbers:

Thierry Zell proved the following result: If A is a family of uniformly bounded compact fibers definable in an \mathcal{O} -optimal expansion of the real field and L is a Hausdorff limit of that family, the rank of each singular homology group of L (Betti numbers) can be estimated in terms of the Betti numbers of simple definable subsets of the Cartesian product of generic fibers.

Saugata Basu, Richard Pollack and Marie-Francois Roy improved the upper bound on the sum of the i -th Betti numbers over certain sign conditions. They also found, using similar methods, a more precise bound for the sum of the Betti numbers of the intersection of Z with a closed semi-algebraic set, defined by a quantifier-free Boolean formula without negations.

Computing various topological invariants of semi-algebraic sets in single exponential time is an active area of research. Several algorithms are known for deciding emptiness, computing the number of connected components of semi-algebraic sets in single exponential time etc. Saugata Basu, Richard Pollack and Marie-Francois Roy developed a new, improved algorithm for computing the Euler-Poincare characteristic (which is the alternating sum of the Betti numbers) of the realization of each realizable sign condition of a family of polynomials restricted to a real variety. The Euler-Poincare characteristic of a

finite set of points is just the cardinality of the set, and the algorithm can be viewed as a direct generalization of the algorithm for counting the real roots of a univariate polynomial satisfying different sign conditions on another family of polynomials. A consequence of their result is that the Euler-Poincare characteristic of any locally closed semi-algebraic set can be computed with the same complexity. The best complexity of any previously known single exponential time algorithm for computing the Euler-Poincare characteristic of semi-algebraic sets worked only for a more restricted class of closed semi-algebraic sets and had a higher complexity.

In important particular cases of semialgebraic and semi-Pfaffian sets defined by qualifier-free formulae with polynomials and Pfaffian functions respectively, upper bounds on Betti numbers of X_V are well known. A. Gabrielov, N. Vorobjov and T. Zell extended the bounds to sets defined with quantifiers, in particular to sub-Pfaffian sets.

A. Gabrielov, N. Vorobjov proved a bound for an arbitrary semialgebraic set defined by an arbitrary Boolean formula.

Results on Kleene Algebra:

Jules Desharnais, Bernhard Moller and Georg Struth proposed Kleene algebra with domain (KAD), an extension of Kleene algebra with two equational axioms for a domain and a codomain operation, respectively. KAD considerably augments the expressiveness of Kleene algebra, in particular for the specification and analysis of state transition systems.

Thorsten Ehm, Bernhard Moller and Georg Struth proposed axioms for Kleene modules (KM). These structures have a Kleene algebra K and a Boolean algebra B as sorts. They arise as algebraic abstractions of relational image and preimage operations. KM is the basis of algebraic variants of dynamic logics. They developed a calculus for KM and its relation to Kleene algebra with domain and to dynamic and test algebras. They applied KM to the reachability analysis in directed graphs.

Bernhard Moller and Georg Struth provided an algebraic background for the formal derivation of greedy-like algorithms. Such derivations have previously been done in various frameworks including relation algebra. They proposed Kleene algebra as a particularly simple alternative. Instead of converse and residuation. They use modal operators that are definable in a wide class of algebras, based on domain/codomain or image/preimage operations. By abstracting from earlier approaches, they arrived at a very general theorem about the correctness of loops that covers particular forms of greedy algorithms as special cases.

Bernhard Moller and Georg Struth enriched Kleene algebra by domain and codomain operators. These abstractions of relational notions give rise to four modal operators. The boxes and diamonds enjoy various symmetries via Galois connections and dualities. Lifting modal statements to modal operator semirings yields a further abstraction and thus a more elegant and concise “statefree” reasoning about modalities.

They used this modal Kleene algebra for calculating soundness and completeness proofs for propositional Hoare logic. While their soundness proof is more direct than related ones, their algebraic completeness proof seems entirely novel. It uses a modal symmetry that relates the wlp predicate transformer with partial correctness assertions and that is beyond the expressibility of formalisms like propositional dynamic logic.

Results on Wavelets:

A reproducing system is two countable families so that the first “analyzes” a function and the second “reconstructs” h.

A variety of such systems have been used successfully in both pure and applied mathematics. They have the following feature in common: they are generated by a single or a finite collection of functions by applying to the generators two countable families of operators that consist of two of the following three actions: dilations, modulations, and translations. The **Gabor Systems**, for example, involve a countable collection of modulations and translations; the **affine systems** (that produce a variety of wavelets) involve translations and dilations.

Considerable amount of research has been conducted in order to characterize those generators of such systems. Eugenio Hernandez, Demetrio Labate and Guido Weiss established a result that “unifies” all of these characterizations by means of a relatively simple system of equalities. One of the novelties here is the use of a different approach that provides a considerably more general class of such reproducing systems; for example, in the affine case, we need not restrict the dilation matrices to ones that preserve the integer lattice and are expanding on \mathbb{R}^n . Another novelty is a detailed analysis, in the case of affine and quasi-affine systems, of the characterizing equations for different kinds of dilation matrices.

Eugenio Hernandez, Demetrio Labate and Guido Weiss obtained a characterization of certain types of reproducing systems. Eugenio Hernandez, Demetrio Labate, Guido Weiss and Edward Wilson applied these results and methods to various affine-like, wave packets and Gabor systems to determine their frame properties. In particular, they study how oversampled systems inherit properties (like the frame bounds) of the original systems. Moreover, their approach allows them to study the phenomenon of oversampling in much greater generality than is found in the literature.

Philip Gressman developed the theory of wavelets on the integers. For this, one needs to first find analogs of translations and dyadic dilations which appear in the classical theory. Translations are defined in the obvious way, taking advantage of the additive group structure of the integers. Dyadic dilations, on the other hand, pose a greater problem. In the classical theory of wavelets on the real line, translation T and dyadic dilation D obey the “commutativity” relations $DT^2 = TD$. Gressman chose to define dyadic dilations on the integers in terms of this functional equation. All such dyadic dilations are characterized and the corresponding multiresolution structures they generate are introduced and examined. He connected multiresolution structures and wavelets on the integers with their counterparts in the line and showed that every wavelet on the integers is an MRA wavelet.

Results on Automated Deduction Systems:

Georg Struth proposed an algebraic calculus for set-based program development. First, he constructed a fragment of set theory via atomic distributive lattices (ADL). Semantically, ADL extends boolean reasoning about sets by element-wise reasoning; it avoids presupposing a universal set. Operationally, ADL yields abstract, concise, elegant proofs from few elementary principles. Second, he developed a focused automated proof-search procedure for ADL with simple deduction and effective reduction and simplification rules. Proof-search is guided by rewriting techniques. The procedure decides several subclasses. The main application is the proof-support for formal methods like B or Z.

Results on Algorithms:

Khaled Elbassioni proved that the problem of extending a given partial order of maximal independent elements of a subset of a product of lattices can be solved in polynomial time.

An n -node forest of trees is called a square-root forest if it has the following structure. For a given positive integer k the forest has $2k$ trees. The first $k+1$ are single nodes. For the other $k-1$ trees, the root of tree r has r single-node children, for all r from 1 to $k-1$. Given a forest of T trees, the rank of a tree is defined to be the number of children of the root of this tree. A phase of operations is defined as first linking the trees in pairs, then replacing the tree with the largest rank with its sub-trees together with a new single-node tree. Amr Elmasry proved combinatorially, that the forest will converge to the square-root forest and bounded the complexity. He also bounded the amortized cost of his pairing strategy.

Amr Elmsary generalized the standard implementation of the pairing heaps. He introduced a new parameter k . When the node with the minimum value is to be deleted from the heap, the operations to combine the resulting sun-trees into one tree depend on the value of k . When the value of k is equal to 2, the implementation will be equivalent to the standard pairing heaps' implementation. He showed that, for any constant k , this general form achieves the same bounds as the standard implementation. He conducted experimental results showing that, by tuning the value of k , the number of comparisons involved in this operation can be reduced.

Amr Elmasry developed Binomialsort, an adaptive sorting algorithm that is optimal with respect to the number of inversions. The bound on the number of comparisons is further reduced by using a new structure called trimonial queues. Experimental results show that their sorting algorithm is practical, efficient and easy to implement.

Amr Elmasry proved that the pairing heap benefits from the presortedness in the input sequence, and sorts adaptively. In particular, he showed that given an input sequence of size n that is already sorted in ascending order from right to left, repeatedly deleting the smallest element, using the pairing heap operation, requires at most $7n$ comparisons. He also showed that starting with any initial heap structure that has the property that the corresponding sequence of the heap is sorted in ascending order, at most $7n$ comparisons are required to produce the sorted sequence by repeatedly deleting the smallest element from the heap. This latter result implies an easy proof of Tarjan's sequential access theorem for splay trees, with a better bound of $3.5n$ rotations. He obtained experimental results supporting the fact that the pairing heap is adaptive by comparing sorting using the pairing heap with Splaysort and Binomialsort.

Amy Stern, graduate student, reports the following research results that were stimulated by the special focus:

For a finite partially ordered set P , let $p(x,y)$ be the probability that $x > y$ in a uniformly chosen linear extension of P , and let $\delta(P)$ be the largest value of δ so that there is some pair x, y in P for which $\delta \leq p(x,y) \leq (1 - \delta)$. In 1969, S.S. Kislitsyn conjectured that if P is not a chain, then $\delta(P) \geq 1/3$. This conjecture, which arises from sorting, is called "The 1/3-2/3 Conjecture." Kahn and Saks (1984) conjectured that if the width of a poset P approaches infinity, then $\delta(P)$ approaches $1/2$.

Jeff Kahn and Amy Stern tried to show that if the width of a poset P approaches infinity, then $\delta(P) \geq 1/e - o(1)$. In the process, they learned things about relations between limiting behaviors of various old and new poset parameters, such as width, variances associated with a uniform linear extension, gaps in the average height sequence, and properties of the order polytope.

New collaborations and research directions

The main goal of the project was to simulate new collaborations and research discussions. The following are examples of such outcomes, many of them interdisciplinary.

As a result of the Workshop on Applications of Lattices and Ordered Sets to Computer Science, Jimmie Lawson had some interesting conversations with Keye Martin regarding his work on natural partial orders occurring in quantum states in physics. These pertained to the possibility of defining partial orders on Lie algebras and possible relationships with Poisson brackets. These have connections to his work, some joint with Coecke, another participant, on quantum states, entropy, and also quantum search methodology (a connection with computer science).

Jimmie Lawson also had conversations with George Markowsky on idempotent analysis, a new branch of mathematics that uses order and lattice theoretic ideas to attack certain types of discrete and continuous optimization problems, both theoretically and computationally.

As a result of the Workshop on Applications of Lattices and Ordered Sets to Computer Science, Michael Huth, Cathy Meadows and Michael Mislove are planning to look at the relevance of Michael's work, which he reported on at the workshop, to the area of security. Cathy is an expert in the area, and after hearing Michael's talk, she suggested they look at possible applications of his work.

As a result of the Workshop on Applications of Lattices and Ordered Sets to Computer Science, Lance Miller, Tim Hannan and Alex Pogel are going to submit a paper for the ICCS (See <http://concept.cs.uah.edu/>) based on further developments of the lattice drawing material and the use of weight functions (for improved representation of data) that were presented during the lattice workshop.

Alex Pogel's work with James Abello developed a lot during that conference, and led to their meetings in August at DIMACS when he proved a number of proposition/theorems on bipartite graphs and concept lattices.

As a result of the Workshop on Applications of Lattices and Ordered Sets to Computer Science, Michael Huth met Glenn Bruns and Patrice Godefroid for the first time. They have exchanged research ideas on multiple-valued model checking that are likely to result a more formal research collaboration next year. Michael Huth also had an exchange with Mike Mislove and Catherine Meadows on how to use the lattice/partial-order ideas of his DIMACS talk in the context of authentication in communities with dynamic topologies. He has read up on the area and should find time to work with them on this matter in Spring 2004.

As a result of the Workshop on Applications of Lattices and Ordered Sets to Computer Science, Ben Worrell, Dusko Pavlovic and Michael Mislove submitted a paper to FOSSACS, one of the component meetings that make up the annual ETAPS series of meetings. The work itself is on the Stone-Gelfand-Naimark duality theory for real C^* -algebras, which gives a duality between labelled Markov processes and tests.

As a result of the Workshop on Applications of Lattices and Ordered Sets to Computer Science, Marc Denecker, Victor Marek and Mirek Truszczyński are working on a research monograph related to topics they presented at the workshop.

In the Workshop on Analysis with Wavelets, Signals and Geometry, an undergraduate (at that time) from Washington University, Phil Gressman, talked about "Wavelets on the integers." His paper on this subject appeared in *Collectanea Mathematica*: "Wavelets on the Integers," *Collect. Math.* 52, 3 (2002), 257-288.

As a result of the Workshop on Analysis with Wavelets, Signals and Geometry, Albert Cohen reports that since that meeting, he has consolidated his collaboration with R.Baraniuk (one of the participants of the wavelet meeting here), of Rice Univ. In particular, he invited one of his students, Justin Romberg, to Paris

where they have worked on geometric multiresolution analyses. They have some new results and an article in preparation.

As a following to the Workshop on Computational Complexity, Entropy and Statistical Physics, Enrico Presutti, visitor to DIMACS, reports that he has studied in collaboration with Joel Lebowitz and Eugene Speer the structure of the measures which describe stationary states outside of thermodynamic equilibrium. They have considered stochastic systems of interacting particles in contact with thermal reservoirs at different temperatures. They have found that the local Gibbsian assumption is not compatible with the properties exhibited by the systems we have examined.

As a result of the Workshop: Algorithmic and Quantitative Aspects of Real Algebraic Geometry in Mathematics and Computer Science, Frank Oles was invited to give talks at both Case Western Reserve University and Cornell University. At Case, he has hopes of future collaboration with G.Q. Zhang. Dexter Kozen, with whom Oles had interesting discussions, hosted his visit to Cornell. Moreover, at Cornell, he made the acquaintance of Claire Cardie, who is well known in computational linguistics (the area in which he anticipated the most significant and immediate applications of the theory on which he spoke at the workshop), and he hopes to invite her to IBM to give a talk. Also, at Cornell he met with Thorsten Joachims, a new faculty member there with an excellent reputation in machine learning, and they discussed Thorsten's interest in extracting rich structures from data using machine-learning techniques. At both institutions he had discussions with a number of graduate students in addition to various faculty members.

As a result of the Workshop on Applications of Lattices and Ordered Sets to Computer Science, Ales Pultr has started cooperation with Mick Adams; the topics will be Kleene algebras and some aspects of Priestley duality.

As a result of the Workshop: Algorithmic and Quantitative Aspects of Real Algebraic Geometry in Mathematics and Computer Science, a collaboration link has been established between Frank Oles and G.Q. Zhang. G.Q. Zhang was invited to the Dagstuhl-Seminar on Spatial Representation: Discrete vs. Continuous Computational Models, Germany, August, 2004.

As a result of the Workshop on Applications of Lattices and Ordered Sets to Computer Science, Mick Adams will be going to Prague to collaborate with Ales Pultr there. Mick is optimistic that there will be some concrete mathematical outcome at that point. Mick is sure that the time that he will spend with Ales will be stimulating mathematically.

As a result of the Workshop on Applications of Lattices and Ordered Sets to Computer Science, Roland Backhouse was an invited speaker at AMAST, 2004.

As a result of the Workshop: Algorithmic and Quantitative Aspects of Real Algebraic Geometry in Mathematics and Computer Science, G.Q. Zhang was invited to the International Workshop on Formal Methods and Security, Nanjing, China, May, 2004 and the 19th Conference on Topology and its Applications, special session on Topology and Computer Science, Cape Town, South Africa, July, 2004.

IV. Project Training/Development

Three Rutgers graduate students, Amy Stern, Amr Elmasry, and David Nacin worked on research projects related to this special focus.

Amy Stern, Mathematics, Rutgers University, worked on the limiting behaviors of various old and new poset parameters, such as width, variances associated with a uniform linear extension, gaps in the average height sequence, and properties of the order polytope.

Amr Elmasry, Computer Science, Rutgers University, proved combinatorially, that a forest of trees will converge to the square-root forest under a certain phase operation. He also generalized the standard implementation of the pairing heaps. He developed Binomialsort, an adaptive sorting algorithm that is optimal with respect to the number of inversions. He proved that the pairing heap benefits from the presortedness in the input sequence, and sorts adaptively. Amr produced four DIMACS Technical Reports describing his research results that stemmed from this special focus.

David Nacin, Computer Science, Rutgers University also participated in this project during the Summer 2002, working on Non-commutative Combinatorics.

In the Workshop on Analysis with Wavelets, Signals and Geometry, an undergraduate (at that time) from Washington University, Phillip Gressman, talked about "Wavelets on the integers." His paper on this subject appeared in *Collectanea Mathematica*: "Wavelets on the Integers," *Collect. Math.* 52, 3 (2002), 257-288. See the section on Project Findings for a further discussion of Phillip's work.

All of the above research is described in more detail in the section on Project Findings.

V. Outreach Activities

Special Focus visitors, graduate students, and senior faculty were available to interact with 2- and 4-year college faculty in the DIMACS "Reconnect" program and with high school teachers in the DIMACS Connect Institute.

VI. Papers/Books/Internet

Algorithmic and Quantitative Real Algebraic Geometry (Sangata Basu and Laureano Gonzalez-Vega, eds.), Workshop Proceedings, AMS-DIMACS Series, Volume 60, 2003.

On the Betti Numbers of Sign Conditions, (S. Basu, R. Pollack, M-F. Roy), submitted 2002.

Algorithms in Real Algebraic Geometry (S. Basu, R. Pollack, M-F. Roy), Springer-Verlag, Berlin and Heidelberg, September 2003.

Computing the Euler-Poincare Characteristic of Sign Conditions, (S. Basu, R. Pollack, M-F. Roy), submitted.

Large Deviation of the Density Profile in the Symmetric Simple Exclusion Process (B. Derrida, J. L. Lebowitz and E. R. Speer), DIMACS Technical Report Series, 2001.
<http://dimacs.rutgers.edu/TechnicalReports/2001/2001-31.ps.gz>

Kleene algebra with domain (J. Desharnais, B. Möller, G. Struth), Institut für Informatik, Universität Augsburg, Report Nr. 2003-07.

Revised version under <http://arxiv.org/abs/cs.LO/0310054>

Pointer Kleene algebra (T. Ehm), in: R. Berghammer, B. Möller (eds.): *Proc. 7th Seminar Relational Methods in Computer Science and 2nd International Workshop on Applications of Kleene Algebra*, Malente, Springer, to appear, 2004.

Kleene modules (T. Ehm, B. Möller, G. Struth), Institut für Informatik, Universität Augsburg, Report Nr. 2003-10. Revised version in: In: R. Berghammer, B. Möller (eds.): *Proc. 7th Seminar Relational Methods in Computer Science and 2nd International Workshop on Applications of Kleene Algebra*, Malente, Springer, to appear, 2004.

An Algorithm for Dualization in Products of Lattices, (K.M. Elbassioni), DIMACS Technical Report Series, 2002.

<http://dimacs.rutgers.edu/TechnicalReports/2002/2002-26.ps.gz>

Generalized Pairing Heaps (A. Elmasry), DIMACS Technical Report Series, 2001.

<http://dimacs.rutgers.edu/TechnicalReports/2001/2001-23.ps.gz>

Optimal Adaptive Sorting (A. Elmasry), DIMACS Technical Report Series, 2001.

<http://dimacs.rutgers.edu/TechnicalReports/2001/2001-24.ps.gz>

Adaptive Properties of Pairing Heaps (A. Elmasry), DIMACS Technical Report Series, 2001.

<http://dimacs.rutgers.edu/TechnicalReports/2001/2001-29.ps.gz>

Scrambled pairing and square-root trees (A. Elmasry), DIMACS Technical Report Series, 2001.

<http://dimacs.rutgers.edu/TechnicalReports/2001/2001-30.ps.gz>

Betti numbers for quantifier-free formulae, (A. Gabrielov and N. Vorobjov), preprint.

Betti numbers of semialgebraic and sub-Pfaffian sets (A. Gabrielov, N. Vorobjov, T. Zell), *J. London Math. Soc.*, to appear, 2003.

Wavelets on the integers (P. Gressman), *Collectanea Mathematica*, **52** (2002), 257-288.

A Unified Characterization of Reproducing Systems Generated by a Finite Family, (E. Hernandez, D. Labate and G. Weiss), *J. of Geom. Analysis*, **12** (2002), 615-662.

Oversampling, Quasi Affine Frames and Wave Packets (E. Hernandez, D. Labate, E.N. Wilson and G. Weiss), *Appl. and Computational Harmonic Analysis*, to appear, 2003.

Wavelets with composite dilations (E. Hernandez, D. Labate, E.N. Wilson and G. Weiss), AMS On-line Research Announcements, to appear.

Greedy-like algorithms in Kleene algebra (B. Möller, G. Struth), Institut für Informatik, Universität Augsburg, Report Nr. 2003-11. Revised version in: R. Berghammer, B. Möller (eds.): *Proc. 7th Seminar Relational Methods in Computer Science and 2nd International Workshop on Applications of Kleene Algebra*, Malente, Springer, to appear, 2004.

Modal Kleene Algebra and Partial Correctness (B. Möller, G. Struth), Institut für Informatik, Universität Augsburg, Report Nr. 2003-8.

Formal concept analysis for web-menu design, (J. Staiger and G.Q. Zhang), submitted.

A calculus for set-based program development (G. Struth), in J.S. Dong, J. Woodcock (eds.): *Formal Methods and Software Engineering: 5th International Conference on Formal Engineering Methods*, Springer, 2003, 541-559.

A calculus for set-based program development I: Mathematical foundations (G. Struth), Technical Report 2003-15, Institut für Informatik; Universität Augsburg, 2003.

A calculus for set-based program development II: Proof search (G. Struth), Technical Report 2003-16, Institut für Informatik; Universität Augsburg, 2003.

Topology of definable Hausdorff limits, (T. Zell), e-Print archive, 2003.

Formal concepts and information systems, (G.Q. Zhang and G. Shen), *Journal of Applied Categorical Structures*, submitted.

VII. Other Products

http://dimacs.rutgers.edu/SpecialYears/2000_MathFound/
Special Focus on Mathematics and the Foundations of Computer and Information Science main web site

<http://dimacs.rutgers.edu/Workshops/Algorithmic/>
Workshop on Algorithmic and Quantitative Aspects of Real Algebraic Geometry in Mathematics and Computer Science

<http://dimacs.rutgers.edu/Workshops/Wavelet/>
Workshop on Analysis with Wavelets, Signals and Geometry

<http://dimacs.rutgers.edu/Workshops/Codescomplexity/>
Workshop on Codes and Complexity

http://dimacs.rutgers.edu/SpecialYears/2000_MathFound/Entropy/
Working Group on Computational Complexity, Entropy and Statistical Physics
Workshop on Computational Complexity, Entropy, and Statistical Physics

<http://dimacs.rutgers.edu/Workshops/Lattices/>
Workshop on Applications of Lattices and Ordered Sets to Computer Science

<http://dimacs.rutgers.edu/Workshops/Codescomplexity/zuckerman-slides.ps>
Codes in Theoretical Computer Science, David Zuckerman, University of Texas

<http://dimacs.rutgers.edu/Workshops/Codescomplexity/weighttalk.ps.gz>
Typical Set Decoding and the Magnetization Enumerator in LDPC - A Statistical Physics View, David Saad, NCRG

http://dimacs.rutgers.edu/Workshops/Codescomplexity/junan_zhang.ps
Finite Length analysis of LDPC codes with large left degrees, Junan Zhang, UCSD

VIII. Contributions within Discipline

We investigated the connections between foundational issues in the computer and information sciences and different branches of mathematics. We established new collaborations between computer scientists and mathematicians.

Theoretical computer science (TCS) is inherently a highly mathematical discipline. Its basic philosophy and methodology -- formulating and analyzing precise models of computational systems that abstract key aspects of their behavior -- has been fundamentally influenced by the methodology of applied mathematics. The role of mathematics in the development of computer and information science, and in particular theoretical computer science, was the major theme of this special focus. At the same time, the algorithmic methods of TCS are increasingly important in the analysis of fundamental problems of mathematics. Exploring this direction of the connection between mathematics and TCS was a second theme of this special focus.

The summary of project findings that resulted from this special focus are just a sample of the research results that we ultimately expect because of the many, many collaborations that were established and that we expect to flourish and be productive for years.

Many of the workshop participants realized that their workshop should be the first meeting of many as they developed their new collaborations. For example, as a result of the Workshop: Algorithmic and Quantitative Aspects of Real Algebraic Geometry in Mathematics and Computer Science, an Oberwolfach Workshop on positive polynomials followed in February 2002.

IX. Contributions -- other Disciplines

This was an inherently interdisciplinary project. Connections between mathematics, computer science, physics, biology, and other disciplines were brought to light. New collaborations between physicists, mathematicians, and computer scientists resulted from the Workshop on Computational Complexity, Entropy, and Statistical Physics; between computer scientists, physicists, and mathematicians from the Workshop on Applications of Lattices and Ordered Sets to Computer Science together with the Workshop on Algorithmic and Quantitative Aspects of Real algebraic Geometry in Mathematics and Computer Science; between statisticians, mathematicians, computer scientists, and industrial researchers from the Workshop on Analysis with Wavelets, Signals and Geometry; between computer scientists, physicists and industrial researchers from the Workshop on Codes and Complexity.

Alex Pogel's return visit to DIMACS included work on an application of lattice concepts to the analysis of data from epidemiology.

X. Contributions -- Human Resource Development

There were three graduate students involved in the project and one undergraduate student. This is described in detail in the section on Project Training/Development.

XI. Contributions to Resources for Research and Education

The PI gave talks on the research in the project to undergraduates at the DIMACS REU program in July 2002 and July 2003. Three of the undergraduate students in our REU program worked on projects closely related to and stimulated by this project. The students, affiliations, and projects were:

Prudence Heck, Rutgers University, Random Matrices

Alexander Olshevsky, Georgia Tech, Expander Codes and Their Applications

Paul Gross, Rose-Hulman Institute of Technology, Extracting a Sequential Algorithm for Solving the Planarity Problem

XII. Contributions Beyond Science and Engineering

As a result of the Workshop on Applications of Lattices and Ordered Sets to Computer Science, Michael Huth, Cathy Meadows and Michael Mislove are planning to look at the relevance of Michael's work, which he reported on at the workshop, to the area of security. Cathy is an expert in the area, and after hearing Michael's talk, she suggested they look at possible applications of his work.