DIMACS Center
Rutgers University

DIMACS Research Experiences for Undergraduates (REU) Program

Report

January 2004

Ia. Participants from the program

Participants:

DIMACS REU Domestic Program:

Jakub Černý, Charles University, Prague, Czech Republic
Sabyasachi Guharay, Princeton University Princeton, NJ
Steven Jaslar, Rutgers University, New Brunswick, NJ
Vít Jelínek, Charles University, Prague, Czech Republic
Jan Kára, Charles University, Prague, Czech Republic
Adam Kirsch, Brown University, Providence, RI
David McCandlish, Swarthmore College, Swarthmore, PA
Aleš Prívetivý, Charles University, Prague, Czech Republic
Christopher Ross, Rutgers University, New Brunswick, NJ
Sterling Stein, Illinois Institute of Technology, Chicago, IL
Ida Švejdarová, Charles University, Prague, Czech Republic
Tomáš Valla, Charles University, Prague, Czech Republic

DIMACS REU Prague Program:

Michael Grabchak, Rutgers University, New Brunswick, NJ
Vishal Gupta, Yale University, New Haven, CT
Victor Kostyuk, Rochester Institute of Technology, Rochester, NY
Alexander Olshevsky, Georgia Tech, Atlanta, GA
Tatiana Yarmola, University of California at Berkeley, Berkeley, CA

Department of Mathematics REU Program:

Prudence Heck, Rutgers University, New Brunswick, NJ
Matthew Kohut, Rutgers University, New Brunswick, NJ
Michael Lesnick, Brown University, Providence, RI
Bernard Mares, Brown University, Providence, RI
John McClain, Rutgers University, New Brunswick, NJ
Alexandra Ovetsky, Princeton University Princeton, NJ
Kevin Phillips, Rutgers University, New Brunswick, NJ
Rupert Venzke, University of Pittsburgh, Pittsburgh, PA
Minh-Tri Vo, Rutgers University, New Brunswick, NJ

Organizers:

Fred Roberts, Director, DIMACS, Rutgers University
József Beck, Dept. of Mathematics, Rutgers University
Jaroslav Nesetril, Director, DIMATIA, Charles University, Prague, Czech Republic
José Torres, Graduate Student Assistant, Rutgers University
Robert Šámal, Graduate Student Assistant, Charles University

Seminar Speakers:

Minh-Tri Vo, Math Student, Rutgers University
Charles Weibel, Director of the Mathematics Graduate Program, Rutgers University
Farid Alizadeh, Professor of Operations Research, Rutgers University
Eric Allender, Professor of Computer Science, Rutgers University
Vadim Lozin, Professor of Operations Research, Rutgers University.
Pavel Valtr, Professor of Mathematics, Charles University

Martin Farach-Colton, Professor of Computer Science, Rutgers University
József Beck, Professor of Mathematics, Rutgers University
Alexander Barg, Associate Director, DIMACS
Siddhartha Sahi, Professor of Mathematics, Rutgers University
Michael Kiessling, Professor of Mathematics, Rutgers University
Ales Pultr, Professor of Applied Mathematics, Charles University
Alexander Kelmans, Professor of Mathematics, University of Puerto Rico
Robert E. Jamison, Professor of Mathematical Sciences, Clemson University
John Gimbel, Professor of Mathematical Sciences, University of Alaska at Fairbanks
Endre Boros, Professor of Operations Research, Rutgers University.

US Faculty Mentors:

James Abello, DIMACS, Rutgers University
Alexander Barg, DIMACS, Rutgers University
József Beck, Department of Mathematics, Rutgers University
Pieter Blue, Department of Mathematics, Rutgers University
Michael Capalbo, DIMACS, Rutgers University
Graham Cormode, DIMACS, Rutgers University
Ovidiu Costin, Department of Mathematics, Rutgers University
William Cuckler, Department of Mathematics, Rutgers University
Khaled Elbassioni, RUTCOR, Rutgers University
Vladimir A. Gurvich, RUTCOR, Rutgers University
Leonid Khachiyan, Department of Computer Science, Rutgers University
Michael Kiessling, Department of Mathematics, Rutgers University
Aaron Lauve, Department of Mathematics, Rutgers University
David Madigan, Department of Statistics, Rutgers University
S. Muthu Muthukrishnan, Department of Computer Science, Rutgers University
Wilma Olson, Department of Chemistry and Chemical Biology, Rutgers University
Shawn Robinson, Department of Mathematics, Rutgers University
Siddhartha Sahi, Department of Mathematics, Rutgers University
Avy Soffer, Department of Mathematics, Rutgers University
Christopher Woodward, Department of Mathematics, Rutgers University
Doron Zeilberger, Department of Mathematics, Rutgers University

Faculty Mentors in Prague:

Martin Klazar, Charles University
Jan Kratochvil, Charles University
Martin Loebl, Charles University
Jiri Matousek, Charles University
Jaroslav Nesetril, Charles University and Director of DIMATIA
Pavel Valtr, Charles University

Ib. Participating Organizations

Telcordia Technologies: Facilities; Personnel Exchanges
Partner organization of DIMACS. Individuals from the organization are participating in the program planning and seminar series.

AT&T Labs - Research: Facilities; Personnel Exchanges
Partner organization of DIMACS. Individuals from the organization are participating in the program planning and seminar series. AT&T Labs hosted a field trip.

NEC Research Institute Inc: Facilities; Personnel Exchanges
Partner organization of DIMACS. Individuals from the organization are participating in the program planning and seminar series.

Lucent Technologies, Bell Labs: Facilities; Personnel Exchanges
Partner organization of DIMACS. Individuals from the organization are participating in the program planning and seminar series.

IBM Research: Facilities; Personnel Exchanges
Affiliated organization of DIMACS. Individuals from the organization are participating in the program planning and seminar series. IBM Research hosted a field trip.

Princeton University: Facilities; Personnel Exchanges
Partner organization of DIMACS. Individuals from the organization are participating in the program planning and seminar series.

Charles University: Financial Support; Facilities; Collaborative Research; Personnel Exchanges
DIMATIA is a major research center at Charles University in Prague, Czech Republic, and our partner in the program.

II. Project Activities

The DIMACS/DIMATIA REU program had the following three parts: (i) A group of students from all across the U.S. participated in an 8-week REU program headquartered at DIMACS; (ii) A second group of five students from all across the U.S. participated in the 8-week domestic REU program and then spent 3 additional weeks at DIMATIA; (iii) a group of students from the Czech Republic participated in the 8-week domestic REU program and then acted as hosts at DIMATIA when the U.S. students went to Prague. We collaborated with the Rutgers Department of Mathematics REU program, with participants from the two programs sharing office and living space, attending the same seminars, and participating in shared mentoring activities.

The overall goal of the program was to provide the participants with an exciting research experience that would help them decide on future educational and career paths. All students, including those participating only in the domestic program, got a taste of the international scientific enterprise. The U.S. students going to Prague got a more direct international experience and benefited from the scientific atmosphere at an international center of research, DIMATIA. The Czech students benefited similarly from exposure to their U.S. counterparts and, moreover, contributed to providing the global perspective that we sought. Our REU program is unique because it is run in the context of two major research centers with many scientific activities. The richness of the intellectual community and the international flavor contributed by the many foreign scientists participating in DIMACS and DIMATIA activities at the same time as the REU students added to the overall atmosphere.

The key to our REU program was the one-on-one research experience under the direction of a mentor. The domestic part of the program officially began when the students arrived on the Rutgers campus in mid-June, moved into campus housing (where the students were housed in a group), and received offices and computer and library accounts at DIMACS. They were met by a graduate student coordinator who

organized activities aimed at getting the REU students to meet each other and introduced them to their mentors to begin a program of directed study and research, including regular student/mentor meetings. There were regular lunches and teas, to which all the mentors and students were invited, as well as our weekly DIMACS REU Seminar Series hosting both local speakers and renowned outside speakers. The seminars were preceded by a pizza lunch and followed by opportunities to interact with the speaker. One of these seminars was devoted to a presentation about careers and graduate school. Another included a discussion of the speaker's experience in the research department of a start up Internet company.

We introduced the students to industrial research by making trips to AT&T Labs and to IBM Research, where our students got tours and there were technical presentations. The trip to IBM Research was a new element of the program, which we included because of the success of the previous trips to AT&T Labs. Just as we feel it is important for the students to be exposed to multiple (and international) academic environments, we also found that the students were served well by the opportunity to explore two different corporate environments. Students were encouraged to take advantage of all of the activities at DIMACS. Many of them attended talks given by members of the DIMACS working group on Algorithms for Multidimensional Scaling and talks given by members of the DIMACS working group on Flu Modeling. A few attended the DIMACS tutorial on Statistical and Other Analytic Health Surveillance Methods given by David Madigan, Department of Statistics, Rutgers University and the DIMACS Workshop on Applications of Lattices and Ordered Sets to Computer Science.

Students were asked to make several presentations about their projects during the course of the program. Near the beginning of the program, each student made a presentation describing his or her research problem. These short presentations, made before the entire REU group plus mentors, were aimed at introducing the whole group to the research topics of other members. This encouraged collaboration and discussion and provided the opportunity, as appropriate, to work on multiple projects or even switch projects. Near the end of the domestic program, students made short presentations about their work to an audience consisting of REU students, mentors, and others in the DIMACS community. Students going to Prague repeated these presentations soon after their arrival at DIMATIA, in order to give the DIMATIA faculty and students an idea of the research activities of the visitors, and made further presentations of their work before they departed Prague. Several of our students were also selected to make presentations during the course of the summer to various DIMACS audiences.

In addition to oral presentations, students were asked to prepare personal websites. These websites described their problem area and were filled in during the program and after the program ended with results that the students obtained.

The DIMACS part of the program did not end with the end of the DIMACS stay. Our U.S. and Czech REU students have been encouraged to stay in touch with their mentors by email, prepare web pages with their work, and keep those pages up to date after they left. Some of them ended up working on a research paper and/or preparing a presentation at a scientific meeting, still under the guidance of their mentor, after they left DIMACS. Many of our students planned or presented talks back at their home institutions and others planned to make their REU project a major piece of their senior thesis or even eventually of a master's thesis.

The goals of the experience in Prague were different from those for the experience in the U.S., and centered around introducing the participating students to a wealth of open problems and questions and appropriate problem-solving techniques and strategies.

We did not assign students going to Prague to an individual DIMATIA faculty mentor in advance. Rather, Jarosolav Nesetril, director of DIMATIA, served at first as acting mentor for each of these students. The DIMATIA faculty were involved in mentoring when the students arrived in Prague

Prior to leaving for Prague, students met several times with DIMACS staff to discuss the experience and the logistics. In addition, the Czech REU students and a Czech graduate student coordinator who accompanied the Czech students to DIMACS were available to offer pre-trip advice.

The Prague experience lasted three weeks. The U.S. graduate student coordinator accompanied the group and acted as the liaison person with the DIMATIA faculty and as a program coordinator while in the Czech Republic. The Czech students who participated in the DIMACS REU program in turn acted as hosts for the U.S. students in Prague. The arrival in Prague was timed to occur the week before the Tenth Prague Midsummer Combinatorics Workshop, a traditional problem oriented workshop attended by 9 international participants and all local DIMATIA members.

Students were met in Prague by the Prague student participants in our domestic program and moved into dormitory housing at Charles University that was provided by DIMATIA (through a companion Czech grant). Funds for meals in Prague were also provided by DIMATIA. During the first week, there were several days of special lectures by DIMATIA faculty Jan Kratochvil, Jiri Matousek, and Jaroslav Nesetril on combinatorics, graph theory, and combinatorial geometry to give students another introduction to the scientific interests of the local faculty, to prepare them for the topics of the Midsummer Workshop, and to present potential research problems. The students also took part in the Summer School in Combinatorics organized by Robert Babilon and Jiri Matousek.

The faculty members presenting tutorial lectures acted as mentors to the students, but we did not make formal assignments and we let the emphasis on problem solving lead to natural connections between students and mentors, often with more than one mentor per student. Students were encouraged to work on research problems as individuals or in groups or both. There were also presentations orienting the students to the culture and history of the city of Prague and the country, with special emphasis on the rich Czech mathematical tradition. The Czech REU students played a central role in this part of the program. The U.S. students also made informal presentations about their domestic REU research. Czech students were invited to attend the tutorials and presentations and joined the U.S. students for lunch and afternoon sightseeing. While at DIMATIA, the REU students participated in the Prague Midsummer Workshop, which has informal problem sessions each morning, aimed at identifying research topics. The Midsummer Workshop is attended by Czech students and faculty, and we involved the Czech students in afternoon sessions with the REU students to maximize the international exposure and contacts for the U.S. students. Two REU student presentations were given by Vít Jelínek and Jan Kára. In the afternoon, the U.S. and Czech coordinators/mentors led discussions on some of these topics, helping the students to focus on research topics identified. The group meetings emphasized approaches to unsolved problems, problem-solving strategies, and group attacks on problems. Students were encouraged to explore, in collaboration with Czech mentors and students, the research questions that arose during the visit to Prague, either through faculty lectures, the Midsummer Conference, or informal discussions, and to pursue the research project begun at DIMACS with Czech mentors.

After the one week Midsummer Workshop, the REU program concluded the next week with more intensive one-on-one sessions between students and mentors and group meetings for problem-solving and for presentations of research results. While the short visit in Prague didn't give students as much time as they had at DIMACS to get deeply into research in pre-defined areas with faculty mentors, the experience taught the students a good deal about how research questions are formulated and pursued. Moreover, with the background gained during the DIMACS portion of the program, the students were able to make a good start on some of the open problems and research topics that they could pursue after leaving Prague. They were encouraged to remain in continuing email contact with their Czech mentors after leaving the Czech Republic.

III. Project Findings

An Improved Upper Bound for the Polygon-Crossing Problem

Finding the maximum number of points of intersection of two or more geometric objects in the plane is one of the basic extremal geometric problems. Jakub Černý (Charles University, Prague, Czech Republic), a 2002 and 2003 REU participant, Zdeněk Dvořák (Charles University, Prague, Czech Republic), a 2003 REU participant, Vít Jelínek (Charles University, Prague, Czech Republic), a 2002 and 2003 REU participant, and Pavel Podbrdský (Charles University, Prague, Czech Republic), a 2002 REU participant, found an improvement on the best-known upper bound for the number of points of intersection of two polygons in the plane. They did this by obtaining an upper bound for the number of points of intersection of two generalized polygons, called k-gons. Their research advisor at DIMACS was Janos Komlos, member of DIMACS and the Department of Mathematics at Rutgers University.

They have submitted their paper for publication and a preprint is available at http://kam.mff.cuni.cz/~kamserie/serie/clanky/2003/s623.ps.gz

A New Bound on the Number of Edges of Certain Geometric Graphs

Determining the number of edges possible in a graph that does not have a given subgraph is a problem that has been studied for over half a century by such eminent mathematicians as Paul Erdos. A geometric graph is a graph that can be drawn in the plane so that no three vertices are on the same line and edges can be represented by line segments. A pair of edges in such a graph is disjoint if the two edges do not intersect as line segments in the plane. Jakub Černý (Charles University, Prague, Czech Republic), a participant in the REU program at DIMACS in both 2002 and 2003, studied geometric graphs that do not have the subgraph consisting of 3 line segments, any two of which are disjoint. He showed that such a graph on n vertices has at most 2.5 n + 1 edges. The history of this result is as follows: In 1989 Alon and Erdos showed that such a graph on n vertices has at most 6n edges. In 1990 this was improved to 3.6n +c by O'Donnell and Perles. In 1993, Goddard, Katchalski, and Kleitman got it down to 3n, with Meszaros modifying this to 3n-1 in 1998. Jakub not only made a significant improvement but also showed that his result has the lowest possible coefficient of n. Jakub's research advisor at DIMACS in 2003 was József Beck and in 2002 was Janos Komlos, both members of DIMACS and the Department of Mathematics at Rutgers University.

Geometric Realizations of Hypergraphs

Steven Jaslar (Rutgers University), a 2002 and 2003 REU participant, and Tatiana Yarmola (University of California, Berkeley), a 2003 REU participant, worked on some geometric realizations of 2-3 vertex hypergraphs in two dimensions. Given a set of points in the unit circle, they looked for ways to efficiently list sets of points whose convex hulls contained 0, which they found can be solved in $n\log n$ time. Also they investigated the possibility of placing points on the unit circle such that the geometric realization of a hypergraph with two and three vertex elements would contain 0 in the convex hull. They found factorial and exponential algorithms for this problem, but conjectured that there could not be a polynomial algorithm.

A Winning Strategy for a Clique Game

Vít Jelínek (Charles University, Prague, Czech Republic), a 2002 and 2003 REU participant, Jan Kára (Charles University, Prague, Czech Republic), a 2003 REU participant, and Robert Šámal, the graduate

student organizer at Charles University, have continued a collaboration on clique games begun during the 2003 REU.

Consider a 'clique game' with two players defined as follows: You are given a clique of n vertices. Each of the players colors one edge in each move - the first player colors by red, the second one by blue. The player who first succeeds in coloring the edge set of a complete subgraph on four vertices is the winner. For n large enough, Ramsey's theorem guarantees a winner. Since there is a winning strategy for at least one player, the first player can win. The winning strategy was not known however. They have come up with a strategy to win this game.

Vít Jelínek gave a talk about their work at the Prague Midsummer Combinatorics Conference in August 2003. They are currently working on a paper and hope to get their results published.

Visibility Graphs of Staircase Polygons

Given a simple polygon P (i.e. P's boundary is not self-intersecting) in the plane, we define the visibility graph of P, Vis(P), as follows: the vertices of Vis(P) are the vertices of P and we let two vertices in Vis(P) be connected by an edge, if and only if, the closed line segment connecting these vertices is completely contained in the boundary of P or lies in P's interior. Vishal Gupta (Yale University, New Haven, CT), a 2003 REU participant, explored the question of whether or not this process can be reversed, that is, given a visibility graph G, can one build a polygon P such that Vis(P) is isomorphic to G? This question is being approached by geometrizing the major steps of a purely combinatorial algorithm, (due to his mentor, James Abello), that constructs an oriented matroid from a visibility graph. Abello's combinatorial algorithm proceeds in many steps that create equivalences between visibility graphs, a certain class of matrices called persistent matrices, and a partial ordering of the symmetric group called the Weak Bruhat Order. For the special class of staircase polygons, Vishal and his mentor developed the corresponding geometric steps that are necessary during the reconstruction process. While in Prague, Vishal prepared a report detailing these findings. The proof of the sufficiency of the proposed geometric steps is currently under revision. An immediate corollary of this theorem will be the characterization of a large class of realizable chains in the Weak Bruhat Order which is in general an NP-Hard problem. Recognizing visibility graphs of polygons is a problem only known to be in PSPACE. It is not known to be NP-Hard and moreover it is not even known to be in NP.

A Result on a Generalization of the Stable Marriage Problem

As previously reported, during the 2002 DIMACS REU program, two undergraduates, Steven Jaslar (Rutgers University) and Daniel Krasner (University of California, Berkeley), settled a conjecture about a generalization of the stable marriage problem. They were working under the mentorship of Professors Endre Boros and Vladimir Gurvich. These results were presented at the Prague Midsummer Combinatorics Conference in August 2002 and have been published as DIMACS Technical Report 2002-34 *Stable Matchings in Three-Sided Systems with Cyclic Preferences* by Endre Boros, Vladimir Gurvich, Steven Jaslar and Daniel Krasner. It has been submitted to the journal **Discrete Math**. Steven Jaslar is planning on giving a talk about the research for the General Honors Program at Rutgers Spring, 2004. The technical report is available at:
ftp://dimacs.rutgers.edu/pub/dimacs/TechnicalReports/TechReports/2002/2002-34.ps.gz
A more complete description of these results can be found in the Report on the 2002 REU program.

IV. Project Training/Development

In a very real sense, the entire project was about training and development, focusing outstanding undergraduates on opportunities in a research career in both academic and business environments and exposing them to the international nature of the research enterprise. The following comments we received from students in the program reflect the impact the program had on training them in this sense:

Comments from 2003 Participants

"I participated in the REU for two summers. Both summers were really rewarding academically and also socially. I think the REU definitely succeeded in its mission of giving me a feeling for what doing research is like."

"It's a great project where I met many interesting people and I realized what is "research" in the US (not only on DIMACS, but the trips to I**BM**, AT&T...). Now I know much more about the facilities of studying/working in the US as a researcher."

"I now see that I enjoy research and want that to be part of my life."

"One of the highlights of the summer was getting to meet Mandelbrot at IBM. AT&T was also interesting."

"I think it is very interesting to see how mathematicians work in other parts of the world and what they do. This is not just for observing since there is a great chance to work with people like that in future."

"Although the main problem was not really solved, the attempt to approach it from different sides and trying a lot of tools was very valuable for me since it showed how to work on a problem."

Recent Comments from 2002 Participants

"I have been accepted to the math graduate department at Columbia University. However, I deferred for one year, and will shortly be heading out to study at Charles University, in Prague, for the spring - thanks to the contacts I have made during the 'REU days.' "

"The research was fun, I learned a lot, and meeting everybody at the REU was very helpful. Since I went to a liberal arts college, there were no graduate students, and not many other undergrads were planning on going to graduate school. Talking to grad students and profs from Rutgers and the other REU students gave me a much better idea of what grad school would be like and what I need to do to prepare for it."

"It helped me understand what the lifestyle was like for someone doing math research."

V. Outreach Activities

A unique aspect of the program was its place within a vital and active research center with many other exciting programs, which were made available to our REU students. The program coincided each summer with the DIMACS Connect Institute (DCI), which provides research experiences for high school teachers and connects them with active researchers in graph theory and its applications. The REU students interacted with participants in DCI and a number of them gave lectures to combined audiences of teachers and researchers at DCI.

The REU students also interacted with the DIMACS Reconnect Program, which is aimed at 2- and 4-year college faculty, and in particular aims to help them bring current research into the undergraduate program. Having a thriving REU program as an example was an important aspect of our summer Reconnect Program.

The students in the REU made connections with a variety of other DIMACS programs, including tutorials, workshops, and seminars, and we encouraged them to get involved with our industrial partners to expose them to industrial research as well as to our other partners such as the Institute for Advanced Study.

VI. Papers

DIMACS Technical Report 2002-34 *Stable Matchings in Three-Sided Systems with Cyclic Preferences,* Endre Boros, Vladimir Gurvich, Steven Jaslar (REU) and Daniel Krasner (REU).

Pancyclicity of strong products of graph, Daniel Král (REU), Jana Maxová, Pavel Podbrdský (REU), and Robert Sámal (REU), **Graphs and Combinatorics**, to appear.

On Pattern Coloring of Cycle Systems, Zdeněk Dvořák (REU), Jan Kára (REU), Daniel Král (REU), O. Pangrác, **Workshop on Graph Theoretical Concepts in Computer Science 2002 (WG'02)**, Lecture Notes Computer Science, Springer-Verlag, to appear.

On the Number of Intersections of Polygons, Jakub Černý (REU), Jan Kára (REU), Daniel Král (REU), Pavel Podbrdský (REU), Robert Šámal (REU graduate student organizer), M. Sotakova, **Commentationes Mathematicae Universitatis Carolinae,** to appear.

Stable Matchings in Three-Sided Systems with Cyclic Preferences, Endre Boros, Vladimir Gurvich, Steven Jaslar (REU) and Daniel Krasner (REU), **Discrete Math,** submitted.

Improved upper bound on crossing number of polygons problem, Jakub Černý (REU), Zdeněk Dvořák (REU), Vít Jelínek (REU), and Pavel Podbrdský (REU), submitted.

Geometric graphs with no three disjoint edges, Jakub Černý (REU), in preparation.

A collection of realizable chains in the Weak Bruhat Order, Vishal Gupta (REU) and James Abello, in preparation.

Vít Jelínek (REU), Jan Kára (REU), and Robert Šámal (REU graduate student organizer) are preparing a paper on their work on a winning strategy for a clique game.

Other students have indicated that they plan on continuing to work on their REU research project and papers may result from these on-going efforts.

VII. Other Products

<u>Presentations</u>

One of the goals of our program was to emphasize student presentations of research. Students returning from our program made presentations in a variety of venues. For instance,

Vít Jelínek gave a talk about the work on a winning strategy for a clique game at the Prague Midsummer Combinatorics Conference in August 2003.

James Abello is planning a talk on the work with his REU student, Vishal Gupta, for Summer, 2004.

Steven Jaslar is planning on giving a talk about the research on Stable Matchings in Three-Sided Systems for the General Honors Program at Rutgers Spring, 2004.

Web Dissemination

http://dimacs.rutgers.edu/REU/

VIII. Contributions within Discipline

Most of the research results obtained were in the fields of discrete mathematics and theoretical computer science, broadly defined. These are areas in which DIMACS is very strong and has a world-wide reputation. Still, it is remarkable that undergraduates could, in at least some of the cases, do cutting edge research and solve some open problems.

The most important contribution to the disciplines of discrete math and theoretical computer science in particular, and to the discipline of the mathematical sciences broadly speaking (including all areas of mathematics, computer science, operations research, statistics, etc.), was the opening up of new horizons to very bright undergraduates, some of whom we hope will become leaders in the field in the future.

IX. Contributions -- other Disciplines

Throughout the program, we emphasized applications of discrete mathematics and theoretical computer science. Probably the biggest impact in terms of applications came from our molecular biology topics. One of our students worked with Professor Wilma Olson, Director of the Rutgers Center for Molecular Biophysics and Biophysical Chemistry. To understand how the genetic code is translated into three-dimensional structures used in biological processes, Professor Olson's group has been analyzing the known geometry of DNA bases in chemical structures and developing mathematical models to incorporate the sequence-dependent bending, twisting, and translation of known fragments in DNA molecules of varying lengths and chemical composition. The REU student was directly involved in the group's research. In addition, there was outreach to other mathematical biology research at Rutgers through connections with DIMACS Special Focus on Computational and Mathematical Epidemiology. Some outstanding students received an exciting introduction to the interface between disciplines.

X. Contributions -- Human Resource Development

The impact on the careers of our students will take awhile to determine. However, we do have some self-assessment feedback from our recent survey of REU participants. When asked whether their participation in this REU program affected their future career and education plans, all but two students answered in the affirmative, indicating an increased interest in research and desire to attend graduate school. The two students who did not answer in the affirmative clearly indicated that they already had intentions of

pursuing a research career before the REU program. More specifically, students who had only a general interest in research mathematics, now had more focused interests, paths they wanted to immediately pursue. Here are some of the comments we received:

"I did not know much about optimization, but as a result of this project, I want to go into operations research and this semester I am taking a linear optimization class."

"I plan on taking a class in random algorithms as soon as I have the opportunity."

We also received some extremely thoughtful comments from past participants about the impact of our program on them. Here is a selection:

"I met many interesting people learned a lot about combinatorial game theory."

"It was great to meet the professors at REU especially our advisor Joseph Beck or Janos Komlos. Many of the REU lectures were very good for improving general 'mathematical outlook'. Also some of the discussions with other REU students working on a project from a different field were enlightening."

"I think that this experience was very interesting and educating for all of us - but even more so for us Czech students. There are three things that I consider the most important about my REU experience:
* I got a chance to see how things are done at a different institution (so far I have only had experience with Charles University)
* I met many interesting people with various backgrounds and could share experience with them
* I learned lots of new things in Graph Theory while attempting to solve the UIR problem."

XI. Contributions to Education and Human Resources

XII. Contributions Beyond Science and Engineering