

DIMACS Center  
Rutgers University

**DIMACS-DIMATIA-Rényi Collaboration on Discrete Mathematics  
and Theoretical Computer Science**

**Annual Report**

June 2006

## **Ia. Participants from the program**

### **Participants who spent 160 hours or more**

PI: Fred Roberts, DIMACS  
Gyula O. H. Katona, Rényi Institute  
Jaroslav Nešetřil, DIMATIA

### **DIMACS/DIMATIA/Rényi Working Group on Algebraic and Geometric Methods in Combinatorics Meeting II**

#### **Organizers:**

Gyula Katona, Alfréd Rényi Institute, ohkatona@Rényi.hu  
Dezso Miklós, Alfréd Rényi Institute  
Attila Sali, Alfréd Rényi Institute

### **DIMACS/DIMATIA/Rényi Working Group on Graph Colorings and their Generalizations Meeting II**

#### **Organizers:**

Fred Roberts, DIMACS  
Gyula Katona, Alfred Rényi Institute

### **DIMACS/DIMATIA/Rényi Working Group on Extremal Combinatorics Meeting II**

#### **Organizers:**

János Komlós, Rutgers University  
Endre Szemerédi, Rutgers University

### **DIMACS/DIMATIA/Rényi Working Group on Graph Colorings and their Generalizations Meeting III**

#### **Organizers:**

Fred Roberts, DIMACS  
Gyula Katona, Alfred Rényi Institute

### **DIMACS/DIMATIA/Rényi Working Group on Extremal Combinatorics Meeting III**

#### **Organizers:**

Jaroslav Nešetřil, Charles University

Pavel Valtr, Charles University

### **DIMACS/DIMATIA/Rényi Working Group on Algebraic and Geometric Methods in Combinatorics Meeting III**

#### **Organizers:**

Jaroslav Nešetřil, Charles University  
Fred Roberts, DIMACS

### **DIMACS/DIMATIA/Rényi Combinatorial Challenges Meeting**

#### **Organizers:**

Brenda Latka, Program Chair, DIMACS  
Gyula Katona, Alfréd Rényi Institute  
Jaroslav Nešetřil, Charles University  
Fred Roberts, DIMACS.

### **Visitors to DIMACS, DIMATIA, Rényi: (students are indicated by \*)**

Imre Barany, Rényi Institute  
Daniel Gerbner, Eötvös Loránd University\*  
Vladimir Gurvich, Rutgers University  
Stephen G. Hartke, University of Illinois  
David Howard, Georgia Tech\*  
Gyula Katona, Alfréd Rényi Institute  
Balázs Keszegh, Rényi Institute\*  
Daniel Král, Georgia Institute of Technology  
Brenda Latka, DIMACS  
Dezso Miklos, Rényi Institute  
Pavel Nejedlý, Charles University  
Jaroslav Nešetřil, DIMATIA  
Ricky Pollack, NYU  
Paul Raff, Rutgers University\*  
Fred Roberts, DIMACS  
Miklós Simonovits, Rényi Institute  
William Steiger, Rutgers

### **Ib. Participating Organizations**

Telcordia Technologies: Collaborative Research  
Partner organization of DIMACS. Individuals from the organization participated in the program planning.

AT&T Labs - Research: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning and research.

NEC Laboratories America: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning.

Lucent Technologies, Bell Labs: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning and research.

Princeton University: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning.

Avaya Labs: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning.

HP Labs: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning.

IBM Research: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning.

Microsoft Research: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning and research.

DIMATIA: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning and research.

Rényi Institute: Collaborative Research

Partner organization of DIMACS. Individuals from the organization participated in the program planning and research.

### **1c. Other Collaborators**

The project involved scientists from numerous institutions in numerous countries. The resulting collaborations also involved individuals from many institutions in many countries.

## II. Project Activities

### Overview

This three-way international research collaboration includes three partners: DIMACS, the Center for Discrete Mathematics and Theoretical Computer Science, headquartered at Rutgers University; DIMATIA, the Center for Discrete Mathematics, Theoretical Informatics, and Applications, at Charles University in Prague, Czech Republic; and the Alfred Rényi Institute of Mathematics of the Hungarian Academy of Sciences in Budapest, Hungary. These three distinguished research centers form three of the most important centers in the international community of researchers in discrete mathematics and theoretical computer science. They have combined their research strengths with the creation of multinational “working groups” in research areas of discrete mathematics and theoretical computer science where the three centers have major strengths and where focused collaboration has the likelihood of leading to major scientific advances. The working groups, consisting of senior and junior faculty and graduate students, are devoted to Extremal Combinatorics (chaired by Gyula Katona, Rényi), Graph Colorings and their Generalizations (chaired by Fred Roberts, DIMACS), and Algebraic and Geometric Methods in Combinatorics (chaired by Jaroslav Nešetřil, DIMATIA). Each group met once at each center in this 3 year project and there was a final four day meeting of all three groups exploring interconnections among the topics. The meetings involved talks and time for collaboration. There were also visitor exchanges involving faculty and students and an international student workshop. This report covers two years, from June 2004 to June 2006.

The project has been successful in increasing the involvement of outstanding junior people in international collaborations by means of visitor exchanges and supporting the attendance of students at the international working group meetings.

#### *DIMACS/DIMATIA/Rényi Working Group on Algebraic and Geometric Methods in Combinatorics Meeting II*

Dates: April 6 - 7, 2004

Organizers: Gyula Katona, Alfréd Rényi Institute, Dezsó Miklós, Alfréd Rényi Institute, Attila Sali, Alfréd Rényi Institute

Location: Alfréd Rényi Institute, Budapest, Hungary

This meeting covered a variety of topics on algebraic and geometric methods. Here are a few examples. Imre Bárány, Rényi Institute, talked about the six circle conjecture of Laszlo Fejes Toth. Bill Cuckler, a graduate student at Rutgers University, talked about his thesis work on Hamiltonian cycles in regular tournaments. Pavol Hell, Simon Fraser University, described matrix partitions of graphs. Gyula Károlyi, Eötvös University, surveyed some new results in combinatorial number theory and the polynomial method. Michal Karonski, Adam Mickiewicz University, discussed work on irregular assignments. Abdelakder Khelladi, CERIST, presented some results on the fibered product of graphs. Dan Král, a graduate student at Charles University, presented his work on equipartite graphs, polytopes, and sets of points. Brenda Latka, DIMACS, gave a talk on well-quasi-orderings and tournaments and presented a finiteness result. Bernardo Llano, Universidad Autonoma Metropolitana, described infinite families of tight 3-uniform hypergraphs. László Lovász, Microsoft Research discussed graph homomorphisms,

statistical physics, and quasi-random graphs. Ryan Martin, Iowa State University, presented a talk on the editing distance in graphs. Fred Roberts, Rutgers University, related the topics of the meeting to the application of role assignments in social networks. Miklós Simonovits, Rényi Institute, talked about progress on the Erdős-T. Sos conjecture on the extremal problem of (large) trees. Endre Szemerédi, Rutgers University, discussed the Dirac theorem for 3-uniform hypergraphs. Gábor Tardos, Rényi Institute, presented material on 0-1 matrices and small excluded submatrices. Pavel Valtr, Charles University, made remarks on the unit distance problem for strictly convex norms. Vince Vatter, a graduate student at Rutgers University, presented his work on well-quasi-ordering and permutations. Nick Weinginger, a graduate student at Rutgers University, described correlation properties of some random colorings.

*DIMACS/DIMATIA/Rényi Working Group on Graph Colorings and their Generalizations Meeting II*

Dates: August 2 - 4, 2004

Organizers: Fred Roberts, DIMACS; Gyula Katona, Alfred Rényi Institute

Location: Prague, Czech Republic

The central topic of the working group is generalization of graph colorings, motivated by practical applications like channel assignment in communications, traffic phasing, fleet maintenance, task assignment, and others. Talks and discussions surveyed and investigated such generalizations of graph coloring as T-colorings, list colorings,  $L(2,1)$ -colorings, and set colorings, with an emphasis on the graph coloring concepts that arise from channel assignment problems, and design of graph coloring algorithms for new models and applications. A few examples of the presentations are described below.

Dan Král, then a graduate student at Charles University, presented joint work with Robert Babilon, Vit Jelinek, and Pavel Valtr on graphs of graphs. Jiri Fiala and Jan Kratochvíl, Charles University, presented their joint work on NP-hardness of the  $L(2,1)$ -labeling problem on regular graphs, giving their proof that for all  $k$  greater than two the decision problem whether a  $k$ -regular graph admits an  $L(2,1)$ -labeling of span  $k+2$  is NP-complete. Vladimir Gurvich, Rutgers University, gave a presentation on complete edge- $n$ -colored graphs, applications and problems. Complete edge- $n$ -colored graphs are related to positional games with perfect information and to box-partitions. Jaroslav Nešetřil, DIMATIA, spoke on the duality of graph homomorphisms. Riste Skrekovski, University of Ljubljana, challenged the group with some coloring problems of planar graphs. Nicholas Weininger, then a graduate student at Rutgers University, presented his thesis work on correlation properties of random homomorphisms.

Please see <http://kam.mff.cuni.cz/conferences/DDR04/> for further information.

*DIMACS/DIMATIA/Rényi Working Group on Extremal Combinatorics Meeting II*

Dates: October 18 - 20, 2004

Organizers: János Komlós, Rutgers University; Endre Szemerédi, Rutgers University

Location: DIMACS Center, CoRE Building, Rutgers University, Piscataway, NJ

This meeting continued the work of the working group on two general topics, extremal graph theory and extremal problems arising from combinatorial search and testing.

Extremal graph theory deals with graphs satisfying specified constraints and optimizing some criteria. Participants investigated a number of specific questions of current research interest in this field, which has played a fundamental role in the history of graph theory. Typically, one is interested in Turán-type extremal problems. Here, a family  $L$  of sample graphs is fixed and one considers various conditions on a graph on  $n$  vertices that does not contain any graph in  $L$  as a subgraph (not necessarily induced). The most famous such problem asks for the maximum possible number of edges  $ext(n,L)$  in a graph  $G$  on  $n$  vertices which does not have any subgraph in  $L$ . Alternatively, we may ask for the maximum possible value  $dex(n,L)$  of the minimum degree of a graph  $G$  on  $n$  vertices which has no graph in the family  $L$  as a subgraph. It is easy to see that  $dex(n,L) < (2/n)ext(n,L)$ . The upper bound is almost achieved in many interesting cases, and an active area of research is to identify conditions under which this occurs. A variant of Turán-type problems is the generalized matching problem: to find many vertex-disjoint copies of a fixed graph in a large graph. Here too, the minimal degree of  $G$  is the natural quantity with which to work. In approaching some problems in extremal graph theory, two standard tools are the Regularity Lemma and the Blow-up Lemma. In the last few years, various promising analogues of these tools were developed for hypergraphs.

The working group also investigated the theory and algorithms of combinatorial group testing, with emphasis on connections to coding theory and combinatorial design. These topics are also of interest to the Working Group on Algebraic and Geometric Methods in Combinatorics. To identify all positive cases in a large population of items, group testing proceeds by grouping the items into subsets, testing if a subset contains at least one positive item, and then identifying all positive items through iteration of group tests. The participants investigated problems involving testing properties of evolving objects. These problems can be formulated as determining whether a function  $f$  has property  $P$  or is *epsilon-far* from any function with property  $P$ . A property testing algorithm is given a sample of the value of  $f$  on instances drawn according to some distribution, and, in some cases, it is also allowed to query  $f$  on instances of its choice. The concept was defined by Goldreich, Goldwasser and Ron. The function  $f$ , viewed as an object itself, can be of combinatorial, geometric, or algebraic nature. Thus far all property tests are designed to work with static objects even though in reality the studied object (e.g., the hyper-link graph of the Internet) can be constantly changing. Participants investigated efficient methods for testing changing objects and determining the frequency with which one has to draw samples if one wants to keep the computed information up to date.

*DIMACS/DIMATIA/Rényi Working Group on Graph Colorings and their Generalizations  
Meeting III*

Dates: April 21 -22, 2005

Organizers: Fred Roberts, DIMACS; Gyula Katona, Alfred Rényi Institute

Location: The Rényi Institute, Budapest, Hungary

The central topic of the working group is generalization of graph colorings, motivated by practical applications like channel assignment in communications, traffic phasing, fleet maintenance, task assignment, and others. Presentations included ones by Fred Roberts, DIMACS, on balanced signed graphs and consistent marked graphs; Zsolt Tuza, SZTAKI, on unique colorings of mixed hypergraphs; Gábor Simonyi, Rényi, on circular, local, and wide colorings; Attila Sali, Rényi, on color critical hypergraphs and forbidden configurations; Peter Mihok, University of P. J. Safarik in Kosice, on critical graphs in generalized graph colourings; Mieczyslaw Borowiecki, University of Zielona Góra, on game generalize colouring; Ela Sidorowicz on game list colouring of graphs; and Gábor Tardos, Rényi, on the local chromatic number of quadrangulations of surfaces.

Please see <http://www.Rényi.hu/conferences/ddr2005.html> for further information.

*DIMACS/DIMATIA/Rényi Working Group on Extremal Combinatorics Meeting III*

Dates: July 30 – August 1, 2005

Organizers: Jaroslav Nešetřil, Charles University; Pavel Valtr, Charles University

Location: Prague, Czech Republic

This was the third meeting of DIMACS/DIMATIA/Rényi Working Group on Extremal Combinatorics. The central two subjects of the working group are extremal graph theory and extremal problems arising from combinatorial search and testing. Zoltan Füredi, Rényi Institute, exhibited a true, short, polynomial proof for the Erdős-Ko-Rado Theorem, joint work with Kyung-Won Hwang and Paul M. Weichsel. Ervin Gyori, Rényi Institute, discussed the question of how bipartite a graph is with no cycle of length  $2k+1$ . Gyula O.H. Katona, Rényi Institute presented his investigations of the maximum size of a family of subsets which does not contain a given poset,  $P$  as a (non-necessarily induced) subposet. There are some  $P$ 's for which this maximum has been exactly determined. In most cases, however, the maximum is known only asymptotically. Jiri Fiala, DIMATIA, Petr A. Golovach, Syktyvkar State University, and Jan Kratochvíl, DIMATIA, presented their work on distance constrained labelings of graphs of bounded treewidth. They gave a proof that the  $L(2,1)$ -labeling problem is NP-complete for graphs of treewidth two, thus adding a natural and well studied problem to the short list of problems whose computational complexity separates treewidth one from treewidth two. Dezsó Miklos, Rényi Institute, spoke on vertex-sets of the hypercube whose span avoids given hyperplanes. Balázs Patkós, Rényi Institute, talked about the distance of FF-free families of hypergraphs omitting a given fixed hypergraph. He considered several collections of forbidden hypergraphs. For some he described exact results on the maximum distance while for others he gave upper and lower bounds on the maximum distance. Pavel Valtr, DIMATIA, explained the main parts of an alternative proof of the Empty Hexagon Theorem.

*DIMACS/DIMATIA/Rényi Working Group on Algebraic and Geometric Methods in Combinatorics Meeting III*

Dates: November 7 - 9, 2005

Organizers: Jaroslav Nešetřil, Charles University; Fred Roberts, DIMACS

Location: DIMACS Center, CoRE Building, Rutgers University



The third meeting of the Working Group on Algebraic and Geometric Methods in Combinatorics included the topics of algebraic methods in the study of homomorphisms of graphs, problems of combinatorial geometry, and applications of algebraic geometry. Some of the highlights of the meeting included the talk by Jarik Nešetřil, DIMATIA, on bounded expansion versus minors and the survey and challenging list of open problems from Van Vu, Rutgers University, involving random matrices with discrete distributions, with main focus being the random sign matrix and its parameters such as determinant, singularity, and eigenvalues.

Benjamin Sudakov, Princeton University, presented his work with P. Keevash on a restricted cross-intersection problem. Suppose  $A$  and  $B$  are families of subsets of an  $n$ -element set and  $L$  is a set of  $s$  integers. The pair  $(A, B)$  is  $L$ -cross-intersecting if the intersection size of every member of  $A$  with every member of  $B$  belong to  $L$ . Sudakov presented an exact bound for for the maximum possible value of  $|A||B|$  among such pairs  $(A, B)$  when  $n$  is sufficiently large, improving earlier work of Sgall. He also discussed several intriguing open problems. Roy Meshulam, Technion, discussed joint work with R. Aharoni and E. Berger on some connections between the expansion constant of a graph and the topology of certain complexes associated with the graph. Applications include a lower bound on the homological connectivity of the independence complex, in terms of a new graph domination parameter defined via vector representations of the graph. This in turn implies Hall type theorems for matchings in hypergraphs. Bojan Mohar, Simon Fraser, presented some joint work with Matt DeVos about local expansion in vertex transitive graphs. The study of expansion in vertex transitive graphs and in groups divides naturally into the study of local expansion, or connectivity, and the study of global expansion (the growth). The expansion properties of a group are those of its Cayley graphs, and vertex transitive graphs are a more general setting.

There are a number of classic results concerning local expansion in vertex transitive graphs. with analogues for digraphs and vertex cuts. They are also meaningful for groups, and have some asymptotic applications. Mohar and DeVos' main theorem gives rough structure of vertex-cuts of bounded size in arbitrary vertex transitive graphs.

Please see <http://dimacs.rutgers.edu/Workshops/Algebraic3/> for further information.

#### *DIMACS/DIMATIA/Rényi Combinatorial Challenges Meeting*

Dates: April 26 - 29, 2006

Organizers: Brenda Latka, Program Chair, DIMACS; Gyula Katona, Alfréd Rényi Institute; Jaroslav Nešetřil, Charles University; Fred Roberts, DIMACS

Location: DIMACS Center, CoRE Building, Rutgers University

The three multinational working groups of the DIMACS, DIMATIA, Rényi partnership were devoted to Extremal Combinatorics, Graph Colorings and their Generalizations, and Algebraic and Geometric Methods in Combinatorics. We observed many interfaces among the work of the three working groups and felt that a meeting where we brought representatives from all three working groups together would have some very interesting possibilities. A major focus of the nine working group meetings has been to identify those combinatorial challenges and research

directions that will guide research in the immediate future. In this meeting we shared these ideas with a broader research community. We invited researchers and their students from outside the three partner institutions with whom we have been interacting to join us. We had invited speakers as well as contributed talks and time for participants to engage in small group discussions extending existing or establishing new international collaborations.

Jaroslav Nešetřil, DIMATIA, opened the meeting with his talk on Homomorphisms: Logic versus Combinatorics. Several properties of graphs (and finite structures) can be alternatively described by combinatorial means as a duality, or by a logical definability and also by properties of homomorphism order. Nešetřil surveyed this recent development and his joint work with C. Tardif, P. Ossona de Mendes, and G. Kun. Applications were part of the discussions as illustrated by the presentation by Jennifer Chayes, Microsoft, on controlling the spread of viruses on power-law networks. The spread of viruses on preferential attachment networks is a problem of relevance both to the spread of viruses and worms on the Internet, and to the spread of communicable diseases in a population. Chayes considered the problem in which the virus may spread to from a site to any neighboring site with some rate, and is spontaneously cured with another rate. She showed that if the ratio of the infection rate to the curing rate is bounded below, then there will be an epidemic. Then she showed that commonly used algorithms, in particular so-called contact tracing, are ineffective in controlling the epidemic. Based on her collaborations with Noam Berger, Christian Borgs, Ayalvadi Ganesh, Amin Saberi and David Wilson, Chayes has developed an alternative algorithm which does control the epidemic and this algorithm is best possible in the sense that no other algorithm is more than a constant factor better. Noga Alon, Tel Aviv University, described several extremal results for tournaments and their connection to voting paradoxes, starting with an old result of Spencer on the existence of large acyclic subgraphs in tournaments on  $n$  vertices. The techniques combine algebraic and geometric tools. Fred Roberts, DIMACS, discussed graph-theoretical problems arising from the competition between a bioterrorist seeking to infect a population (turning infected vertices red) and a public health defender seeking to vaccinate enough people to minimize the resulting epidemic (turning protected vertices green). He described the same problems from the point of view of placing firefighters at the vertices of a graph so as to minimize the number of trees burned in a forest fire. Jerry Griggs, University of South Carolina, described work that has been very much part of this project involving generalized graph coloring problems, which arise in connection with efficient channel assignments for networks of radio transmitters, when conditions are imposed due to different levels of interference. The network is represented by a simple graph, possibly an infinite one. Each vertex must be labelled by a nonnegative number, and avoiding interference leads to minimum separation requirements between labels. Griggs surveyed the problems and progress in this area.

Balazs Keszegh, Alfréd Rényi Institute, described his work, joint with Gabor Tardos, on excluded submatrices in 0-1 matrices. Paul Seymour, Princeton University, presented his work on testing for a theta, a graph consisting of two nonadjacent vertices joined by three vertex-disjoint paths. It has been an open question of some interest whether we can test in polynomial time whether a graph contains a theta as an induced subgraph since a number of very similar questions are known to be NP-complete. It turns out that this is polynomially solvable. The algorithm consists of a number of applications of a subroutine to test whether three given vertices are in an induced tree. This question in turn has a surprisingly neat answer: there is a

structural characterization of the graphs in which no such tree exists. Dan J. Kleitman, MIT, showed that the edge set of any girth 6 planar graph can be partitioned into edges forming a forest and edges at most two incident to any vertex. The similar question with girth 8 and the “two” above replaced by “one” seems to be open. Kleitman also talked about the three variable case of Rado's boundedness conjecture. and his work with MIT undergraduate Jacob Fox. Some linear equations (or sets of linear equations) always have non-trivial monochromatic solutions among rational numbers whenever these are colored in a finite number of colors. Others always have monochromatic solutions when the number of colors is below some finite threshold but not beyond it. Rado in 1933 conjectured that there is a finite upper bound on the magnitude of this threshold number of colors for any specific number of variables in the equation. Kleitman and Fox showed that for the three variable homogeneous equations, 24 is such an upper bound. Joel Spencer, Courant Institute, gave a survey of results related to the beauty of certain mathematical proofs. Paul Erdős was always pleased when a conjecture was resolved but when the argument was “ugly” he would say “Well, very good, but now let’s look for the Book Proof.” Spencer gave two cases where his own original arguments have been replaced by gems of others and one case in which he prefers his arguments. Vera Sos, Alfréd Rényi Institute, talked about convergent graph sequences and generalized quasi random graphs. She discussed problems on the maximal size and structure of solution-free sets for linear equations in different additive structures, focusing on the structure of sumsets of Sidon sets of maximal size. Gabor Tardos, Simon Fraser University, closed the meeting with his presentation on local chromatic number of quadrangulation of surfaces, representing joint work with Bojan Mohar and Gabor Simonyi.

Please see <http://dimacs.rutgers.edu/Workshops/CombChallenge/program.html> for details on the program.

### **III. Project Findings**

#### New collaborations and research directions

From the list of papers included in this report, it is clear that many fruitful collaborations have developed among the participants from the three institutions, DIMACS, DIMATIA, and Rényi. A description of a selection of these is included.

#### *Real number graph labellings with distance*

Jerrold Griggs, University of South Carolina, learned from his participation in this project that researchers in Prague were considering models for graph labelling with real numbers that were closely related to his own work. He had previously been working with integer labels, in a series of problems suggested by Fred Roberts, DIMACS. Griggs eventually formulated several general conjectures for graph labellings with distance conditions with his Ph.D. student, Xiaohua Teresa Jin. The theory of integer lambda-labellings of a graph, introduced by Griggs and Yeh, seeks to model efficient channel assignments for a network of transmitters. To prevent interference, labels for nearby vertices must be separated by specified amounts  $k_i$  depending on the distance  $i$ . Griggs and Jin expanded the model to allow real number labels and separations. The main finding (“D-set Theorem”) is that for any graph, possibly infinite, with maximum degree at most

$\Delta$ , there is a labelling of minimum span in which all of the labels have the form of a  $\sum a_i k_i$ , where the  $a_i$ 's are nonnegative integers. Griggs and Jin showed that the minimum span is a continuous function of the  $k_i$ 's, and conjecture that it is piecewise linear with finitely many pieces. Their stronger conjecture is that the coefficients  $a_i$  can be bounded by a constant depending only on  $\Delta$  and  $p$ . They found results in strong support of the conjectures, and developed formulas for the minimum spans of several graphs with general conditions at distance two.

### *Colorings of Cartesian products of classes of graph*

Stephen Hartke, University of Illinois and in earlier years of the project a graduate student at Rutgers, Hemanshu Kaul, University of Illinois at Urbana-Champaign, Dan Král, Georgia Tech and formerly DIMATIA, and Doug West, University of Illinois – Urbana, collaborated on the two problems posed by Mario Szegedy during the DIMACS/DIMATIA/ Rényi meeting held at DIMACS in April 2006. The problems were inspired by some problems arising in computational complexity and deal with coloring of a Cartesian-like product of graphs. Hartke, Kaul, Král, and West resolved one of the problems completely and obtained interesting bounds for the other one. Let  $G_0$  be a class of graphs, e.g., a class of all bipartite graphs. The  $d$ -dimensional Cartesian product  $G_0^d$  of  $G_0$  is the set of graphs obtained from the  $d$ -dimensional rectangular grid with  $n_1 \times \dots \times n_d$  vertices by placing a graph of  $G_0$  on each of the lines parallel to one of the axes, i.e., each vertex of the grid is contained in  $d$  graphs of  $G_0$ . If the chromatic number of graphs contained in  $G_0$  does not exceed  $k$ , the chromatic number of the graphs contained in  $G_0^d$  does not exceed  $k^d$ . Hartke, Kaul, Král, and West showed that this bound is best the possible by showing the  $d$ -dimensional Cartesian product of  $k$ -colorable graphs contains a graph with chromatic number  $k^d$ . They also showed that the  $d$ -dimensional Cartesian product of single-edge graphs contains a graph with chromatic number approximately equal to the square root of  $d/\log d$ .

### *Mixed hypergraphs and other coloring problems*

A mixed hypergraph is a triple  $(V, C, D)$  where  $V$  is the vertex set and  $C$  and  $D$  are families of subsets of  $V$  called  $C$ -edges and  $D$ -edges, respectively. A proper coloring of a mixed hypergraph  $(V, C, D)$  is a coloring of its vertices such that no  $C$ -edge is polychromatic and no  $D$ -edge is monochromatic. Dan Král, Georgia Tech and formerly DIMATIA, obtained results relating coloring mixed hypergraphs to other graph coloring problems. In particular, he proved that mixed hypergraphs can be used to efficiently model several graph coloring problems including homomorphisms of simple graphs and multigraphs, circular colorings,  $(H, C, \leq K)$ -colorings,  $(H, C, K)$ -colorings, locally surjective, locally bijective and locally injective homomorphisms,  $L(p, q)$ -labelings, the channel assignment problem,  $T$ -colorings and generalized  $T$ -colorings. These results were inspired by problems on mixed hypergraphs that Král learned from Zsolt Tuza during the DIMACS/DIMATIA/ Rényi meeting held at DIMACS in 2003.

### *Local chromatic number of quadrangulations of surfaces*

A quadrangulation of a surface is a graph embedded in the surface such that every face is a quadrilateral. Clearly, such graphs in the plane are bipartite, but some quadrangulations of the torus are 3-chromatic. A surprising result of Youngs states that quadrangulations of the projective plane are either bipartite or 4-chromatic. A generalization of this statement by Mohar,

Seymour, and Archdeacon et al. states that if a quadrangulation of a non-orientable surface satisfy a certain parity constraint then it is not 3-colorable. The local chromatic number of graphs was introduced by Erdős et al. It is the minimum number of colors in the most colorful closed neighborhood of a vertex in a proper coloring of the graph. E.g., a graph is locally 3-colorable if there is proper coloring with any number of colors where the neighbors of any vertex have at most two different colors. Locally 3-chromatic graphs exist with arbitrarily large (ordinary) chromatic number. Gábor Tardos, Simon Fraser University, Bojan Mohar, Simon Fraser University, and Gábor Simonyi, Rényi Institute, considered the generalization of the above mentioned result on quadrangulations of surfaces from chromatic number to local chromatic number. Surprisingly, this is only possible for non-orientable surfaces of genus up to 4. They demonstrated the existence of a locally three-chromatic quadrangulation of the genus 5 non-orientable surface satisfying the required parity constraint. The local three-coloring in the above example uses six different colors. For local three-colorings using only five different colors, they showed that the threshold lies at genus 7.

### *Inequalities for the First-Fit chromatic number*

The First-Fit (or Grundy) chromatic number of  $G$  is defined as the maximum number of classes in an ordered partition of  $V(G)$  into independent sets so that each vertex has a neighbor in each set earlier than its own. The well-known Nordhaus-Gaddum Inequality states that the sum of the ordinary chromatic numbers of an  $n$ -vertex graph and its complement is at most  $n+1$ . M. Zaker suggested finding the analogous inequality for the First-Fit chromatic number. Zoltan Furedi, UIUC, and Rényi Institute, together with Gyarfás, Sarkozy, and Selkow, has found a sharp upper bound for the case  $n=9$ . They extended the problem for multicolorings as well and have asymptotic results for infinitely many cases. Furedi, Gyarfás, Sarkozy, and Selkow also showed that the smallest order of  $C_4$ -free bipartite graphs with First Fit chromatic number equal to  $k$  is asymptotically  $2k^2$ .

### *Local chromatic number, Ky Fan's theorem, and circular colorings*

The local chromatic number of a graph is in between the chromatic and fractional chromatic numbers. This motivates the study of the local chromatic number of graphs for which these quantities are far apart. Such graphs include Kneser graphs, their vertex color-critical subgraphs, the Schrijver (or stable Kneser) graphs; Mycielski graphs, and their generalization; and Borsuk graphs. Gábor Tardos, Simon Fraser University and Rényi Institute, Bojan Mohar, Simon Fraser University, and Gábor Simonyi, Rényi Institute found more or less tight bounds for the local chromatic number of many of these graphs.

### *On graphs whose maximal cliques and stable sets intersect*

A graph  $G$  has the CIS-property (and  $G$  is called a CIS-graph) if every maximal clique and independent set in  $G$  intersect. Vladimir Gurvich, Rutgers, and Endre Boros, Rutgers, have been working on the problem of finding necessary and sufficient conditions for the CIS-property to hold. In general, the problems of efficient characterization and recognition of the CIS-graphs remain open. The CIS-property can be generalized as follows. A  $d$ -graph is a complete graph whose edges are colored by  $d$  colors. A  $d$ -graph  $H = (V; E_1, \dots, E_d)$  is called a CIS- $d$ -graph if the

intersection of maximal vertex sets whose vertices are not connected by color  $i$ , is not empty. A  $d$ -graph without 3-colored triangles is called a Gallai  $d$ -graph. It has been conjectured since 1978 that every CIS- $d$ -graph is a Gallai  $d$ -graph. Gurvich and Boros showed that the Gallai CIS- $d$ -graphs can be efficiently characterized in terms of CIS-graphs. A survey paper will appear.

### *Channel assignment problem with variable weights*

Distance constrained labelings of graphs form an important model for radio frequency assignment problems. An  $L(p_1, \dots, p_k)$ -labeling of a graph  $G$  for integers  $p_1, \dots, p_k$  is a labeling of its vertices by non-negative integers such that the labels of two vertices at distance  $i$  differ by at least  $p_i$ . The least number  $K$  for which there is a proper  $L(p_1, \dots, p_k)$ -labeling by integers between 0 and  $K$  is denoted  $\lambda_{\{p_1, \dots, p_k\}}(G)$ . Note that  $\lambda_{\{1, \dots, 1\}}(G) + 1$  is the chromatic number of the  $k$ -th power of  $G$ . Jerrold Griggs, University of South Carolina, and his Ph.D. student, Xiaohua Teresa Jin studied the dependency of  $\lambda_{\{p_1, \dots, p_k\}}(G)$  on the parameters  $p_1, \dots, p_k$  (when  $G$  is a fixed finite or infinite graph). Daniel Král, Georgia Tech and formerly DIMATIA, addressed this problem in a more general setting using a notion of lambda-graphs and proved several conjectures posed by Griggs and Jin: Piecewise Linearity Conjecture, Coefficient Bound Conjecture and Delta Bound Conjecture.

### *The Firefighter Problem*

Graduate student Paul Raff, Rutgers, Mathematics, began a collaboration with DIMACS post doc Kah Loon Ng, after Raff attended a talk by Ng on the firefighter problem. DIMACS visitor James Abello and Rutgers graduate Stephen Hartke, UIUC, have also participated in this research. Raff and Ng investigated a special case of the so-called firefighter problem, which involves the following scenario: in a given graph a finite number of vertices are initially “on fire” or diseased. At each turn  $t$ , a number  $f(t)$  of firefighters (or vaccines) are able to be positioned at unburned vertices to prevent the fire from spreading there. Once a vertex is on fire or defended it stays that way permanently. The main goal of the firefighter problem is to determine if it is possible to have a scenario where there are vertices that are not defended yet can never catch fire. Raff and Ng’s work concentrated on the two-dimensional grid as the underlying graph. It was previously known that having exactly one firefighter per turn would not be sufficient to contain the fire, whereas having exactly two firefighters per turn would. Raff and Ng extended these results and proved that if more than 1.5 firefighters on average are available per turn then the fire could be contained. This work was jointly supported by the NSF grant for Special Focus on Computational and Mathematical Epidemiology.

### *Modeling spread of disease through networks*

The spread of disease through social networks can be modeled using graph theory, where colored vertices represent those who have the disease (red) and those who do not (green). In one simple model, if there are  $k$  red neighbors of a green vertex, the disease spreads to that green vertex in the next time period. An irreversible  $k$ -conversion set is a set of vertices such that if they are all colored red, then all vertices will end up being red -- i.e., all will be infected. Paul Dreyer (RAND Corporation) and Fred Roberts (DIMACS) found necessary and sufficient conditions for a set to be an irreversible  $r$ -conversion set in an  $r$ -regular graph and found both exact values and bounds for the size of the smallest irreversible  $k$ -conversion sets in rectangular and toroidal

grids, trees, and other special graphs. They also obtained results on strategies for “vaccinating” green vertices so that they could not turn red, a problem closely related to the firefighter problem described in the finding called *The Firefighter Problem*. This work was jointly supported by the NSF grant for Special Focus on Computational and Mathematical Epidemiology.

There has been fruitful interaction between this project and the DIMACS/DIMATIA Research for Undergraduates program.

#### *Group-valued edge-colorings of cubic graphs*

Daniel Král, Georgia Tech and formerly DIMATIA, E. Macajova, graduate student at Comenius University, Bratislava, Slovakia, O. Pangrac, Charles University and former graduate student coordinator in the DIMACS-DIMATIA REU program of 2001, Andre Raspaud, Laboratoire Bordelais de Recherche en Informatique, J. S. Sereni, graduate student, Nice, and M. Skoviera, Comenius University, Bratislava, studied edge-colorings of bridgeless cubic graphs by non-zero elements of finite Abelian groups that satisfy that the colors of the three edges incident with the same vertex are different and their sum is zero. Such a coloring always exists for some groups while for other groups, its existence is equivalent to 3-edge-colorability or is implied by the conjecture of Berge and Fulkerson on the existence of six perfect matchings covering each edge twice.

#### *Construction of large graphs with no optimal surjective $L(2,1)$ -labelings*

An  $L(2,1)$ -labeling of a graph  $G$  is a mapping of the vertices of  $G$  to a  $\{0, \dots, K\}$  such that the labels of two adjacent vertices differ by at least two and the labels of vertices at distance two differ by at least one. A hole of  $c$  is an integer in  $\{0, \dots, K\}$  that is not used as a label for any vertex of  $G$ . The smallest integer  $K$  for which an  $L(2,1)$ -labeling of  $G$  exists is denoted by  $\lambda(G)$ . The minimum number of holes in an optimal labeling, i.e., a labeling with  $K=\lambda(G)$ , is denoted by  $\rho(G)$ . Georges and Mauro showed that  $\rho(G)$  is less than or equal to  $\Delta$ , the maximum degree of  $G$  and conjectured that if  $\rho(G)$  is equal  $\Delta$  and  $G$  is connected then the  $G$  has at most  $\Delta(\Delta + 1)$  vertices. Daniel Král, Georgia Tech and formerly DIMATIA, Riste Skrekovski, University of Ljubljana, Slovenia, and Martin Tancer undergraduate at Charles University, disproved this by the construction of a counterexample.

#### *Diameters of duals are linear*

Jaroslav Nešetřil, DIMATIA, and Ida Svejdarova, undergraduate at Charles University, proved that dual graphs and relational structures are connected. Given an oriented tree  $T$ , one can construct a graph, the dual of  $T$ , such that  $T$  does not homomorphically embed in a graph  $G$  if and only if  $G$  does homomorphically embed in the dual of  $T$ . Though the dual of  $T$  may have an exponential in  $|V(T)|$  number of vertices, Nešetřil and Svejdarova proved that these exponential structures have a linear diameter, which they determined up to a constant. This collaboration led to the application of Svejdarova to the PhD program at University of Illinois, Urbana Champaign where she is now pursuing graduate work.

#### *The circular chromatic index of flower snarks*

A graph is  $k$ -edge-colorable if its edges can be colored using  $k$  colors in such a way that no two adjacent edges receive the same color. By a classical theorem of Vizing every cubic graph is 4-edge-colorable, and hence cubic graphs fall into two categories: those that are 3-edge-colorable, and those that require four colors. Those of the latter kind that satisfy a mild connectivity requirement (cyclic 4-connectivity) are called snarks. Snarks are of great interest because the nonexistence of planar snarks is equivalent to the Four Color Theorem, and it is known that a minimal counterexample to several important conjectures must be a snark. M. Ghebleh, graduate student at Simon Fraser University, Dan Král, Georgia Tech and formerly DIMATIA, S. Norine, graduate student at Georgia Tech, and Robin Thomas, Georgia Tech, determined the circular chromatic index of flower snarks.

#### **IV. Project Training/Development**

The meetings offered many students the opportunity to travel to and participate in international collaborations. One of these opportunities was specifically for students: participation in DIMATIA's Spring School. Two students, David Howard, Georgia Tech, and Paul Raff, Rutgers, were selected to participate, based on the recommendation of their advisors. David Howard describes the School and its impact.

“The program worked as such. The Spring School was an 11 day program that consisted of about 30-40 lectures ranging in many different topics in Combinatorics. The Spring School took place in the small village of Borova Lada (population <400 people). This was very good because there were few distractions to take away from the lectures. I enjoyed many of the talks given over the 11 day span.

“One of my favorite talks was on a theorem of Erdős which said that given a large enough minimum degree and no  $r+1$  clique exists then the graph is  $r$ -colorable. I found this very intriguing because one would think that increasing the minimum degree should increase the number of colors. Another good talk introduced a notion of  $(\sigma, \rho)$  dominating sets and though this would be difficult to describe here I mention it because I am interested in perhaps trying to tackle some of the open problems listed from the talk. There were many other speakers who presented other interesting material but these two were my favorites. I also gave a talk at the conference which I think is very good for me, since I hope one day I will finish my doctorate and continue in research and will have to give many more talks along the way. Thus this was great practice for things like that. Also there were a few students that were very interested in my talks and I left them a few open problems for them to think about. I hope they will have new ideas on the subject so that maybe these problems can get solved.

“I met many people in the Czech Republic who I would now consider friends. Most of the contacts I made were also students but I made a couple of contacts with professors as well. I actually lack many last names but I do have their emails for contact information. Most of the student I met were from the Czech Republic although I also met two graduate students from Norway, two students from Bordeaux, France (Louis Esperet was one of the students whose last name I do have). I met Professor Petr Golovach, from Russia who gave the talk on  $(\sigma, \rho)$



dominating sets. I met a post doc, Dirk Schlatter, from Germany, and finally I reaffirmed ties with my friend Paul Raff at Rutgers University.

“As a result of this trip I am hoping to apply for a Fulbright scholarship to Charles University in Prague to study Combinatorics there. I am hoping to study with professor Nešetřil. I have contacted him already and he has said that he hopes I will be selected to come.

“This conference was excellent in all aspects and I am very thankful to have been given the opportunity to have gone to this Spring School. I believe I am more knowledgeable in my area of research. I made many contacts as I said above, and on top of all that I had a lot of fun in the process. I also had a chance to present material that other students were very interested in looking into, and hopefully progress will be made on some of these problems.” Dave Howard, graduate student, Georgia Tech

Paul Raff, graduate student, Rutgers University, was the other student who attended the Spring School in April, 2006. He also had an excellent experience.

“When I was first given the opportunity to go to the Spring School of Combinatorics in Borova Lada, Czech Republic, I was initially cautious. There was only one other American going on the trip, and this was my first truly independent academic-related trip, and it was across the ocean. Now that the Spring School is over, I hope I can go back next year, and possibly even sooner!

“The Spring School of Combinatorics is a program that was set up approximately 10 years ago in the Czech Republic, through grants given by the European equivalent of the NSF. It was, however, through NSF funding that I was able to go. The structure of the program is fairly informal: four talks are given every day, and there is no restriction on the subject of the talk. At the 2006 Spring School, there were two distinct groups of participants: the first group consisted of students, mostly from the Czech Republic, who were at the level of a junior or senior in a university here in America. The second group of academics were people who were well into graduate school or beyond. As such, the talks given by these two groups were different - the talks in the first group were usually talks about a paper that was read and studied by the student, whereas the talks given by the second group usually involved original research by the speaker. The structure was beneficial for everyone, as the younger students were able to get a broad glimpse of mathematical research at the graduate level and above, and the second group (myself included) was able to explore other areas of combinatorics.

“At the Spring School, I gave a talk on the so-called firefighter problem and the research I and Dr. Kah Loon Ng did during the past year with assistance from DIMACS. I was fascinated from a talk given by Louis Esperet at the Laboratoire Bordelais de Recherche en Informatique in France on  $(d; 1)$ -total labeling of sparse graphs. I have been reading through his paper and hope to do further work with him, as his paper leaves a lot of solvable open problems. I also became very interested in scheduling problems posed by Kjartan Hoie and Shuang Wang at the University of Bergen, in Norway. Above all, though, I was stunned by the possibility of a new lower bound on the diagonal Ramsey number  $R(k; k)$  claimed by Gabor Hegedus at the Budapest University of Technology and Economics. I am surprised that it hasn't been heard of back here in the states, but my peers were also very excited to learn of the new lower bound.

“I was very pleased with the response I received from the talk I gave at the Spring School, and it actually helped shape the direction I would like to head with my future research. Part of my first paper on the firefighter problem involved so-called containment certificates, which wasn't really a focus of the paper but a nice tool to achieve new results. Based on the response from my talk, I decided to put more focus into the structure and theory behind containment certificates, and in my initial work with them I realized that they have great power in them, mainly to prove negative results (most of my results so far have been positive). Now that my first paper is submitted to the SIAM Journal of Discrete Mathematics, I plan on using this summer and beyond to work more with containment certificates, with the help of Dr. Ng and Elizabeth Gillaspay, an REU student this summer that will be working with us.” Paul Raff, graduate student, Rutgers University

There has been a close connection between this project and the DIMACS/DIMATIA Research Experience for Undergraduates (REU), with several of the participants serving as mentors to REU students on research related to this project. Vladimir Gurvich, Rutgers, worked on the topic of graphs whose maximal cliques and stable sets intersect with Bianca Viray, 2004 REU-student, and later with Steven Jaslár, a former REU student.

Several students from our various partners, who participated in this project, have or almost finished their theses (in part stimulated by the project). Three examples are Stephen Hartke, University of Illinois, a student of Fred Roberts of Rutgers, Daniel Král, Georgia Tech, a student of Jaroslav Nešetřil, DIMATIA, Nick Weininger, a student of Jeff Kahn, Bill Cuckler, a student of Jeff Kahn, and Diogo Andrade, 2006 expected graduation, a student of Vladimir Gurvich, Rutgers.

## V. Outreach Activities

Project visitors, graduate students, and senior faculty were available to interact with 2- and 4-year college faculty in the DIMACS “Reconnect” program, and with high school teachers in the DIMACS Connect Institute and the DIMACS Bio-Math Connect Institute.

## VI. Papers/Books/Internet

R. Babilon, V. Jelinek, D. Král, and P. Valtr, “Labelings of graphs with fixed and variable edge-weights,” KAM-DIMATIA Series 2004-701, 2004.

M. Balek (ed.), *X. Midsummer Combinatorial Workshop*, KAM-DIMATIA Series 2004-682, 2004.

I. Bárány, and J. Matoušek, “Berge's theorem, fractional Helly, and art galleries,” *European J. Comb.*, to appear,

- I. Bárány, and P. Valtr, “Planar point sets with a small number of empty convex polygons,” *Studia Math Hung.* **41** (2004), 243-266.
- M. Bodirsky and J. Kara, “The Complexity of Equality Constraint Languages,” KAM-DIMATIA Series 2005-743, 2005.
- E. Boros and V. Gurvich, “On graphs whose maximal cliques and stable sets intersect,” in preparation.
- M. DeVos and R. Samal, “High Girth Cubic Graphs Map to the Clebsch Graph,” ITI Setries 2006-282, Charles University, Prague, 2006.
- Z. Dvorak, D. Král, and O. Pangrac, “Locally consistent constraint satisfaction problems,” *Theoretical Computer Science*, **348** (2005), 187-206.
- J. Fiala, D. Král, R. Škrekovski, “Brooks-type theorem for generalized list T-Coloring,” *SIAM Journal of Discrete Mathematics*, **19** (2005), 588-609.
- J. Fiala, and R. Škrekovski, “Generalized list T-colorings of cycles,” *Discrete Applied Mathematics*, **148** (2005), 13-25.
- M. Ghebleh, D. Král, S. Norine, and R. Thomas, “The circular chromatic index of flower snarks,” *KAM-DIMATIA Series 2005-731*, 2005.
- J. R. Griggs and X. T. Jin, "Real number graph labellings with distance conditions," *SIAM J. Discrete Math.*, to appear.
- B. Grunbaum, T. Kaiser, D. Král, M. Rosenfeld, “Equipartite polytopes and graphs,” *ITI Setries 2004-201*, Charles University, Prague, 2004.
- S. Hartke, H. Kaul, D. Král, and D. West, “Colorings of Cartesian products of classes of graphs,” in preparation.
- J. Hladky, M. Krcal, and B. Lidicky (eds.), “Spring School on Combinatorics,” *KAM-DIMATIA Series 2005-720*, 2005.
- V. Jungic, J. Nešetřil and R. Radoicic, “Rainbow Ramsey Theory,” *KAM-DIMATIA Series 2004-664*, 2004.
- V. Jungic, J. Nešetřil and R. Radoicic, “Rainbow Ramsey Theory,” *Integers: Electronic Journal of Combinatorial Number Theory*, **5** (2005).
- D. Král, “Coloring powers of chordal graphs,” *SIAM J. Discrete Math.*, **18** (2004), 451-461.
- D. Král, “An exact algorithm for the channel assignment problem,” *Discrete Applied Mathematics*, **145** (2005), 326–331.

D. Král, “Channel assignment problem with variable weights,” *SIAM Journal on Discrete Mathematics*, to appear.

D. Král, “Mixed Hypergraphs and Other Coloring Problems,” *Discrete Mathematics*, to appear.

D. Král, E. Macajova, O. Pangrac, A. Raspaud, J. S. Sereni, and M. Skoviera, “Group-constrained edge-colorings of cubic graphs,” in preparation.

D. Král, R. Skrekovski, and M. Tancer, “Construction of large graphs with no optimal surjective  $L(2,1)$ -labelings,” *KAM-DIMATIA Series 2005-720*, 2005.

J. Nešetřil and I. Svejdarova, “Diameters of duals are linear,” *KAM-DIMATIA Series 2005-720*, 2005.

### **Talks**

Z. Dvorak and V. Jelinek, “On the Complexity of the  $G$ -Reconstruction Problem,” Algorithms and Computation: 16th International Symposium, ISAAC 2005, Sanya, Hainan, China, December 19-21, 2005.

D. Howard, “Strange Combinatorial Connections,” DIMATIA Spring School, Borova Lada, Czech Republic, April 27, 2006.

P. Raff, “The Firefighter Problem,” DIMATIA Spring School, Borova Lada, Czech Republic, April 28, 2006.

F. Roberts, “Consensus List Colorings and Physical Mapping of DNA,” Combinatorial Challenges 2006 (A Meeting in Celebration of Pavol Hell's 60th Birthday), Victoria, BC, Canada), May, 2006.

F. Roberts, “Competition Graphs of Semiorders,” Conference on Probabilistic Combinatorics and Algorithms: A Conference in Honor of Joel Spencer's 60th Birthday, April, 2006.

### **VII. Other Products**

Main web site for DIMACS-DIMATIA- Rényi Collaboration on Discrete Mathematics and Theoretical Computer Science  
<http://dimacs.rutgers.edu/Research/partnership/>

Main web site for DIMACS/DIMATIA/Rényi Working Group on Algebraic and Geometric Methods in Combinatorics II  
<http://dimacs.rutgers.edu/Workshops/Algebraic2/>

Main web site for DIMACS/DIMATIA/Rényi Working Group on Graph Colorings and their Generalizations II

<http://dimacs.rutgers.edu/Workshops/GraphColor2/>

Main web site for DIMACS/DIMATIA/Rényi Working Group on Extremal Combinatorics II

<http://dimacs.rutgers.edu/Workshops/Extremal2/>

Main web site for DIMACS/DIMATIA/Rényi Working Group on Graph Colorings and their Generalizations III

<http://dimacs.rutgers.edu/Workshops/GraphColor3/>

Main web site for DIMACS/DIMATIA/Rényi Working Group on Extremal Combinatorics III

<http://dimacs.rutgers.edu/Workshops/Extremal3/>

Main web site for DIMACS/DIMATIA/Rényi Working Group on Algebraic and Geometric Methods in Combinatorics III

<http://dimacs.rutgers.edu/Workshops/Algebraic3/>

Main web site for DIMACS/DIMATIA/Rényi Combinatorial Challenges Meeting

<http://dimacs.rutgers.edu/Workshops/CombChallenge/>

## **VIII. Contributions within Discipline**

This report has already addressed the many problems that have been solved and new areas of research and collaboration that have been developed. While many of these collaborations are documented in this report, we expect that this project will lead to substantially more in the future. Here is some of the feedback we have received about the impact of such collaborations.

During the DIMACS/DIMATIA/Rényi Combinatorial Challenges Meeting (April 26 - 29, 2006) Noga Alon, Tel Aviv University and IAS, Princeton learned from Benny Sudakov, Princeton, about a conjecture of Ahlswede, Cai and Zhang on cross-intersecting families, raised in 1989, and a related conjecture of Sgall. Together with Eyal Lubetzky, who also attended the meeting, they have been working on this conjecture and managed to obtain an asymptotic solution that may even lead to a precise solution. The work on the project is still in progress, and a paper is in preparation.

“Attending the DIMACS/DIMATIA/Rényi Combinatorial Challenges conference had the following positive influence on my research: besides hearing many interesting talks and meeting with several significant researchers in the field of Discrete Mathematics as a general benefit, we discussed several issues about the minor crossing number and how general lower bounds on the ordinary crossing number could be translated or newly developed for this graph invariant with László Székely. The questions seem interesting and tractable, and will eventually result in stronger bounds for this graph invariant for some graph families.” Drago Bokal, graduate student, Simon Fraser University

“As a result of the [Combinatorial Challenges] conference, I have potential collaboration with: Gyula Katona, in the interface of adaptive coding theory and set systems; and Milan Bradonjic and Nicole Immerlica on theory and application of random geometric graphs, especially in determining graph diameter for general random geometric graph models.

“There were several other talks beyond these discussions which I found particularly instructive and will most likely impact the direction of my future research, especially those of Van Vu and Vera Asodi.” Robert Ellis, Illinois Institute of Technology

“I attended the April 26-29, 2006 meeting. As a result of that, I am currently working with Prof. Van Vu on an algorithmic lower bound on the wake-up problem in asynchronous radio networks. Our results are currently of a preliminary nature. We have partial results on the lower bound for network topologies having specific diameters only.” Rohan Fernandes, graduate student, Rutgers University

“I attended the graph coloring workshop in 2003, and also the big workshop held in April. I learned in the first workshop that researchers in Prague were considering models for graph labelling with real numbers that were closely related to my own proposals. I eventually formulated several general conjectures for graph labellings with distance conditions with my Ph.D. student, Xiaohua Teresa Jin.

“One of the Czechs at the 2003 workshop, Daniel Král, eventually proved all of the conjectures in a more general setting, called lambda graphs. I met him again at the recent workshop, and we are working on a survey article spanning the remarkable recent progress in this field of real number labellings.

“At the recent workshop, I was most pleased to renew my connections with Gyula Katona and Zoltan Furedi of the Rényi Institute. We are in the process of considering new projects in extremal set theory. I visited them recently in Budapest, and am anxious to see them again soon to push our ideas further.

“As far as training and development, my own student, Teresa Jin, spoke at the 2003 workshop, and went on to complete a worthwhile dissertation. The workshop gave her good exposure and allowed her to make new contacts. It influenced her research.

“I believe the idea of bringing together researchers on these topics from the three countries (institutes) is wise and fruitful. Another round of meetings like this would be a great idea.” Jerry Griggs, University of South Carolina

“The focus of the conference April 26-29 was strongly on extremal and probabilistic combinatorics, while I am an algebraic combinatorialist. But in addition to being very interesting for me to see what is going on in other branches of the subject, the conference has directly influenced my research. In particular, Gabor Tardos gave a talk where he discussed an analogue of the chromatic number of a graph. This analogue concerns certain proper colorings of a graph with restrictions on the number of colors used on the neighborhood of any vertex. At the end of the talk, I asked if anyone had considered using this notion to define an analogue of the

chromatic polynomial and Tardos did not know of one. Searching the web turned up nothing as well, so I have started to develop this idea myself and expect it to turn into one or more publications.” Bruce Sagan, Michigan State University, DIMACS visitor.

During Ricky Pollack’s visit at the Rényi Institute, he had extensive scientific discussions with Geza Toth, Gabor Tardos, Karoly Borozsky Jr., Gabor Fejes Toth, and Janos Pach. With Gabor Fejes Toth Pollack discussed the recent solution of the Kepler conjecture by Thomas Hales and the various options for publishing it. Pollack and Janos Pach discussed several problems in Graph Drawing. One in particular considers drawings where the edges are portions of algebraic curves defined by polynomials of fixed degree. This is very much related to Pollack’s work on Betti number bounds, applications and algorithms. Pollack also collaborated with Karoly Borozsky Jr. on packing in hyperbolic space.

### **IX. Contributions -- other Disciplines**

This is an inherently interdisciplinary project. Connections between computer science, mathematics, statistics, and other disciplines were brought to light. New results about  $L(2,1)$ -labeling of graphs have applications to the assignment of frequencies to transmitters that avoid interference. Fred Roberts’ work on consensus list colorings of graphs has applications to biology, specifically to the physical mapping of DNA.

### **X. Contributions -- Human Resource Development**

One of the key components of the meetings was the strong participation of students and postdoctoral fellows. The meetings included many talks that were presented by students or reflected their work. The meetings also provided substantial travel support for many students.

Ian Levitt, a graduate student at Rutgers, attended the 2004 Budapest meeting early in his graduate student career. He reports the impact of the meeting as follows:

“I would like to begin by heartily thanking Dr. Latka, Dr. Roberts, and all of the organizers and administrators who made it possible for me to attend this conference. It was an invaluable experience for me and I am deeply grateful for the opportunity. It came at an important time in my graduate education. I have recently passed the general requirements for the PhD program and am now beginning to focus my attention. This conference was the perfect way to "shop around" for ideas and to see what is happening in a range of fields. Indeed, the conference helped solidify my commitment to doing Hungarian-style combinatorics, i.e. extremal and probabilistic. I was able to have excellent conversations with some mathematicians that I otherwise would not have been able to meet. For example, I spent a day with two of Szemerédi's former students, Ryan Martin and Gabor Sarkozy, during which we had much excellent mathematical discussion. Gabor had some open problems that we discussed, and Ryan had some suggestions for my summer reading list. Some of the talks which were especially interesting to me were those given by Lovasz, Rucinski, Simonovits, and Tardos. Lovasz's talk was specially exciting, and I am looking for ways in! These four talks have given me much to research and become acquainted with. Finally, I would like to note that another important benefit of this

conference was that it enabled me to become familiar with the Hungarian language. It may seem like a small thing, but being in Hungary is the best way to know Hungarian, and it is important to me to be comfortable with the language since I expect to be involved with its speakers.

“I wanted to add that another benefit was that the trip provided a great opportunity to bond with my fellow graduate students. Probably the best conversations (mathematical and otherwise) during the whole trip came between Vince, Nick, Bill, and myself, and these came rather frequently.” Ian Levitt, graduate student, Rutgers University

William Cuckler, a graduate student at Rutgers, attended the Extremal Combinatorics meetings in Budapest in both 2003 and 2004. Earlier reports described the impact of the 2003 meeting. He found the experience to be very valuable and had this to say about the 2004 meeting:

“The title of my talk was Hamiltonian cycles in regular tournaments, which was about recent work on a lower bound of the above. The talk generated some interest from the participants of the conference, including Dr. Rucinski who asked for a copy of the paper. The talk could prove invaluable to my career because it allowed me to establish such contacts and made my result known to many prominent discrete mathematicians. Also, I got the opportunity to practice my speaking skills in a more formal environment and the comments that the participants offered me will help me to improve on my ability to speak at future seminars and conferences.

“Besides my talk, there were other aspects of the conference that were valuable to my development as a discrete mathematician. The talks showcased the recent developments of combinatorics; many open problems and recent techniques were presented that could prove valuable to me in the future. One open problem that I spent some time on and may spend more time on in the future is an extremal problem presented by Dr. Gabor Tardos at the problem session. The new ideas presented were a stimulant to learn more about some different areas of combinatorics that I knew little of before. For example, Dr. Lovasz gave an inspiring talk on graph homomorphisms, statistical physics and quasirandom graphs that demonstrated the use of semidefinite matrices to solve many combinatorial problems. I intend to learn more about the ideas contained in this talk as perhaps they could prove important in the future direction of research.” William Cuckler, graduate student, Rutgers University

Nick Weininger was another Rutgers graduate student who attended the 2004 Budapest meeting. Here are his comments on that meeting and how it related to earlier meetings:

“I gave a 30-minute talk entitled ‘Correlation properties of some random colorings.’ This was the first talk I’d ever given at a conference outside the United States, and also the first time I’d used computer-projected slides; thus it was an educational experience in several respects. After the talk, I had a couple of productive discussions with audience members (Ryan Martin, Gabor Simonyi, Jeff Kahn) about one of the open problems I’d mentioned at the end of the talk. As a result of these discussions, I realized that one of the conjectures I’d made was not true in its full generality, and managed to prove some small yet not totally trivial special cases of another.

“Among the talks I attended, Laszlo Lovasz’s stands out as particularly interesting and useful. The results he presented are potentially applicable to several of the problems I’m working on—



the random-homomorphism problem I talked about, for one, and also an inequality involving Ising model partition functions. Jeff Kahn, Aart Blokhuis, and I had a useful discussion of a problem Jeff presented at the end of his talk.

“The talks at the first Prague conference in December '02 introduced me to the notion of graph homomorphisms, a topic I find really interesting, and led me eventually to formulate one of the problems I've been working on for much of the last year. This is the question of correlation properties of random homomorphisms which I mentioned as an open problem at the end of my talk in Budapest in April.

By the time of the second Prague meeting in August of this year (2004), I'd made some progress on this problem-- thanks in part to conversations in Budapest-- and my talk at that meeting focused on it. I thought that talk was very well received, and I had a number of useful conversations about the problem afterward with Jan Kratochvíl, Jarik Nešetřil, and Matt DeVos. In particular, Prof. Nešetřil pointed me to some relevant recent papers by him and others. One of these contained a lemma which I was able to generalize in order to solve some nontrivial special cases of the homomorphism correlation problem; this will form part of the thesis I am now beginning to prepare.

So, all in all, these conferences have been a demonstrable, significant aid to my research efforts.”  
Nick Weininger, graduate student, Rutgers University

Christine Carroll, a graduate student at Georgia Tech, attended the meeting in Prague in 2005. She found it to be a very rewarding experience.

“My time in Prague was amazingly brief, but I gained a lot from it. The collection of speakers was a very diverse talented group. I made several valuable networking connections, which I am maintaining well after the conference has ended. I met Enrique Esteva, from Cinvestav University, now a faculty member at West Virginia Wheeling Jesuit University. We have met up at several conferences since then and have been sharing information about job and conference opportunities, as well as a variety of random graph problems.

“Though all of the talks were quite good, one of the talks in particular made a large impression on me. Katona's talk on techniques for counting the number of Boolean set systems which exclude certain graph structures was extremely informative. One of the main research problems I am working on is counting the number of union closed set systems in various contexts, including in the Boolean Cube. This problem can be viewed as the exclusion of the complete bipartite graph with two vertices in the bottom set and one in the top. Katona discussed the example where we want to exclude this structure when it is not necessarily induced. Though my context needs the stricter requirement of excluding only induced copies of this subgraph, but his techniques and view of this problem have given me intuition on how to proceed.” Christina Carroll, graduate student, Georgia Tech

Phil Matchett Wood had this to say about the April, 2006 meeting that focused on the interconnections among the themes of the project:

“The Combinatorial Challenges meeting at DIMACS was a great opportunity to meet the giants in the field of probabilistic combinatorics. I am studying for my oral quals at the moment, and it was illuminating to hear outstanding mathematicians---Noga Alon, Joel Spence, Janos Pach, and Paul Seymour to name a few---talk about discovering results that I am now studying. The conference brought together a dynamic mix of people who had worked in industry---at RAND Corp. and Microsoft, for example---and people who worked in academic institutions around the world. In my career, I hope to study mathematics that is useful to industry and interesting to academics, and I was excited to meet people at this conference who had done just that.” Philip Matchett Wood, graduate student, Rutgers University

Michael Richter, graduate student, Rutgers, attended the DIMACS/DIMATIA/Rényi Working Group on Extremal Combinatorics meeting in July, 2005 in Prague. He reported:

“I attended the third working group on extremal combinatorics and gained from what I saw while I was there. The talk that I enjoyed the most was the organizer's, Pavel Valtr's talk on the empty hexagon theorem. This was a large outstanding problem in combinatorial geometry on whether having  $n$  points in the plane, for big enough  $n$ , would force there to be a subset of six points that were the vertices of a convex empty hexagon. The problem was answered in the affirmative this year, with a proof by Gerken, and Valtr presented here an updated proof which we were able to see extremely shortly after it had been done. I am now working on a combinatorial geometry problem with a professor, Rados Radoicic at Rutgers University and was lucky to see such an intriguing problem's solution right after it had been solved. So, the trip to Prague provided the perfect introduction to a subject that I am now taking a class in and working with.” Michael Richter, graduate student, Rutgers University

## **XI. Contributions to Resources for Research and Education**

## **XII. Contributions Beyond Science and Engineering**