DIMACS

Center for Discrete Mathematics &
Theoretical Computer Science

DIMACS EDUCATIONAL MODULE SERIES

MODULE 09-1
Geometry of Power Indicess
Date prepared: August 5, 2008

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Module Description Information

- **Title:**

  Geometry of Power Indices

- **Authors:**

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- **Abstract:**

  This module provides an overview of power indices for weighted voting systems with three voters, from a geometric viewpoint, using 2-dimensional simplices to represent both the domain and range of the power index function. Students learn to work with barycentric coordinates; to partition the domain into regions based on the combination of winning coalitions; and to partition the range into regions based on the relative power rankings of the voters. Power indices are also interpreted in terms of linear combinations of basis vectors so that a large family of power indices falls within the convex hull of the basis vectors projected onto the range. The geometry is then used to explore paradoxes and non-simple weighted games.

- **Informal Description:**

  Section 1 provides an overview of weighted voting systems and power indices. In sections 2 and 3, the power index is thought of as a function on coalitions. Section 2 investigates the geometry of the domain when examining a three person voting system, while section 3 investigates the geometry of the range (i.e. power index of each voter). The concept of a power index can be generalized to include situations where coalitions are not just winning or losing. This is introduced in section 4, where vectors and linear algebra plays an important role in the formulation of a power index in this generalized situation. Section 5 gives an example of a voting system where the ranking of the different players, based on their power, depends on the power index used. Potential student projects projects include investigating the paradoxes associated with power indices and analyzing the geometry of power indices of 4-person voting games. Exercises occur throughout the text, to encourage students to make connections among the new concepts as they are introduced. Selected solutions are included.

- **Target Audience:**

  This module is designed for students taking courses at a wide variety of levels. It could be used in courses such as linear algebra, finite mathematics, discrete mathematics, or any kind of “introduction to proof” course. The module becomes progressively more difficult with each section.

- **Prerequisites:**

  This module assumes a very basic knowledge of 3 dimensional rectangular coordinate system, including plotting points in 3 dimensions, and graphing lines and planes.

- **Mathematical Field:**

  Game Theory, Voting Theory

- **Application Areas:**

  Social Sciences

- **Mathematics Subject Classification:**

  Primary: 91B12, 91A12, Secondary: 91F10

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- ## Other DIMACS Modules related to this module:

  None