Selection of Surveillance Centers for Foot and Mouth Disease (FMD): Wisconsin Dairy Farms

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Contents

1 Module Description Information

- **Title: Selection of Surveillance Centers for Foot and Mouth Disease (FMD): Wisconsin Dairy Farms**
- **Author:** Kim A. S. Factor
- **Abstract:** This module investigates the use of dominating sets to determine testing sites for Foot and Mouth Disease (FMD) in dairy cows. It is intended to be used for early warning. An optional segment is included for those wishing to expand the mathematical approach to include some computer science.
- **Information Description**: Dairy cattle are susceptible to a highly contagious disease known as Foot and Mouth Disease. Although it is not currently within the United States, it could be reintroduced at any time. Once it has been identified within an animal population such as dairy cattle, all of the cattle must be euthanized. Additionally, susceptible animals in neighboring areas are also eliminated in order to avoid spreading the disease. The loss of animal life and the economic devastation to the community can be great. It is, therefore, desirable to find places where animals can be tested at regular intervals so that FMD can be identified at the earliest possible time. Additional ideas for expanding the module are also provided.
- **Target Audience**: The intended target audience is sophomore college students who are in their first "proofs" class and/or a discrete mathematics class. It provides a project that will allow the student to personalize the research, make decisions about measurement values, support the decisions in a manner compatible with the material they have been learning in class, and conduct group research into a new area. This is also appropriate for the same level of computer science students who are taking a discrete mathematics course, and suggestions for expanding to this audience are included.
- **Prerequisites:** Students should be at a mathematical maturity level where they understand the idea of a viable argument. The project associated with this module is designed to make use of the logical thinking. There is no need to know graph theory, as the small amount that is actually used in the process is covered in the module itself.
- **Mathematical Field:** Graph theory
- **Application Area:** Biosurveillance
- **Mathematics Subject Classification:** 05C90, 05C69
- **Contact Information:**

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- **Other DIMACS modules related to this module:** None at this time.
- **Keywords:** biosurveillance, graph theory, domination in graphs, domination sets, domination number, foot and mouth disease, dairy farms

2 Abstract

This module investigates the use of dominating sets to determine testing sites for Foot and Mouth Disease (FMD) in dairy cows. It is intended to be used for early warning. An optional segment is included for those wishing to expand the mathematical approach to include some computer science.

3 Introduction

This module is primarily for the use of college instructors who are teaching students being introduced to mathematical proof, problem solving, and/or discrete mathematics. Although primarily designed for mathematics students, it can easily be extended to include computer science students who may also be in the class. It offers a good opportunity for mixed groups of mathematics and computer science students to work together on an application. This is a module that should require approximately two lecture sessions to prepare students for an out-of-class research project. Background material on the necessary mathematics, as well as material on the application are included in the following two sections. A detailed presentation of the project is given after that. Suggestions for extensions of the projects, as well as graduate-level research is included in the conclusion at the end of the module.

The ability to identify diseases at the earliest possible opportunity is an important subject. Since the author lives in Wisconsin, and there are many dairy cows in Wisconsin, Foot and Mouth Disease (FMD) is the disease of interest for this module. It must be noted, however, that FMD affects many cloven-hoofed animals, and the material does not need to be focused as narrowly as it is presented. The particular focus is in determining what farms might be used as testing centers to enable earliest possible detection of FMD in the area. This is done using a graph theoretic model. Students will be responsible for some literature review, modeling, metric selection, and a variety of other tasks designed to continue the mathematical maturation process begun in the course.

As a disclaimer, I need to add that any time I refer to "mathematics" students I include statistics majors, secondary mathematics education majors and elementary education majors with an emphasis in mathematics. At Marquette, we have a joint department, and our students take classes together. This module is appropriate for all students who meet the suggested prerequisites.

4 Graph Theory Concepts

To supplement the following definitions, texts such as Goodaire and Parmenter [1] and Grimaldi [2] are recommended. We begin with the elementary definitions, and then move to specific definitions that will be required for the module. Exercises for the students are included, with solutions supplied in Appendix A.

A *graph* $G = (V, E)$ has a nonempty set of vertices $V(G)$, and a set of *edges* $E(G) = \{uv \mid u, v \in V(G)\}$. Figure 1 illustrates this basic concept. We say that vertices u and v are *adjacent* if uv is an edge in G. Note that edge uv can also be written as vu. The edge uv is *incident* with vertices u and v, which are also incident with the edge.

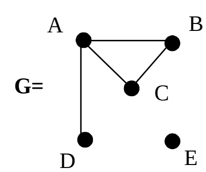

Figure 1: Graph $G = (V, E)$ with $V(G) = \{A, B, C, D, E\}$, and $E(G) = \{AB, AC, AD, BC\}$.

To give examples of some of the preceding definitions, notice that in graph G of Figure 1, vertex A is adjacent to vertices B, C, and D. Vertex E is not adjacent to another vertex. It is called an *isolated vertex*. The edge between vertices A and B is the edge AB. Edge AB is incident with both of its end vertices, A and B.

The following exercise can be provided to students in order to test their understanding of the new terminology. Pay close attention to make sure that they use the term "vertex" when speaking of one vertex, and "vertices" when there are more. Students love to say "vertice" and "vertexes." Also, note that when one edge crosses another, it does not create a vertex.

Exercise 1 *Use the graph H below to answer the following questions.*

1. *Find $V(H)$.*
2. *Find $E(H)$.*
3. *List the vertex or vertices adjacent to vertex z.*
4. *Are there any isolated vertices? List any isolated vertex if yes. If not, explain why.*
5. *With which vertices is edge xu incident?*
6. *Is vertex v incident with any edge? List any edges with which v is incident.*

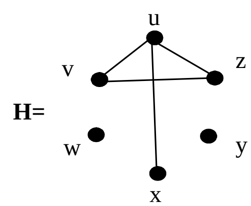

Figure 2: Graph H for use with Exercise 1.

Now we have a common language that can be used as we go forward into the definitions that relate directly to the project at hand. Given a graph G, consider a subset of the vertices of G and call that set S. Now check all vertices v in $V(G)$. If v is either in S or is adjacent to a vertex in S for all vertices $v \in V(G)$, then S is said to be a *vertex domination set* of G. We will call it a *dominating set* or *domination set* for short. For example, consider graph G_1 in Figure 3. Vertices a, c and e have been circled. Let $S = \{a, c, e\}$. Then S is a domination set of G_1. For vertices a, c, and e of G_1, they are in S. Vertex b is adjacent to a and c, which are vertices in S. Note that a and c are said to *dominate* vertex b. Vertices d and f are likewise adjacent to vertices in S. Since all vertices in G_1 are either in S, or adjacent to vertices in S, S is a dominating set. Notice also, that if one of the vertices in S were removed, the resulting set would no longer dominate. For example, if $S = \{a, c\}$, the vertex e would neither be in S nor be adjacent to a vertex in S. Similar scenarios result if vertex a or vertex c were removed from S. Since S is as small as it can be and still dominate, it is a *minimal dominating set*.

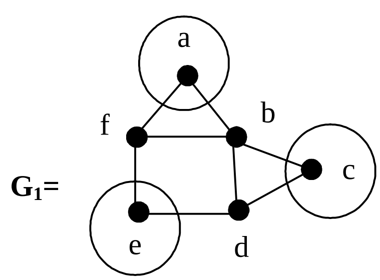

Figure 3: Graph with minimal dominating set circled.

Although G_1 in Figure 3 is shown with a minimal dominating set circled, this is not a "minimum" dominating set. It does not contain the fewest elements of all dominating sets. Consider sets $S_1 = \{a, d\}$ and $S_2 = \{c, f\}$. Both S_1 and S_2 have two elements, and both are dominating sets. A dominating set of G with the fewest elements is called a *minimum dominating set of* G. The size of a minimum dominating set is known as the *domination number of* G, and is denoted $\gamma(G)$. Thus, for graph G_1 in Figure 3, $\gamma(G) = 2$.

The following exercise has several graphs with which to test the student's understanding of domination and domination number. Graphs (A) - (C) follow a theme that will hopefully build the student's understanding.

Exercise 2 *For each of the following graphs,* (1) *if possible, find a minimal dominating set that is not a minimum dominating set, or explain why it is not possible if none exists;* (2) *find a minimum dominating set; and* (3) *determine* $\gamma(G)$.

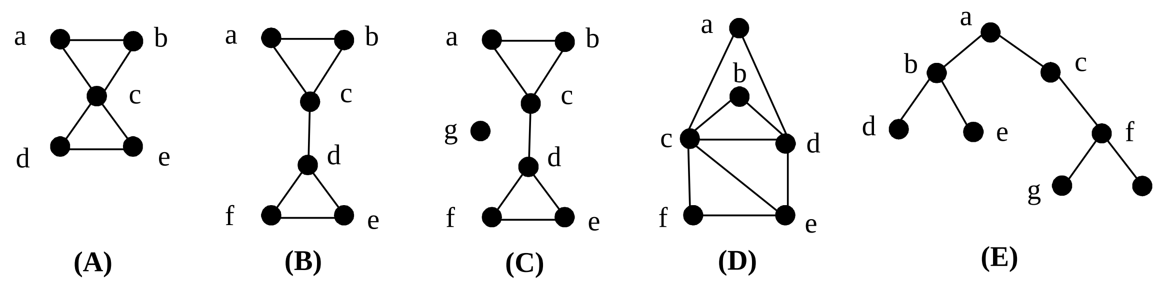

Figure 4: Graphs to use to find dominating sets.

5 Foot and Mouth Disease (FMD)

The following information is exactly what I gave my students for their project. I was unable (because of time constraints) to have my veterinarian come in and talk about FMD. Next year, however, she will be talking to the students and giving them the facts about this disease from a clinical viewpoint. The information below is just a sketch of the information that is out there. My students found out even more during their research, and I like that they were able to do that. Therefore, I will not give additional written information next time. The written background follows.

One of the most feared diseases that affects animals is the Foot and Mouth Disease (FMD). Although it has not been in the United States for decades, it is still found in the world. The Center for Disease Control (CDC) puts a high priority in keeping it out of the U.S. It is part of the biosurveillance that is done by the CDC. Any protocols for early detection are extremely useful, therefore, when they are developed. As a class, we are embarking on a new way of thinking about the biosurveillance. What we will try has not been done before. There is more on that later.

What is FMD?

> "Scientists say that it is a viral disease of cloven-footed mammals, which causes painful blisters to appear in mouths and on feet and udders, and leads to lameness, drooling at the mouth, a loss of appetite and reduced milk production. They explain that, in the short term, many diseased animals will suffer. Some, especially the weak and the young, will die, and even animals recovering from the disease will show costly long-term reductions in growth rates and milk production. They also point out that FMD is one of the most contagious diseases known to man and can spread in almost every way imaginable." (Woods [5])

In the preceding definition, note that the spread of the disease is mentioned. Actually, it is spread very easily, and a herd can be infected quickly. The spread can happen by animals being in near proximity (through the air or water), dirt from an infected herd being tracked on a shoe or tire to another herd, and in many other ways. Once it has entered an area it is extremely difficult to eradicate.

Figure 5 shows the infection rate of a 1000-cow dairy. The pink line shows the number of infected cattle. The yellow line shows how many of the cattle were showing signs of the disease. Notice that most of the herd was already infected by the time the first signs were noticed. Thus, this disease is very hard to diagnose early.

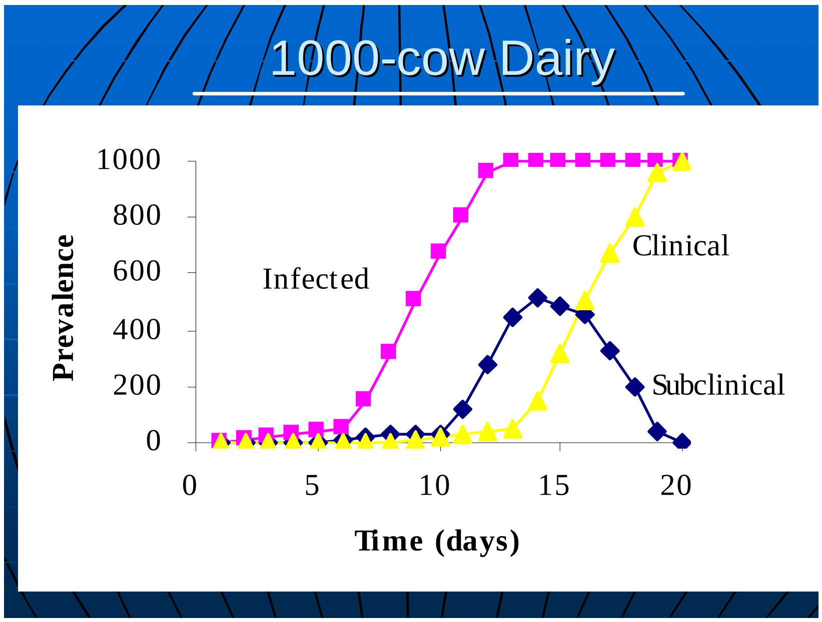

Figure 5: Graph showing that by the time the clinical signs can be observed, the entire herd is infected with FMD. *(Tim Carpenter, 2008)*

Currently, blood tests are not reliable for detecting the disease. Portable sniffers can be used, but they are expensive and it takes a long time to test many cattle. Plus, the cattle have to already be infected for a couple of weeks for the sniffers to work. Thus, it is difficult to detect FMD before it infects a herd, but it is easy to bring the virus in.

How do the farmers feel about the disease? (This is why you must NOT contact any of the farms that are on your list.)

> "Farmers view FMD from a very different perspective (from the scientists). They say that it is one of the diseases they fear the most. It stops them from moving livestock around and sending them to market, restricts their social lives and leaves financial hardship in its wake. An unfortunate minority can recall the day when with a heavy heart, they rang the vet to report suspicious symptoms among their stock. They tell of their pain at seeing their animals slaughtered, a lifetime's work destroyed; of the silence the next morning, and knowing that life would never be quite the same again." (Woods [5])

So how is the disease controlled once it has been found? At the present (this is current as of 2001 when an epidemic in the United Kingdom broke out), every animal in an infected herd is slaughtered. In addition, the animals of all surrounding farms are slaughtered. Any movement of possibly infected animals to places such as slaughter houses, feed lots, 4-H competitions, etc. is traced and the animals (those being traced and any herds with which they have had contact) are slaughtered.

The UK used a circle with a 10 km radius to contain an "infected" area. A circle with a 20 km radius was used to create the "surveillance zone" where animals were quarantined for three weeks to be certain they were not infected. Any animals (cloven-footed animals such as cows, pigs, sheep, etc.) within the 10 km radius were killed whether they were showing signs of the disease or not. You can see from the previous chart that if they waited for the clinical signs of the disease to appear, it would be too late.

Can the disease be cured? The sad truth is that, yes, the disease can be cured. There is even a vaccine for FMD. The problem with vaccinating a herd is that it is impossible to tell if the herd has FMD using a blood test because the vaccine looks like FMD. Economics are the main reason (in my opinion) for the fear of the disease and the killing of all of the animals.

A dairy cow that has had FMD may not produce milk in the same quantity as before. Beef cattle, pigs, and other meat animals still have perfectly good meat, but countries will not buy meat or animal products from infected animals. You may think that it is because people can get the disease. However, no one has ever gotten FMD. Thus, entire herds and farmers' livelihoods are destroyed.

Needless to say, being able to detect the disease as early as possible has extreme importance in saving animal life as well as monetary benefits. In your projects, you will assume that a new detection device is available that can detect FMD within 5 days of infection. Be advised that this is not the case at this time.

If you are interested in learning more about Foot and Mouth Disease, or how animal disease is detected, I suggest Morpurgo [3] for a fictional account that is easy to read, Woods [5] to obtain the view of someone who does not agree with current protocol, and Salman [4] for information on all sorts of surveillance and survey systems for many diseases. Of course, the Internet has a vast amount of information.

6 The Project

6.1 The Rationale

Students in math classes, and especially in "proofs" classes, tend to miss the connection between what they are doing in class and anything outside of the classroom. Additionally, it is often not until something is used that it is understood. Therefore, this project is designed in such a way that (hopefully) the student will be able to use the reasoning tools provided in the class to work on a very important and immediate concern regarding the welfare of animals and the economy involving these animals.

As a way to help the students personalize the research, I have chosen dairy cows. That is because Marquette University is in Milwaukee, Wisconsin. Many of the students come from Wisconsin or nearby states. Since Wisconsin is known as the "Dairy State", and FMD affects dairy herds, it is a natural selection for me.

FMD was chosen because under current detection techniques, the entire herd has been infected by the time the first cow shows signs of the disease. The entire herd, and usually all neighboring herds deemed within a "close" distance, must be euthanized. Not only is this sad, it helps remove a lot of variables we might have to introduce mathematically should the incubation time, percentage of herd infected once the disease is identified, etc. be greatly varied. This disease was also chosen because it is not currently in the United States, so detection becomes paramount when trying to assure a quick response and containment.

In this project as well as most other projects that I assign, I want to be able to see a variety of communication techniques. That is why a formal paper is required, as well as a presentation. I want to make certain the students are able to communicate their findings in writing. I expect this to be much more technical and grade the mathematics in the paper instead of in the presentation. The presentation is an opportunity for students to have some fun with visual aids. I encourage mine to make lots of pictures, be prepared to explain, and to develop a different method of delivery from the paper.

This project forces students to use the definitions that they are given and apply them to something else. For example, the students are introduced to vertices and edges. In the project, they are told in mathematical terms what the vertices and edges must be in this application. Students will also want to be told what "close" is. This may be the first time they have been called upon to design something by themselves. It is a very good instrument for helping students develop some independent thought and an ability to justify their choices.

Note that the blood test at the beginning of the Project Description is a figment of my imagination. It may exist, and it may not.

6.2 Information to Prepare Ahead of Time

The students will be doing a lot of the work in this project, but it is a good idea to have at least two things prepared. If possible, find out where the dairy farms are. If the information is not readily available (from either the Internet or a state institution), determine "reasonably" sized regions within which students will be identifying dairy herds. As long as there are 8 to 12 herds within close proximity, that should be deemed a "reasonable" region. If the class also has computer science students, larger regions may be desirable. We want the computer science students to determine an algorithm that will help find the minimum dominating sets of the region. If there are too few farms, the students will not have much to do. If there are too many farms, then the task of finding the domination number becomes excruciating.

In Appendix B, I have included a list of some Wisconsin farms. I used some of these farms for my own group project. Since there is not a "list of Wisconsin dairy farms" that I could find, I pieced together a very small list using a variety of online resources. Please feel free to use the list and group the farms in whatever order is most useful for your classroom. Also in Appendix B is a Wisconsin map that shows all of the counties.

6.3 Structure and Assessment

1. This is designed for groups of 2 or 3 students. I suggest different data sets (see the following subsection) for each group so that everyone has something that is unique, and that they will be able to share with the class. There is always the question of group self-selection. Since this project takes place later in the course, it would be a good candidate for instructor-selected groups. This is especially true when there are computer science students that we want to incorporate into the groups. In my class, I will allow the groups to self-select, as my choices have not necessarily been better in the past than my students'. However, I still mandate the integration of the computer science students into the groups of mathematics students. (When I implemented this project, I let the groups self-select, as all of my students were mathematics majors. It worked well.)

2. Allow four weeks for the project to be completed. Some time will be needed for students to map out their regions, do associated literature searches, create a viable metric, analyze data, create a new model, write a paper and prepare a presentation.

3. Assessment tools (note that the following supplements are included in Appendix C: Guidelines for groups, rubric used for grading presentations, and the individual student assessment of the group):

   (a) Technical paper: Students are asked to turn in a technical paper, written using correct grammar and spelling. Only one is required per group. They are graded on the writing portion as well as their mathematical and biological arguments. Papers should have figures, charts, and tables where appropriate, and contain a reference page. They should have at least three outside references (internet references are acceptable, but there must be at least one reference that is not from the Web).

   (b) Presentation: Everyone in the group will participate in presenting their findings and recommendations. This occurs during the last week of class. They may use Power Point, overhead transparencies, or other forms of visual communication. Presentations are between 10 and 20 minutes depending upon the number of groups and the time allotted to the presentations. Students are graded on their ability to communicate their findings, the seriousness with which they answer questions, the evidence of time spent preparing the presentation, and the overall techniques used. It is important to grade on things here that will not be graded concurrently in the paper. I strongly advise my students to do something different than reading their paper. I want to see a different communication ability during the presentation than in the paper. Students should aim more for pictures and explanations than in trying to put their findings in small writing on a transparency.

   (c) Individual Student Assessment: (Note that although I give a grade for the group paper and a grade for the group presentation, there will be individual grades that are given that may not be the same as the group grade. Two measurements are used to help with this decision.)

i. Meet with one student from each group at the end of the first, second, and third weeks. They must be able to explain what is happening in the group, and what steps have been taken to that period of time. In this way, it is possible to better judge who knows what is going on, an who is having problems. This is also an excellent time to help groups narrow their focus if they have decided to solve all of the world's problems in four weeks, or to point students in a better direction if they seem to be floundering. Mandatory meetings takes some time (although not really that much), but is a wonderful way to keep your finger on the pulse of the class.

ii. Have each student submit a peer assessment sheet for the project as a whole. It can be very simple, with one column for names, one column for percent of work done, and one column for comments. It allows students to assess the work that they and their group-mates did. I find that most students are very honest, even if they did not do much of the work. This is an important tool when determining grades for the individuals. So that no one but myself sees the sheets, I attach them as a last page to the comprehensive final.

6.4 Project Description

The following are instructions to the students.

Because of your expertise in mathematics, the government has contacted you to suggest farms where surveillance centers can be set up to detect (as early as possible) the contraction of Foot and Mouth Disease (FMD) by dairy cattle in your region. A new blood test has been developed that can identify the disease as early as five days after a cow has been infected. Therefore, they would like to set up testing stations at farms where it would make the most difference in livestock life and economic stability. To help in this endeavor, you will be completing the following tasks, (plus any others that you find help with the analysis) and making your report. You have been broken into groups to look at different regions in Wisconsin. Your region will be delivered to your group in a separate communiqué. Write up your results in a paper. You will also present the results to the entire group.

1. For the dairy herds in the region, create a mileage table, as shown below. The mileage can be approximate. **Herds within the same town or city should be considered to have a distance of 0 miles for the purpose of this analysis.

	Herd A	**Herd B**	**Herd C**
Herd A	0	15	8
Herd B	15	0	0
Herd C	8	0	0

2. Develop a definition for what you will use as a measure of "closeness" between two herds. Remember that once the disease has been detected, the entire herd and the herds of all contiguous (close) herds must be euthanized. We must define "close" so that we do our best to realistically determine all herds that may be infected just by being within the proximity of the diagnosed herd, without having such a large measurement that herds are needlessly destroyed. For example, consider a farm where the next closest farm containing a dairy herd is on the other side of a national forest. Do you count that farm as "close" just because it is the closest? Perform a literature search to see what has been used in the past (if any such measure is available—note that I have included some information in the "Background" section). You may use it, while citing the source, or you can define what you mean by "close." In either case, you need to provide reasons why you choose what you do. These reasons need to convince me.

3. Using your definition for "close" create a graph $G = (V, E)$, where $V(G) = \{\text{dairy herds}\}$ and $E(G) = \{uv \mid u \text{ and } v \text{ are close}\}$.

4. Find $\gamma(G)$.

5. Using $\gamma(G)$, find all minimum dominating sets. Discuss how these sets can be used to determine a site-selection for early detection of FMD in dairy herds for your region.
6. Once FMD has been detected at one of your test sites from part (5), determine what response could minimize the loss of dairy herds. Support every suggestion you make, and include answers to such questions as:
   (a) When would this make a big difference?
   (b) When would it make no difference?
   (c) Discuss any assumptions you are making in this model.
   (d) Will a suggestion for response that minimizes economic loss be the same as the one that minimizes the loss of livestock? Why or why not (think about this one carefully).
7. What additional information would be helpful in creating an early detection (surveillance) strategy? Discuss how the information would help you better choose the detection centers.
8. How might your suggestions be merged with suggestions of the teams from other counties in the state? Would you need to consult with them before implementation? Why? (Note that I am not suggesting you consult with the other groups, just think about what it would entail and if it would be necessary for a statewide protocol.)

If you would like to write this entire project up as a response to the government, that is a great idea. (It is not necessary for those of you who would rather do a traditional paper.) However, be certain that all information that I need is included in the report, and be certain that it contains the names of everyone in your group. For groups that write a traditional paper, I do not want to see answers numbered to correspond to the numbers in the project. This should read like a paper that you turn in as part of an English writing assignment.

6.5 Extensions of the Project

There are so many things that can be done with this project, depending upon what the desired outcome may be. I have tried to group together ideas based upon my own set of similarity rules.

1. Changes to the given project
   (a) If time and gas prices permit, send the groups out to map out the farms in predetermined areas. This will take the place of using prelisted farms such as those in Appendix B, and will further personalize this project.
   (b) Use a different animal that is susceptible to FMD. If desired, all animals can be used, but it is much more difficult to identify the farms. With dairy cows, the farms display placards with the company's name to which they sell their milk. Any farm with such a placard is a dairy farm. In this way, small farms are not included, but we do include the majority of the herds.
   (c) More computation can be included for mixed groups of math and computer science students (engineering students also work well in these groups). This can be done by requiring a computational element in the project. While the sizes of the regions should lend themselves to manual calculations, they will also allow a computer science student to develop an algorithm that can be checked on the smaller sizes and used for much larger environments. For example, algorithms can be made to find $\gamma(G)$. Other algorithms could be designed to find all minimum dominating sets, and to determine the number of farms affected should one of the test sites detect an infected animal.
2. Extensions to the given project

(a) Students may become excited about this and want to do an independent study. Hurray! One suggestion is that they take all of the research done in the class and apply the different metrics to the regions as an aggregate whole. This will necessarily require the creation of a computer program to find some sort of domination number. If that becomes too hard, have the student estimate a "reasonable" number for which they can find a collection of farms to act as the sites for testing. This could not only be a great opportunity for the student, but provide a viable option for a biosurveillance strategy of early detection.

(b) Given the region, find out the travel patterns for the farms (this will not necessarily be easy). Since FMD is transmitted by just about anything that is close to an infected area, travel patterns are of extreme importance when tracking the disease spread. For example, where do the milk trucks go after they leave a farm? When a calf is sold, what stops does it make along the way? Are cattle from the farm shown in 4-H? These are some of the questions that can be posed to extend the lessons learned in the project.

7 Appendix A: Solutions to Exercises

Following are solutions to the exercises in Part 4 of the module.

Exercise 1.1: $V(G) = \{u, v, w, x, y, z\}$ Note that set notation must be used.

Exercise 1.2: $E(G) = \{uv, ux, uz, vz\}$ Set notation must be used, but edges can be listed in any order. End vertices of an edge can also be listed in any order, e.g. uv and vu.

Exercise 1.3: Vertices u and v are adjacent to z since each shares an edge with z.

Exercise 1.4: Both vertex w and vertex y are isolated. Neither is incident with an edge.

Exercise 1.5: All edges are incident with their end vertices. Thus, edge xu is incident with vertices x and u.

Exercise 1.6: Vertex v is incident with two edges: uv and vz.

Exercise 2.A.1: $\{a, e\}$, $\{a, d\}$, $\{b, d\}$, or $\{b, e\}$ are minimal since no vertex can be removed from any of the sets and still have a dominating set of vertices.

Exercise 2.A.2: $\{c\}$ is the only minimum dominating set.

Exercise 2.A.3: $\gamma(G) = 1$ since the smallest dominating set contains one vertex.

Exercise 2.B.1: There is no minimal dominating set that is not maximum. This is because any time we pick one of a, b, or c, all of the three vertices are dominated and the same is true when picking d, e, or f. In order to dominate all of the vertices one of each of these must be chosen. If we add another vertex, it can be removed and the set of vertices will still dominate. For example, if I have $S = \{a, b, d\}$, all of the vertices are dominated. However, I can remove vertex b and still have a dominating set, $\{a, d\}$. Therefore, S is not minimal.

Exercise 2.B.2: Any pair of vertices with one from $\{a, b, c\}$ and one from $\{d, e, f\}$.

Exercise 2.B.3: $\gamma(G) = 2$

Exercise 2.C.1: This has the same answer as 2.B.1. The only difference is the isolated vertex g. It must be in every dominating set (or else it will not be dominated), and the remaining graph has the same restrictions as the graph in part (B).

Exercise 2.C.2: Any triple of vertices such as $\{a, d, g\}$ where one vertex is from $\{a, b, c\}$, one vertex is from $\{d, e, f\}$ and one vertex is g.

Exercise 2.C.3: $\gamma(G) = 3$

Exercise 2.D.1: This is an interesting one because minimal dominating sets can be made with different cardinality. For example, both $\{a, b, f\}$ and $\{d, f\}$ are minimal dominating sets. There are others possible. Just be certain that any set of vertices does not dominated when any of the vertices is removed.

Exercise 2.D.2: $\{c\}$ is the only minimum dominating set.

Exercise 2.D.3: $\gamma(G) = 1$

Exercise 2.E.1: Trees (connected graphs that have no cycles) provide another opportunity to find minimal dominating sets of a variety of sizes. In this graph, dominating sets such as $\{a, d, e, g, h\}$ and $\{b, c, g, h\}$ are minimal since the removal of any of the vertices results in a set that does not dominate.

Exercise 2.E.2: The only minimum dominating set is $\{b, f\}$.

Exercise 2.E.3: $\gamma(G) = 2$

8 Appendix B: List of farms and Wisconsin county map

Figure 6 is a map showing the counties of Wisconsin. It is shown with the permission of Thompson Communications

Figure 6: Counties of Wisconsin. Source: http://www.wicounties.org/WS_County_Directory.asp

The following list of farms with their cities and counties was compiled using a combination of online resources. Many of the farms are quite a ways apart. That is okay, because students need to reason through what "close" means, and justify placement of their surveillance centers. Combine farms (some farms can be in more than one grouping) so that some are in the same town. Try to get contiguous counties. The following list is given in the form: **Farm/Town/County**, and is alphabetical by county.

1. Jerian Holsteins/Barron/Barron
2. New Horizons B/Rice Lake/Barron
3. Morning Glory Dairy/DePere/Brown
4. Wayside Dairy/Greenleaf/Brown

5. Burnett Dairy/Alpha/Burnett
6. Faber/Chilton/Calumet
7. LaClare/Chilton/Calumet
8. Chilton/Chilton/Calumet
9. Hilbert/Hilbert/Calumet
10. E-8984/Boyd/Chippewa
11. Polzin/Cadott/Chippewa
12. Scientific Holsteins/Chippewa Falls/Chippewa
13. Chippewa Falls A/Chippewa Falls/Chippewa
14. Chippewa Falls B/Chippewa Falls/Chippewa
15. E-8979/Stanley/Chippewa
16. New Horizons H/Dorchester/Clark
17. F803/Hendren/Clark
18. Loyal-View & Royal-Mist/Loyal/Clark
19. New Horizons F/Loyal/Clark
20. 512/Spencer/Clark
21. Holland Family/Thorp/Clark
22. 507/Unity/Clark
23. Tetzner Dairy/Washburn/Clark
24. Wargo Acres/Lodi/Columbia
25. Bleu Mont/Blue Mounds/Dane
26. Hinchley's/Cambridge/Dane
27. UW-Madison Dairy Cattle/Madison/Dane
28. Femrite/Stoughton/Dane
29. Car-Bon/Beaver Dam/Dodge
30. Colfax/Colfax/Dunn
31. Bolen Vale/Downing/Dunn
32. Gingerbread Jersey/Augusta/Eau Claire
33. F801/Bridge Creek/Eau Claire
34. Homeland Dairy/Brandon/Fond du Lac
35. Rickert Brothers/Eldorado/Fond du Lac
36. Joas/Oakfield/Fond du Lac
37. Crystal Ball/Osceola/Fond du Lac

38. Pollack-Vu/Ripon/Fond du Lac
39. Vande Holsteins/Waupun/Fond du Lac
40. Alto/Waupun/Fond du Lac
41. New Horizons C/Fennimore/Grant
42. UW-Plattteville Pioneer/Platteville/Grant
43. Sugar River/Albany/Green
44. Brodhead/Brodhead/Green
45. Decatur Dairy/Brodhead/Green
46. Blue Haven/Juda/Green
47. 2002/Monroe/Green
48. Zimm-View/New Glarus/Green
49. Grand View/Berlin/Green Lake
50. Greenfield Hilltop/Markeson/Green Lake
51. Swanke's Dairy/Princeton/Green Lake
52. Blue Marble/Barneveld/Iowa
53. Grass/Dodgeville/Iowa
54. Gildale Holsteins/Hollandale/Iowa
55. Eachibon/Melrose/Jackson
56. Lundy /Jefferson/Jefferson
57. Dolph/Lake Mills/Jefferson
58. Crave Brothers Dairy/Waterloo/Jefferson
59. Rosy-Lane Holsteins/Watertown/Jefferson
60. Bumble B/Benton/Lafayette
61. 2001/South Wayne/Lafayette
62. New Horizons D/Antigo/Langlade
63. 5613/Merrill/Lincoln
64. Goehring's Clarks Mills/Cato/Manitowoc
65. Grotegut/Manitowoc/Manitowoc
66. Fitz-Pine/Newton/Manitowoc
67. Miltrim/Athens/Marathon
68. 519/Dorchester/Marathon
69. New Horizons I/Marathon/Marathon
70. Marathon City/Marathon City/Marathon

71. New Horizons E/Marathon City/Marathon
72. Bernicks/Spencer/Marathon
73. Rosedale/Oxford/Marquette
74. Chapman Brothers/Tomah/Monroe
75. Dewgood Holsteins/Oconto/Oconto
76. Shady Lawn/Sobieski/Oconto
77. Lamers Dairy/Appleton/Outagamie
78. Caprine Supreme/Black Creek/Outagamie
79. Arla/Kaukauna/Outagamie
80. R-R Letters/Seymour/Outagamie
81. 1878/Shiocton/Outagamie
82. Betzoldvale/Amery/Polk
83. 1871/Amherst Junction/Portage
84. 510/Kennan/Price
85. Davis/Kennan/Price
86. New Horizons G/Lone Rock/Richland
87. Larson Acres/Evansville/Rock
88. Janesville/Janesville/Rock
89. Honeycrest/Spring Valley/Rock
90. New Horizons A/Loganville/Sauk
91. Frozene/Westfield/Sauk
92. Drake Dairy/Elkhart Lake/Sheboygan
93. A-OK/Sheboygan Falls/Sheboygan
94. Elm Park/Sheboygan Falls/Sheboygan
95. Highland Crossing/Sheboygan Falls/Sheboygan
96. Ever-Green-View/Waldo/Sheboygan
97. 492/Gilman/Taylor
98. 513/Medford/Taylor
99. F206/Osseo/Trempealeau
100. Castle Rock Organic Dairy/Osseo/Trempealeau
101. Butler/Whitehall/Trempealeau
102. Wubbenhorst/Westby/Vernon
103. Krusen Grass/Elkhorn/Walworth

104. Van Dell/Sharon/Walworth

105. Sugar Creek/Elkhorn/Walworth

106. Springbrook Organic/Springbrook/Washburn

107. Cozy Nook/Waukesha/Waukesha

108. 1858/Marion/Waupaca

109. Railane Holsteins/Scandinavia/Waupaca

110. 1880/Plainfield/Waushara

111. Gateway-Acres/Poy Sippi/Waushara

112. White Clover/Menasha/Winnebago

113. Omro Dairy/Omro/Winnebago

114. Von Holzen/Winneconne/Winnebago

115. Majestic Oak/Marshfield/Wood

9 Appendix C: Project supplements for use with groups

I post the following instructions for the groups in an attempt to try and avoid as much misunderstanding as possible.

Team Work:

- Make every effort to have team meetings when everyone can be there. If someone on the team habitually "cannot make it," please see me.
- Everyone is expected to attend all team meetings.
- Participate in group discussions. When work is split up, be sure and have your portion done by the next meeting. If someone else needs to do it, you will not receive full credit for the project.
- Each member of each team is important. It is possible that someone on the team will see a solution before anyone else does. Let the other people also have input. There may be more than one way to view this.
- This is not a race.
- Group participation does not mean that everyone discovers every answer. But everyone should give a good amount of time to trying. There are only three weeks until papers are due and presentations will be given. In research, you must begin early. I will expect reports on your findings as the days go by.
- Each member of the group will have the opportunity (during the final exam) to evaluate the participation level of each member of their group (including themselves). It is important, therefore, that you do your best at all times, and that others know you are working on things.
- I am the complaint department (Factor's Grievance Solutions, Inc.). Bring personnel problems to me if they get too bad. Try to work them out as a group first.
- Treat each member of your group with respect. No one should be made to feel stupid or unwanted. Disrespect in any form is one thing that I absolutely will not tolerate. Act professionally.
- Above all, try to enjoy what you are doing. This is truly a unique experience. No one has done it before. Do your best!

DESCRIPTION	POINTS POSSIBLE	POINTS EARNED	COMMENTS
I. PRESENTATION			
A. Audio: Can the class understand what you are trying to tell them? Do you say what you mean?	2		
B. Visual: Do you have poor, fair, good or excellent visual aids? Do they help the class understand what you are presenting?	2		
C. Interaction with the class and maturity of presentation.	1		
II. PROBLEM			
A. Understanding the problem: How well do you understand the problem you have been given? How well do you explain the problem to the class?	2		
B. Explanation of your approach to solving the problem, and how you got to your solution.	3		
C. Correctness of the mathematics and conclusion(s).	3		
III. EVIDENCE OF TIME SPENT			
A. On solving the problem (this is independent of whether you actually solved the problem or not).	1		
B. On preparing the presentation.	1		
TOTAL	**15**		

Figure 7: Rubric for Group Presentations

The chart shown in Figure 7 is available for the students to see at least one week prior to presentations. This allows them to plan their presentations around what they know will be observed.

At the end of the final exam, I attach the page shown in Figure 8. Students fill it out and I am the only one who sees it. Most students welcome the chance to debrief.

RESEARCH PROJECT EVALUATION SHEET (consider all of the work done for the research, paper, and the presentation)

NAMES OF GROUP MEMBERS (including yourself)	**% of group work done (should add up to 100% for total in column)**	**Comments**
1. Self:		
2.		
3.		
4.		

Figure 8: Individual Assessment of Group Work done.

Bibliography

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