Mathematical Challenges in Human-Environment and Climate Systems.

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Introduction

The 2010 report *Toward a Science of Sustainability* [1] outlines a broad framework of interdisciplinary research questions focused on complex systems that arise as human-environment systems. The 2008 report *Foundations for Complex Systems Research in the Physical Sciences and Engineering* [2] identifies gaps in our understanding of complex systems and suggests research directions to correct this. Both reports point to our need to develop hierarchies of model systems. In such a hierarchy, each model can exploit structures at particular, interacting scales of the system to capture key behaviors with optimal clarity. An overarching research challenge for the community is that models at different scales need to be collaboratively developed so that they can interact with each other, and be systematically mapped into more complicated models.

Structures in Fishery Systems

In the context of sustainability questions, there is often a yawning gap between the models we currently use to try to draw insight, and the true complexity of the question. One research approach for making some headway into that gap is to identify structures that characterize human-environment systems at a variety of scales. We need concrete examples to help us identify more general structures. The following examples of interacting structures inherent in fishery systems have counterparts in many other human-environment systems, and underscore the need for broadly interdisciplinary research teams to develop appropriate hierarchies of model systems.

Heterogeneity in reproductive value of individuals. The classic Gordon-Shaefer bioeconomics model describes a fish population with logistic growth harvested at rate proportional to Ex, where x represents the fish biomass density and E represents the effort level of harvesting. The model predicts that an open access fishery will suffer the *tragedy of the commons*, in which effort will increase to the point where the total costs of effort balance total revenue, so that a) no profit is made, and b) the fish population is driven down to a low level, with consequences throughout the ecosystem. The model also predicts that regulation designed to reduce total effort can lead to equal or greater harvest (and hence greater profit) while restoring the fish population to more robust size. However, reducing effort carries the social cost of reducing employment, so the model suggests that difficult trade-offs may be required.

By contrast, Neubert and Herrera [3] use a simple extension of the Gordon-Shaefer model to show that including natural heterogeneity in the reproductive value of fish can reverse the model's prediction. They exhibit conditions under which regulation designed to protect the fish with the greatest reproductive potential can lead to increased profit, increased population size *and* increased employment. Such scenarios with the potential for triple benefit are extremely important to sustainability efforts, providing compelling arguments for management decisions. An important research challenge is to understand when such model behavior persists as model complexity is increased.

Neubert and Herrera use spatial structure to characterize heterogeneity in reproductive value [3]. Fish in the interior of a habitat are assumed to be safer from predation than fish on the periphery, and therefore have higher reproductive potential. Other relevant spatial heterogeneity may result from variation in the protective nature of the ocean bottom, nutrient availability or temperature gradients. More generally, reproductive heterogeneity can arise from a great variety of biological or environmental factors, such as size, life-stage, sex, season, etc. Several of these factors are already used in fishery management, of course. For example, the Maine lobster fishery has regulations on minimum and maximum legal size, a system of notching the tails of females with eggs, and a prohibition on harvesting notched females. Identifying the structure of reproductive heterogeneity that results from a combination of factors for any particular fishery, and understanding how to make optimal use of that structure in managing the fishery, will require interdisciplinary teams including modelers, economists, biologists, fishermen and managers.

One complex system (humans) managing another complex system. This is an inescapable component of many human-environment systems, but what structure does it confer on the system? As noted by Wilson *et al* [4], collective action toward management of a resource is easier to implement when it is consistent with the self-interest of the affected individuals. This raises the question: how do structures within the resource system shape collective behavior in the self-interested human system? Wilson *et al* are using adaptive agent-based models simulating the learning process and evolving interactions among competing fishermen to gain insight into the emergent social structures. When applied to a lobster fishery, the sedentary behavior of lobsters and the type of interactions between fishermen enabled by the fishing technology (traps, in this case), leads to the social structure of management observed in the Gulf of Maine lobster fishery [4]. Clusters of fishermen balance collaboration within clusters and competition between clusters to create group ``territories'' with well-defined boundaries. Thus, behavior at the fine scale of individuals generates emergent structure at a broader scale. Such clusters do not emerge in all fishery systems. To explore the dependence of emergent social structure on resource dynamics, Wilson *et al* are currently using the same modeling approach to compare three Gulf of Maine fisheries with strikingly different resource dynamics (lobster, sea urchins, and cod).

Matching management scale to resource scale. Ames [5] uses historical records and interviews with fishermen to reconstruct the spatial dynamics and critical habitats of cod and other species in the Gulf of Maine before the fisheries were so severely depleted. One interesting result from the study is the existence of distinct subpopulations of fish associated with individual inshore spawning grounds. The fish migrate offshore and back inshore with the seasons, but each subpopulation occupies roughly distinct territories. Interviews with fishermen reveal that the fishery associated with each subpopulation collapsed when the spawning grounds were discovered and fished. How do these insights into the multi-scale temporal and spatial structure of the resource system inform fishery restoration and sustainable management? Ames [6] proposes a collaborative management plan with a multi-scale structure matching that of the resource, including fine-scale inshore layers encompassing the spawning grounds, mid-scale layers encompassing the associated coastal shelf migration routes, and a large-scale offshore layer. These intriguing suggestions raise many general questions about a) how scales of resource dynamics and management structure can or should interact, b) whether the approach of Wilson *et al* could be used at distinct scales, or to investigate the impact of the management structures at different scales on each other and c) how the bifurcation or ``tipping point'' behaviors at each scale interact.

Climate Change

The 2007 *Fourth Assessment Report* of the Intergovernmental Panel on Climate Change (IPCC) [7] states that warming of the climate is "unequivocal". Modeling of almost any human-environment system also needs to consider the role of the changing climate in that system. Regional changes in temperature, sea level, fresh water supply, precipitation, extreme weather events, ocean and soil acidification, etc., collectively impact every aspect of our goal to live sustainably on this planet, from agriculture to natural resource management, preservation of biodiversity, renewable energy, infrastructure development, and alleviation of poverty and disease. Relevant environmental changes forced by the changing climate need to be included in our modeling of human-environment systems to inform management decisions likely to have long term impact.

It is also critical that we address the mathematical challenges facing climate modeling and prediction as we develop a research agenda for sustainability. Mathematical modeling is our only vehicle for experimenting with hypotheses about climate processes, and for extrapolating into the future. Thus, the mathematics community has a fundamental role to play in furthering our understanding of climate. The following examples list a few of the mathematical challenges that pervade climate change research.

Definition of Climate. Climate is currently defined by the average and variation in temperature and weather processes over a time scale of, typically, 30 years. Are there other potential definitions, exploiting multiple time-scales of the climate system, the inherently chaotic dynamics of the weather, and possible strange attractors of the system that might help characterize climate dynamics?

Climate Process Modeling. Many processes relevant to climate change remain poorly understood, and poorly modeled. These include: the carbon cycle on geological and biological time scales, sea-ice structure and mechanics, ocean circulation, glacier melting, cloud formation, and extreme weather, such as hurricanes, tornadoes and flooding. We are especially far from including human behavior and biological adaptation in the models. Phase transitions are observed in many physical climate processes. Examples include the break-up of ice sheets, the on-off nature of fluid flow though sea ice (a key process in understanding ice-albedo feedback), and the threshold where a moist column of air starts to precipitate (a key process in understanding cloud formation). How do phase transitions in individual processes impact the climate system? More abstractly, in what ways can bifurcations in a subsystem impact the dynamics of a higher dimensional system?

Climate Process Interactions. Feedback among nonlinear climate processes adds more mathematical richness. This is exhibited quite dramatically in the latest generation of climate models. As more feedback interactions have been incorporated, intuition for model behavior has dropped severely. We can draw from our experience with networks of systems in mathematical biology and elsewhere to investigate the interplay between the dynamics of individual systems and their coupling. As usual, one of the research challenges is to find models with optimal levels of simplicity to gain insight at different scales. Within the landscape of climate models it may be helpful to return to simplified models for each climate process, in order to better focus on the network coupling and its role in the resulting dynamics.

Paleoclimate Dynamics. Proxies for global temperature and atmospheric CO_2 now reach back 70 million years, showing considerable variation in planetary response to Milankovitch forcing, among other things. Several abrupt changes in climate are also visible in the paleoclimate record. Models that can capture past variation, and particularly, abrupt changes, help to dissect climate mechanisms beyond those of our immediate experience, that could potentially come into play as we continue to increase atmospheric greenhouse gas concentrations, for example.

Predicting Abrupt Change. In [8], Sheffer discusses tools for predicting critical transitions or bifurcations. One such tool is to monitor the rate of decay of perturbations of a system from its stable state. While we cannot conduct controlled experiments of this type with the climate system, we may be able to use natural variation to investigate the changing response of processes to perturbation. Focusing on seasonal variation, for example, Artic sea-ice coverage is changing more in summer than in winter. If we view summer as a perturbation from the more stable winter state, does the greater loss of coverage in the summer suggest a slower recovery from perturbation, indicating loss of resilience and pending destabilization in the sea-ice system? More generally, what is the right mathematical framework for analyzing seasonal variation as response to perturbation?

Many more mathematical challenges facing climate modeling are described in the 2007 MSRI report *Mathematics of Climate Change* by Mackenzie [9]. Issues range from numerical challenges and efficient assimilation of data to the myriad questions around quantification of uncertainty for decision support. They need to be included in the sustainability research agenda for our community.

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References.

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