Incidence functions and population thresholds

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Outline

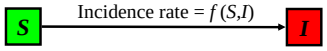
Incidence functions

Density dependence

Population thresholds

- Invasion
- Persistence
- Thresholds and host extinction

The incidence rate



Incidence term in models describes the rate that new infections arise.

$f(S,I)$ = Force of infection × S

Force of infection, $\lambda = c(N)\, p\, I/N$

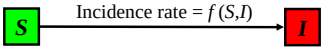
$c(N)$ = contact rate (possibly density-dependent)

p = probability of transmission given contact

I/N = prob. that randomly-chosen partner is infectious

So $$\boldsymbol{f(S,I) = c(N)\, p\, \frac{SI}{N}}$$

Density-dependent transmission



$$\boldsymbol{f(S,I) = c(N)\, p\, \frac{SI}{N}}$$

If contact rate is linearly density-dependent:

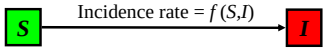
$c(N) = kN$

Then $f(S,I) = kN\, p\, SI/N$

$= \beta_{MA}\, SI$ where $\beta_{MA} = kp$

→ **"Mass action" transmission**. Also known as density-dependent or, confusingly, "pseudo-mass action" (see McCallum et al, 2001)

Frequency-dependent transmission



$$\boldsymbol{f(S,I) = c(N)\, p\, \frac{SI}{N}}$$

If contact rate is constant with respect to density:

$c(N) = c_0$

Then $f(S,I) = c_0\, p\, SI/N$

$= \beta_{FD}\, SI/N$ where $\beta_{FD} = c_0\, p$

→ **"Frequency-dependent" transmission**. Also known as the standard incidence or, confusingly, "true mass action" (see McCallum et al, 2001)

How should pathogen transmission be modelled?

Hamish McCallum, Nigel Barlow and Jim Hone

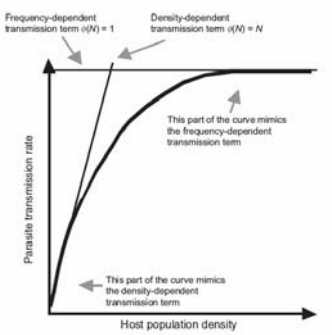
Table 1. Some proposed forms for the transmission function

Number	Function[a]	Comments
1	βSI	Mass action
2	$\beta SI/N$	Frequency-dependent transmission
3	$\beta S^q I^p$	Power relationship; Constants: $0 < p < 1$, $0 < q < 1$. Phenomenological
4	$\beta I(N - I/q)$; $I < qN$ 0; $I \geq qN$	Constant: $0 < q < 1$. Embodies a refuge effect (q = proportion of the population potentially susceptible, because of spatial or other heterogeneities)
5	$kS \ln\left(1 + \frac{\beta I}{k}\right)$	Negative binomial. Small k corresponds to highly aggregated infection. As $k \to \infty$, expression reduces to βSI (mass action)
6	$\frac{N}{1-\varepsilon+\varepsilon N} \frac{F(S,I)}{N}$	Asymptotic contact function separated from the mixing term $F(S,I)$, which may be any of those above. If constant $\varepsilon = 0$, contacts are proportional to N. If $\varepsilon = 1$, contacts are independent of N
7	$\frac{\beta SI}{c+S+I}$	Asymptotic transmission. c is a constant

[a] I is the density of infected hosts, S is the density of susceptible hosts, and N is the total host density. β is the transmission rate. Oth parameters, where necessary, are identified under comments.

McCallum et al (2001) Trends Ecol Evol 16: 295-300.

Saturating transmission



Deredec et al (2003) Ann Zool Fenn 40: 115-130.

Many choices – what to do?

Classically it was assumed that transmission rate increases with population size, because contacts increase with crowding.

→ mass action (βSI) was dominant transmission term

Hethcote and others argued that rates of sexual contact are determined more by behaviour and social norms than by density, and favoured frequency-dependent transmission for STDs.

Since the 1990s, this has been a topic of active research using experimental epidemics, field systems, and epidemiological data.

Detecting density dependence

How can we test for density dependence in transmission?

- Fit models with different transmission functions to epidemic time series.
- Look at indicators for transmission $\propto N$ in epidemiological data:

With increased transmission rate, we expect:

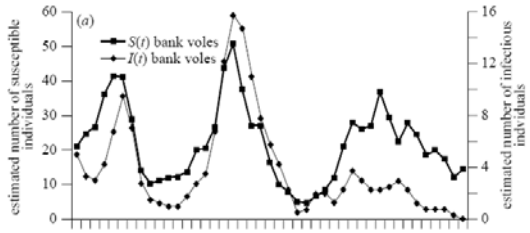
↑ estimates of R_0

↑ exponential growth rate of epidemic, r

↓ proportion susceptible following epidemic, or at steady state

↓ mean age of infection in endemic setting

Evidence for FD vs MA transmission



Fitting models to data from **cowpox in bank voles and wood mice**

→ FD model is better fit than MA (though neither is perfect)

Begon et al (1999) Proc Roy Soc B 266: 1939-1945.

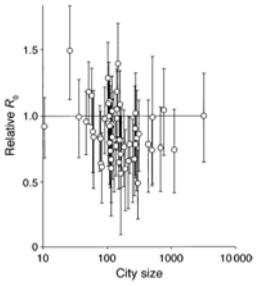
Evidence for FD vs MA transmission

Measles in England and Wales

• R_0 is ~ constant vs population size

→ roughly FD transmission

(recall that MA predicts that $R_0 \propto N$)



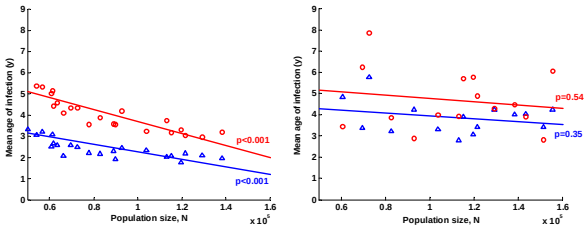
Bjornstad et al (2002) Ecol Monog. 72: 169-184

Evidence for FD vs MA transmission

Model results:

density-dependent transmission

Lepto data



Leptospirosis in California sea lions

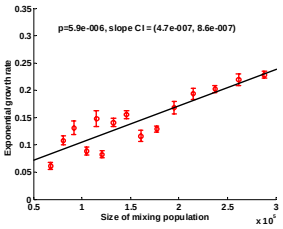
Mean age of infection does not decrease with N

→ transmission not density-dependent.

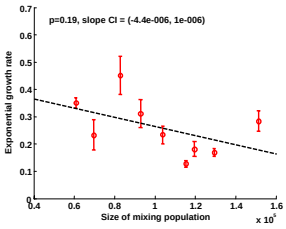
Evidence for FD vs MA transmission

Model results:

density-dependent transmission



Lepto data

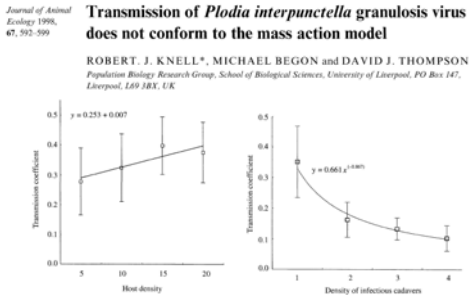


Leptospirosis in California sea lions

Epidemic growth rate does not increase with *N*

→ transmission not density-dependent.

Evidence for FD vs MA transmission → neither?



Journal of Animal Ecology 1998, **67**, 592–599

Transmission of *Plodia interpunctella* granulosis virus does not conform to the mass action model

ROBERT. J. KNELL*, MICHAEL BEGON and DAVID J. THOMPSON

Population Biology Research Group, School of Biological Sciences, University of Liverpool, PO Box 147, Liverpool, L69 3BX, UK

PiGV in *Plodia* (Indian meal moth)

Transmission rate is not FD or MA – need complex functional forms.

Interpret in terms of host heterogeneity and effects of density on behaviour.

So what should we do?

Despite its fundamental importance, the issue of how to formulate the transmission term in simple models is unresolved.

Some pointers:

- FD transmission is generally thought to be more appropriate than MA in large well-mixed populations.
- In quite small populations, transmission is generally thought to exhibit some density dependence and MA is acceptable.
- Think about population structure and mechanisms of mixing at the scales of space and time you're thinking about.
  Is a very simple model appropriate?

  (more on this in the next lecture)

Population thresholds in epidemic dynamics

R_0 has been the central concept in epidemic dynamics since ~1980, thanks largely to the work of Anderson & May.

(see the history of R_0 by Heesterbeek 2002, *Acta Biotheoretica*)

Long before this, people studying epidemic dynamics have focused on population thresholds.

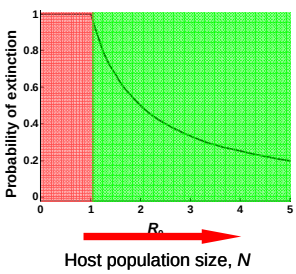
- Population threshold for invasion (Kermack & McKendrick 1927): host population size below which parasite cannot invade.
- Population threshold for persistence, or the critical community size (Bartlett 1957, Black 1966): host population size below which parasite cannot persist long-term.

Population threshold for disease invasion

Under density-dependent transmission, $R_0 = \beta N D$

or in fact R_0 any increasing function of *N*.

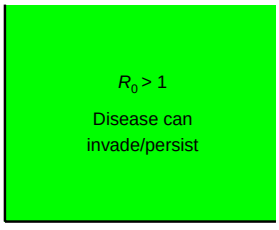
→ R_0>1 corresponds to a population threshold $N>N_T$.



Population threshold for disease invasion

Under frequency-dependent transmission, $R_0 = \beta D$.

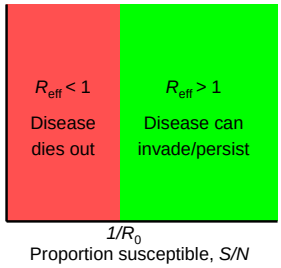
→ No threshold *N* for R_0>1.



Susceptibility threshold for disease invasion

Recall: under any form of transmission, $R_{effective} = R_0 \times S/N$.

→ For $R_{effective} > 1$, must have $S/N > 1/R_0$.

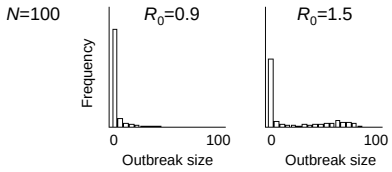


This phenomenon is the basis for herd immunity.

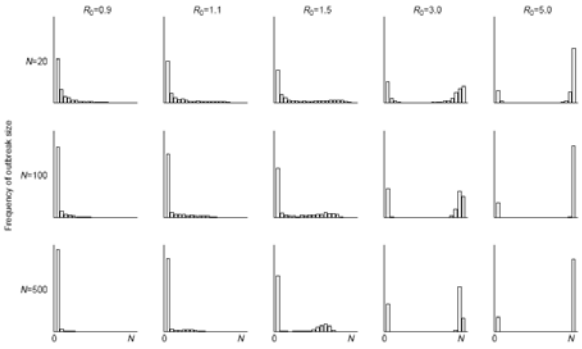
Population thresholds for invasion: evidence

Despite its conceptual simplicity , real-world evidence for invasion thresholds is hard to find, for several reasons

- failed invasions are difficult to observe
- demographic stochasticity leads to variation in outbreak sizes
  - when R_0<1, limited chains of transmission can still occur
  - when R_0>1, epidemic can still die out by chance.

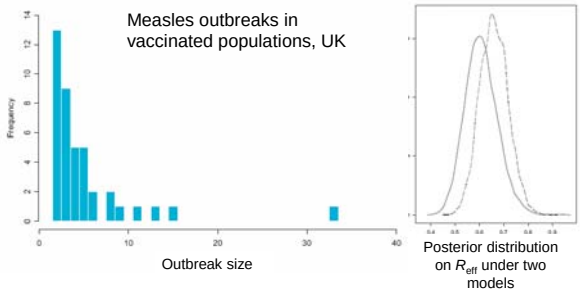


Stochastic variation in outbreak size



Lloyd-Smith et al (2005) Trends Ecol Evol 20: 511-519.

Stochastic variation in outbreak size



Branching process models allow analysis of outbreak size to make inference about the effective reproductive number.

Farrington et al (2003) Biostatistics 4: 279-295.

Population thresholds for persistence

Even if parasite is able to invade (R_0>1), this does not guarantee its persistence in the long term.

There are two broad mechanisms whereby a disease can fail to persist, or fade out:

Endemic fadeout: random fluctuations around the endemic equilibrium can cause extinction of the parasite.

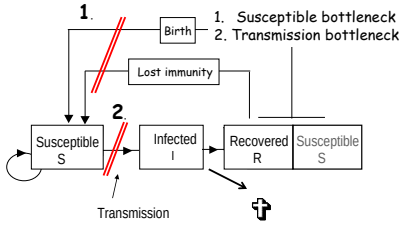
Epidemic fadeout: following a major epidemic, the susceptible pool is depleted and the parasite runs out of individuals to infect.

Critical Community Size is population size above which a disease can persist long-term (yes, this definition is vague).

Persistence thresholds – another view

Courtesy of Ottar Bjornstad

Broken chains of transmission can arise in two ways:



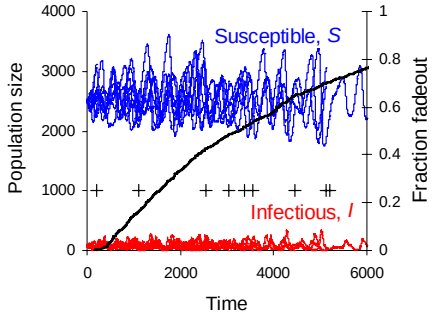
Epidemic fadeout: Parasite extinction occurring because susceptible numbers are so low immediately following an epidemic that small stochastic fluctuations can remove all parasites. (**Susceptible bottleneck**)

Endemic fadeout: Parasite extinction occurring because endemic numbers of infected individuals are so low that small stochastic fluctuations can remove all parasites. (**Transmission bottleneck**)

Endemic fadeout

Stochastic SIR model with FD transmission and R_0=4.

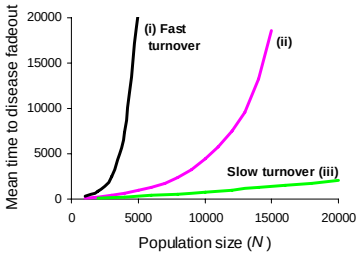
10 simulations are shown. + signs show times when disease fades out.



Endemic fadeout

Mean time to endemic fadeout for stochastic SIR model with R_0=4 and different rates of demographic turnover.

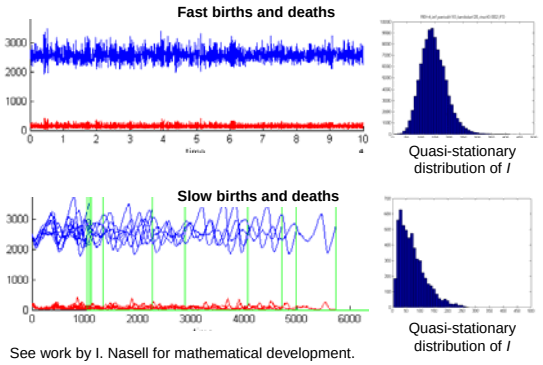
1. No sharp threshold in *N*; there's a gradual trend of longer persistence.
2. Demographic rates are as important as *N*, if not more so.



Lloyd-Smith et al (2005) Trends Ecol Evol 20: 511-519.

Endemic fadeout

Quasi-stationary distribution: distirbution of *I* conditioned on non-extinction.



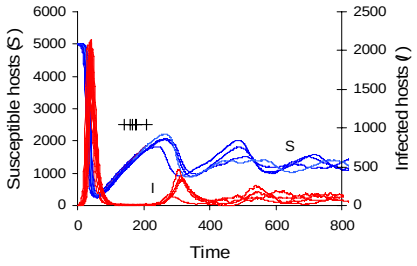
See work by I. Nasell for mathematical development.

Epidemic fadeout

Stochastic SIR model with FD transmission and R_0=4

(exact same model as for endemic fadeout, but now started from I=1 instead of I=I^*.)

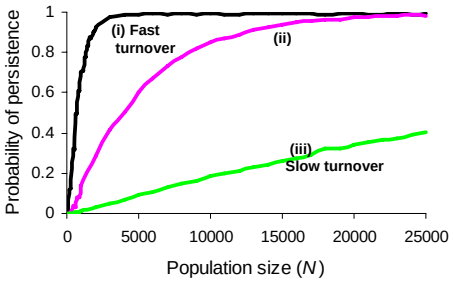
10 simulations are shown. + signs show times when disease fades out.



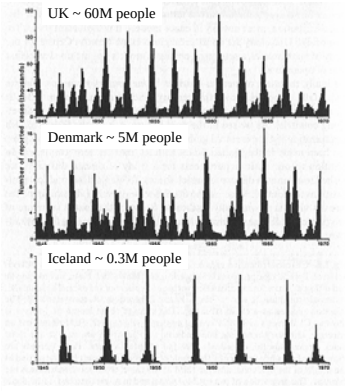
Epidemic fadeout

Probability that disease persists through the first post-epidemic trough

1. No sharp threshold in *N*; there's a gradual trend of longer persistence.
2. Demographic rates are as important as *N*, if not more so.

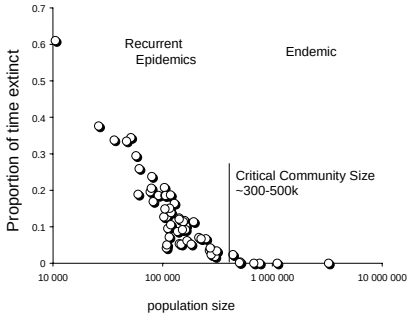


The classic example of epidemic fadeout: measles



Note how measles is not endemic in Iceland, but instead has periodic outbreaks dependent on re-introduction of the virus.

The classic example of epidemic fadeout: measles

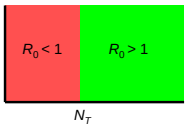


Measles in England and Wales

Extinction risk: can a disease drive its host extinct?

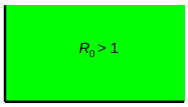
Density-dependent transmission

Population threshold N_T protects host from disease-induced extinction.



Frequency-dependent transmission

No threshold → disease-induced extinction is possible (Getz & Pickering 1984).



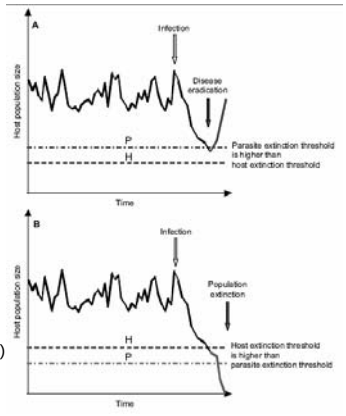
→ Direct extinction due to disease in single host is unlikely, but diseases can cause bottlenecks such that genetic diversity and Allee effects become important.

Extinction risk: can a disease drive its host extinct?

What if the host population itself has a threshold density below which it cannot persist?

Then the outcome depends on the relative values of the threshold population size for disease extinction vs host extinction.

Deredec & Courchamp (2003) Ann. Zool. Fenn. 40:115-130.



Extinction risk: multiple host species and spillover

Spillover from reservoir can threaten endangered populations

Table 1. Extinctions and near-extinctions of threatened host populations caused by disease

Host species	Population size		Pathogen	Source of infection	References
	Before outbreak	After outbreak			
Black-footed ferret *Mustela nigripes*	58	5[a]	Canine distemper virus	Badgers/coyotes?	1
Bighorn sheep *Ovis canadensis*	60 not reported[b]	0 0	*Pasteurella* *Pasteurella*	Domestic sheep Domestic sheep	2 2
African wild dog *Lycaon pictus*	50–70 4 12	0 0 3[a]	Rabies[c] Rabies Rabies	Domestic dogs Jackal Jackal	3, 4 5 6
Ethiopian wolf *Canis simensis*	53 23	12 11	Rabies Rabies[c]	Domestic dogs Domestic dogs	7 7

Woodroffe, 1999

1997 Mediterranean monk seal die-off in Mauritania.

>100 monk seals died (~1/3 of global population), probably due to dolphin morbillivirus (a relative of measles) that spilled over from another species.