Bilinear Optimality Constraints for the Cone of Positive Polynomials and Related Cones

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Abstract

For a proper cone $\mathcal{K} \subset \mathbb{R}^n$ and its dual cone $\mathcal{K}^*$ the complementary slackness condition $\mathbf{x}^T \mathbf{v} \mathbf{s} = 0$ defines an n-dimensional manifold $C(\mathcal{K})$ in the space $\{\ (\mathbf{x}, \mathbf{s}) \mid \mathbf{x} \in \mathcal{K},\ \mathbf{s} \in \mathcal{K}^*\ \}$. When $\mathcal{K}$ is a symmetric cone, this fact translates to a set of n bilinear optimality conditions satisfied by every $(\mathbf{x}, \mathbf{s}) \in C(\mathcal{K})$. This proves to be very useful when optimizing over such cones. Therefore it is natural to look for similar optimality conditions for non-symmetric cones. In this paper we examine several well-known cones, in particular the cone of positive polynomials $\mathcal{P}_{2n+1}$ and its dual, the closure of the moment cone $\mathcal{M}_{2n+1}$. We show that there are exactly four linearly independent bilinear identities which hold for all $(\mathbf{x}, \mathbf{s}) \in C(\mathcal{P}_{2n+1})$, regardless of the dimension of the cones. For nonnegative polynomials over an interval or half-line there are only two linearly independent bilinear identities. These results are extended to trigonometric and exponential polynomials.

(Joint work with Nilay Noyan, Gabor Rudolf, and David Papp)