Fast approximation schemes for structured programs and matrix games.

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Abstract: We review exponential and logarithmic potential reduction methods for block angular convex programs with K blocks and M coupling constraints. When embedded in the classical Lagrangian decomposition paradigm, these yield fast approximation schemes with low memory requirements for large min-max and max-min convex feasibility problems.

The coordination complexity of such methods is independent of the dimension of the blocks, but running times depend on problem class and block type. The algorithms use approximate but standard *block solvers*, ranging from convex programming routines, to LPs, minimum-cost flows, shortest paths, knapsacks, etc. Such models capture diverse applications, including network design, cutting stock, scheduling, assignments, K-commodity generalized flows with side constraints, and others. For example, for a fixed relative error tolerance $\varepsilon \in (0, 1)$, the standard K-commodity concurrent flow problem in an n-vertex m-edge graph can be computed in roughly $\tilde{O}(\varepsilon^{-2}Knm)$ time, and its *minimum-cost* variant within polylogarithmic factors. Computational experiments with large instances support the practical viability of the approach, which also lends itself to coarse-grain parallel implementations. While it is possible compute a pair of ε-optimal strategies for an $m \times n$ matrix game A in polylogarithmic time on a quadratic number of processors, an improved randomized parallel algorithm for any $A = [a_{ij}] \in [-1, 1]$ and fixed accuracy $\varepsilon > 0$, runs in sublinear expected time on an $(n + m)/\log(n + m)$-processor EREW PRAM, ensuring a nearly quadratic expected speedup relative to any deterministic algorithm for $m = n$.

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