# Audit Games

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## **Repositories of Personal Information**





















# Healthcare Privacy



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# A Research Area

- Formalize Privacy Policies
  - Precise definitions of privacy concepts (restrictions on information flow)
    - Information used only for a purpose
    - All disclosure clauses in HIPAA & GLBA
- Enforce Privacy Policies
  - Audit and Accountability
    - Detect violations of policy
    - Identify agents to blame for policy violations
    - Resource allocation for inspections and punishments (economic considerations)

Project page: Privacy, Audit and Accountability





# Play in Three Acts

- 1. Rational Adversary Setting
- 2. Byzantine Adversary Setting
- 3. Research Directions

#### [Blocki, Christin, Datta, Procaccia, Sinha; 2013]

## Audit Game Model [BCDPS'13]



If a violation is found, adversary is fined

Price of punishment [Becker'68]

- Utility when target t<sub>i</sub> is attacked
  - Defender:  $p_i U_{a,D}(t_i) + (1 p_i)U_{u,D}(t_i) ax$
  - Adversary:  $p_i (U_{a,A}(t_i) x) + (1 p_i)U_{u,A}(t_i)$

# Stackelberg Equilibrium Concept

- Defender commits to a randomized resource allocation strategy
- Adversary plays best response to that strategy
- Appropriate equilibrium concept
  - Known defender strategy avoids security by obscurity
  - Predictable adversary response
- Goal
  - Compute optimal defender strategy

## Related Work

- Security resource allocation games [Tambe et al. 2007-]
  - Computes Stackelberg equilibrium
  - Deployed systems for resource allocation for patrols at LAX airport, federal air marshals service; under evaluation by TSA, US coast guard
- Audit games generalize security resource allocation games with the punishment parameter
  - Computing Stackelberg equilibrium becomes more challenging
  - Applicable to similar problems

Computing Optimal Defender Strategy

Solve optimization problems  $P_i$  for all  $i \in \{1,..,n\}$ and pick the best solution

$$\begin{array}{l} \max p_{i} \ U_{a,D}(t_{i}) + (1 - p_{i}) U_{u,D}(t_{i}) - ax \\ \text{subject to} \\ p_{j}( \ U_{a,A}(t_{j}) - x \ ) + (1 - p_{j}) U_{u,A}(t_{j}) \leq \\ p_{i} \ ( \ U_{a,A}(t_{i}) - x \ ) + (1 - p_{i}) U_{u,A}(t_{i}) \\ \forall j \in \{1,...,n\} \\ p_{i} \text{'s lie on the probability simplex} \\ 0 \leq x \leq 1 \end{array} \right$$

# 1. Quadratic constraints

- p<sub>i</sub>x terms
- 2. Non-convex optimization problem
  - Constraints representable as x<sup>T</sup> A x + Bx + c
     ≤ 0
  - A is not positive semi-definite

Properties of Optimal Point

• Rewriting quadratic constraints  $p_{i}(-x - \Delta_{i}) + p_{n}(x + \Delta_{n}) + \delta_{i,n} \le 0$ 

$$\Delta_{j} = U_{u,A}(t_{j}) - U_{a,A}(t_{j})$$



# Overview of Algorithm



- Iterate over regions
- Solve sub-problems EQ<sub>i</sub>
  - Set probabilities to zero for curves that lie above & make other constraints tight
- Pick best solution of all EQ<sub>i</sub>

Solving Sub-problem EQ<sub>i</sub>

- 1.  $p_j(-x \Delta_j) + p_n(x + \Delta_n) + \delta_{j,n} = 0$ 
  - Eliminate  $p_j$  to get an equation in  $p_n$  and x only
- 2. Express  $p_n$  as a function f(x)
  - Objective becomes a polynomial function of x only
- Compute x where derivative of objective is zero & constraints are satisfied
  - Local maxima

# 4. Compute x values on the boundary a Found by finding intersection of p<sub>n</sub> = f(x) with the boundaries a Other potential points of maxima

 Take the maximum over all x values output by Steps 3.4 Steps 3 & 4 require computing roots of 13 polynomials

# Computing Roots of Polynomials

- Using existing algorithms
  - Splitting circle method [Schonage 1982] can approx. irrational roots to precision K in time polynomial in K
    - Steps 3 and 4 take imprecision into account
  - LLL [Lenstra et al. 1982] can find rational roots exactly

#### Main Theorem

- The problem can be approximated to an additive ε factor in time O(n<sup>5</sup> K + n<sup>4</sup> log(1/ε)) using only the splitting circle method, where K is the bit precision of inputs.
- Using LLL the time is still polynomial O(max{n<sup>13</sup>K<sup>3</sup>, n<sup>5</sup> K + n<sup>4</sup> log(1/ ∈)}), and if the solution is rational the exact solution is found.

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#### [Blocki, Christin, Datta, Sinha; CSF 2011]



# Audit Model



# Repeated Game Model for Audit



- Typical actions in one round
  - Emp action: (access, violate) = ([30,70], [2,4])

Imperfectio

Org action: inspection = ([10,20])

## Game Payoffs

Organization's payoff



- Audit cost depends on the number of inspections
- Reputation loss depends on the number of violations caught
- Employee's payoff unknown

# Audit Algorithm Choices



Choose allocation probabilistically based on weights



# Property of Effective Audit Mechanism



- Audit mechanism should be <u>comparable</u> to <u>best</u> <u>expert in hindsight</u>
- Audit: Experts recommend resource allocations

## Low Regret

- Low regret of s w.r.t. s1 means s performs as well as s1
- Desirable property of an audit mechanism
  - Low regret w.r.t all strategies in a given set of strategies

*regret*  $\rightarrow$  0 as  $T \rightarrow \infty$ 

#### Audit setting

 Audit mechanism recommended resource allocation performs as well as best fixed resource allocation in hindsight

# Challenges in Audit Setting

- Sleeping experts
  - Not all experts available in each audit round (e.g., [300,10] in Figure 1)
- Imperfect information
  - In each round, only one expert's advice is followed and associated loss observed
  - Requires loss estimation for outcome for all other experts





#### Regret Minimizing Audits (RMA) New audit $w_s = 1$ for all cycle starts. strategies s Find AWAKE Pick s in AWAKE with Update weight\* of probability $D_t(s) \propto w_s$ strategies s in AWAKE Estimate payoff vector Violation caught; Pay using Pay(s) obtain payoff Pay(s)

\* $W_s \leftarrow W_s$ .  $\gamma^{-Pay(s) + \gamma \cdot \sum_{s'} D_t(s') Pay(s')}$ 



Guarantees of RMA

• With probability  $1 - \epsilon$  RMA achieves the regret bound

$$2\sqrt{\frac{2\ln N}{T} + \frac{2\ln N}{T} + 2\sqrt{\frac{2\ln\left(\frac{4N}{\epsilon}\right)}{T}}}$$

- N is the set of strategies
- T is the number of rounds
- All payoffs scaled to lie in [0,1]

#### Related Work

Weighted Majority Algorithm [LW89]:

- Average Regret: O((log N)/T)<sup>1/2</sup>
- Defender cannot run this algorithm unless he observes the adversaries moves (perfect information setting)
- Imperfect Information Setting [ACFS02]:
  - Average Regret: O(((N log N)/T)<sup>1/2</sup>)
  - Regret bound converges to 0 much slower
- Our regret bounds are of the same order as the perfect information setting assuming loss estimation function is *accurate* and *independent*

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# **Research Directions**

- Augmenting model and algorithm
  - Repeated interaction
  - Multiple defender resources constrained by audit budget
  - Multiple heterogeneous targets attacked by adversary
  - Information flow violations
  - Combining rational and byzantine adversary model
- Acquiring parameters of model
  - Ponemon studies, Verizon data breach reports

Initial

results in

[BCDS'12]

- From risk management to privacy protection
  - Why should organizations invest in audits to protect privacy?
  - What public policy interventions are most effective in encouraging thorough audits (e.g., HHS audits, data breach notification law)?

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#### Thanks! Questions?

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Proof of Property of Optimal Point

• Quadratic constraints  $p_n(x + \Delta_n) + \delta_{j,n} \le p_j(x + \Delta_j)$  where  $\Delta_j \ge 0$ 

Fact 1: p<sub>i</sub> is 0 or the j<sup>th</sup> constraint is tight

- Fact 2a: if p<sub>n</sub>(x + Δ<sub>n</sub>) + δ<sub>j,n</sub> ≤ 0 then p<sub>j</sub> is 0
   p<sub>j</sub>(x + Δ<sub>j</sub>) ≥ 0, thus the constraint cannot be tight, so p<sub>j</sub> is 0
- Fact 2b: if  $p_n(x + \Delta_n) + \delta_{j,n} > 0$  then tight constr

p<sub>i</sub> cannot be 0, so constraint has to be tight

# Problem $P_n$

Fortunately, the problem  $P_n$  has another property that allows for efficient methods. Let us rewrite  $P_n$  in a more compact form. Let  $\Delta_{D,i} = U_D^a(t_i) - U_D^u(t_i)$ ,  $\Delta_i = U_A^u(t_i) - U_A^a(t_i)$ and  $\delta_{i,j} = U_A^u(t_i) - U_A^u(t_j)$ .  $\Delta_{D,i}$  and  $\Delta_i$  are always positive, and  $P_n$  reduces to:

$$\begin{aligned} \max_{\substack{p_i, x \\ \text{subject to}}} & p_n \Delta_{D,n} + U_D^u(t_n) - ax , \\ \text{subject to} & \forall i \neq n. \ p_i(-x - \Delta_i) + p_n(x + \Delta_n) + \delta_{i,n} \leq 0 , \\ & \forall i. \ 0 \leq p_i \leq 1 , \\ & \sum_i p_i = 1 , \\ & 0 \leq x \leq 1 . \end{aligned}$$

# Problem $Q_{n,i}$

 $\begin{array}{ll} \max_{\substack{x,p_{(1)},\ldots,p_{(i)},p_n \\ \text{subject to}}} & p_n \Delta_{D,n} - ax \;, \\ p_n(x + \Delta_n) + \delta_{(i),n} \geq 0 \;, \\ \text{if } i \geq 2 \; \text{then} \; p_n(x + \Delta_n) + \delta_{(i-1),n} < 0 \;, \\ \forall j \geq i. \; p_n(x + \Delta_n) + \delta_{(j),n} = p_{(j)}(x + \Delta_j) \;, \\ \forall j > i. \; 0 < p_{(j)} \leq 1 \;, \\ 0 \leq p_{(i)} \leq 1 \;, \\ \sum_{k=i}^{n-1} p_{(k)} = 1 - p_n \;, \\ 0 \leq p_n < 1 \;, \\ 0 < x \leq 1 \;. \end{array}$ 

# Problem $R_{n,i}$

$$\begin{array}{l} \max_{x,p_n} \quad p_n \Delta_{D,n} - ax \ , \\ \text{subject to} \\ p_n(x + \Delta_n) + \delta_{(i),n} \geq 0 \ , \\ \text{if } i \geq 2 \ \text{then } p_n(x + \Delta_n) + \delta_{(i-1),n} < 0 \ , \\ p_n\left(1 + \sum_{j:i \leq j \leq n-1} \frac{x + \Delta_n}{x + \Delta_{(j)}}\right) = 1 - \sum_{j:i \leq j \leq n-1} \frac{\delta_{(j),n}}{x + \Delta_{(j)}} \ , \\ 0 \leq p_n < 1 \ , \\ 0 < x \leq 1 \ . \end{array}$$