

The Over-Concentrating Nature of Simultaneous Ascending Auctions

Charles Z. Zhèng

Department of Economics
Northwestern University

September 23, 2004

Motivation

1. Applications in US and European spectrum auctions.
2. Natural model for decentralized markets
 - (a) When there is no central coordination on the sales of multiple goods separately owned by different entities, so VCG mechanisms is unavailable, it is natural to assume that separate owners sell their goods separately.
 - (b) To capture the interactions among different sectors of an economy without artificially ranking one sector over another, it is natural to assume that these separate auctions start simultaneously.
 - (c) The open-outcry ascending-bid feature of these auctions provides a transparent setup to understand the process of price formation.
3. The question:

How do *decentralized* markets allocate *complementary* goods?

Literature

1. Efficiency when goods are substitutes:
Gul-Stacchetti (1999); Milgrom (2000).
2. Problems when goods are not substitutes:
Gul-Stacchetti (2000); Milgrom (2000).
3. Some centralized mechanisms:
Ausubel (2002); Ausubel-Milgrom (2002);
Bikhchandani-deVries-Schummer-Vohra (2001).
4. Some asymmetric-information analysis:
Krishna-Rosenthal (1996); Rosenthal-Wang (1996).
5. Optimal auction with multiple goods:
Levin (1997).
6. Optimal auction with resale (single good):
Zhèng (2002).

Needed—

a theory of simultaneous ascending auctions
of possibly complementary goods
without any central coordination.

1. The question:

It is not surprising that these decentralized auctions are inefficient. The question is: In what pattern are they inefficient?

2. The sticking point:

Inefficiency may take various forms, all parameter-dependent, so it is difficult to make predictions.

(a) Reason: Exposure problem:

A bidder may have bought an item at a price above its standalone value and fail to buy its complements.

(b) Over-concentration:

The goods go to a single bidder at equilibrium, while efficiency requires that they go to different bidders.

(c) Over-diffusion:

The goods go to different bidders at equilibrium, while efficiency requires they go to a single bidder.

This Paper

1. The prediction is qualitatively unambiguous:

Inefficiency of these auctions takes the form of probable over-concentration and never over-diffusion.

2. Reasons

(a) Jump-bidding

- i. Some bidders strictly want to jump-bid;
- ii. jump-bidding eliminates the exposure problem;
- iii. the only remaining inefficiency is:
 - the threshold problem:
single-item bidders cannot fully cooperate to combine their bids against multi-item bidders.

(b) Resale

- i. Inefficiency leads to resale;
- ii. a multi-item bidder becomes the middleman;
- iii. the middleman chooses to under-sell the goods.

The Primitives

1. Two items: A and B.
2. Three bidders:
 - (a) local bidder α who values only item A;
 - (b) local bidder β who values only item B;
 - (c) a global bidder γ who views A & B as complements.

	\emptyset	A	B	A & B
local α	0	t_α	0	t_α
local β	0	0	t_β	t_β
global γ	0	0	0	t_γ

3. For each $i \in \{\alpha, \beta, \gamma\}$, t_i is a random variable whose realized value is bidder i 's the private information and is independently drawn from a distribution F_i , with continuous positive density f_i and support $[0, \bar{t}_i]$.
4. A bidder's payoff is equal to his valuation of the set of items he buys minus his total payment.
5. Results of this paper can be extended to have multiple individuals of the same kind of bidders.
6. *Equilibrium:* pBe with undominated strategies.

The Plan for the Rest of the Talk

1. The basic mechanism

that bans jump-bidding, cross-bidding, and resale:
Exposure problem leads to various kinds of inefficiency.

2. Jump-bidding eliminates the exposure problem.

- (a) Strict incentive of jump-bidding.
- (b) Amend the basic mechanism to allow jump-bidding.
- (c) Unique allocation outcome of jump-bidding.
- (d) Unambiguous prediction: over-concentration.
- (e) Simultaneous auctions mimic package auctions.

3. Extension to cross-bidding (α can bid for B; β can bid for A). (Skip. Please see the paper.)

4. Extension to resale

- (a) The dynamic mechanism-selection game.
- (b) Endogenous separation between primary and resale markets.
- (c) At equilibrium: over-concentration.

The Basic Mechanism

1. The items are auctioned off via separate English auctions held simultaneously.
2. Bidder α can bid only for item A, bidder β only for B, and γ can bid for both items.
3. Prices start at zero. For each item k , the price p_k for item k rises continuously at an exogenous positive speed \dot{p}_k until k is sold.
4. Quitting/exit/dropout is irrevocable.
5. The auction of an item ends when all but one bidder has quit the item; immediately the remaining bidder buys the item at its current price.
6. The good cannot be returned for refund.
7. Bidders' actions are commonly observed.

Equilibrium in the Basic Mechanism

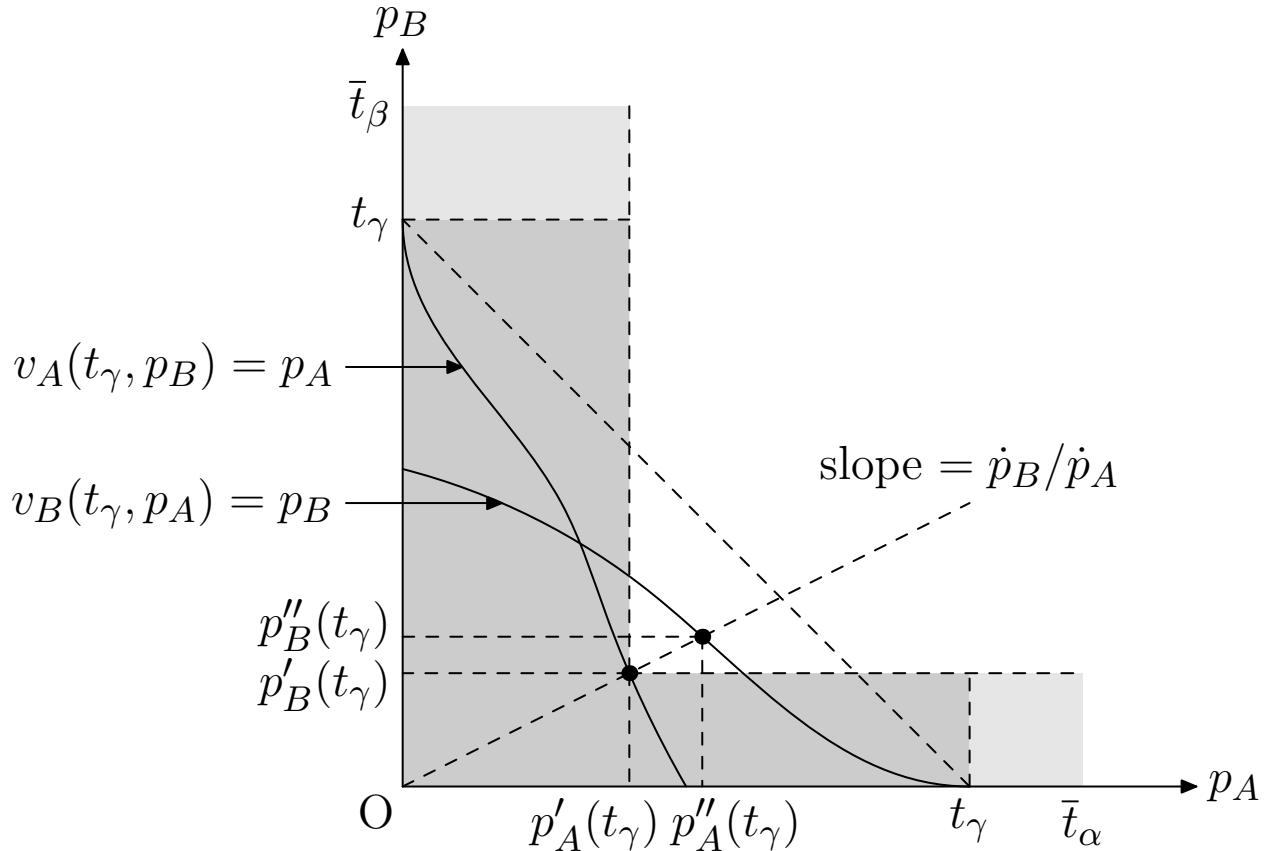
For any $(p_A, p_B) \in [0, \bar{t}_\alpha] \times [0, \bar{t}_\beta]$ and $t_\gamma \in [0, \bar{t}_\gamma]$, let

$$v_A(t_\gamma, p_B) := E[(t_\gamma - t_\beta)^+ | t_\beta \geq p_B]; \quad (1)$$

$$v_B(t_\gamma, p_A) := E[(t_\gamma - t_\alpha)^+ | t_\alpha \geq p_A]. \quad (2)$$

1. Local bidders: straightforward.
2. Global bidder γ : Given any current prices (p_A, p_B) —
 - (a) if neither A nor B has had a winner, continue bidding for both items if
$$v_A(t_\gamma, p_B) > p_A \text{ and}$$
$$v_B(t_\gamma, p_A) > p_B,$$
and quit both auctions if one of the inequalities fails;
 - (b) if item A or B has been won by someone else, quit from both auctions immediately;
 - (c) if item A (or B) has been won by bidder γ , continue bidding for item B (or A) until its current price p_B (or p_A) reaches $t_\gamma.$

Various Inefficiency in the Basic Mechanism



1. The allocation:

- (a) dark area: $\{A, B\} \rightarrow \gamma$;
- (b) grey area: $\{A, B\} \rightarrow \alpha$ or β ;
- (c) white area: $A \rightarrow \alpha$ & $B \rightarrow \beta$.

2. Inefficiency:

- (a) over-concentration;
- (b) over-diffusion;

The Incentive for Jump Bidding

1. Let us consider the moment when local bidder α is quitting at p_A . Now global bidder γ is on the verge of buying A without knowing how much he will have to pay for its complement B.
2. Suppose the other local bidder β could credibly reveal his value t_β to bidder γ at this moment. Then bidder γ would know that the price for item B will be t_β .
3. If $t_\gamma < p_A + t_\beta$, γ 's profit will be negative if he is to buy both items, and he would not be able to avoid such loss if he buys A now, because he will bid for B up to t_γ once he has bought A.
4. Thus, if $t_\gamma < p_A + t_\beta$, bidder γ wishes to quit both items immediately and yield the right for item A to bidder α . Then the global bidder could avoid the exposure problem, and local bidder β 's winning event could be expanded from $\{t_\gamma : t_\beta > t_\gamma\}$ to $\{t_\gamma : t_\beta > t_\gamma - p_A\}$.
5. Such arrangement would need local bidder β to reveal his type credibly. That can be done by a jump bid for item B.

Amendments to Allow Jump-Bidding

1. Dropout: *stop* or *withdraw*.
Bidding: *continue* or *jump-bid*.
2. Dropout/jump-bid for an item \Rightarrow the price clock for this item pauses for at most δ seconds.
3. If i stops from an item at p , then, during the pause, the other active bidder j for the item can withdraw. If j withdraws, the good goes to i at price p . (If i also withdraws, the good is not sold and each bidder pays half of the penalty p .) If j does not withdraw during the pause, he buys the good at price p .
4. If i jump-bids b for an item, every active bidder either stops or matches b or tops b .
 - (a) If someone tops b with b' ($> b$), repeat step 4 with the new jump bid b' .
 - (b) If someone matches b but no one tops it, the pause ends and the price clock resumes from b .
 - (c) If all but the jump-bidder drops out, the jump-bidder buy the item at his jump bid.

Jump-Bidding in the Decisive Moment

1. If a local bidder, say α , is the first to dropout (from A)...
2. The *decisive moment*: the tiny interval after α 's dropout and before global bidder γ has decided whether to withdraw from item A.
3. Since γ 's maximum willingness-to-pay (*MWTP*) for item B jumps when he buys A , local β wants to influence γ 's decision in this moment through jump bidding.
4. Jump bidding eliminates the exposure problem:

Proposition 1:

Assume that it takes less than half of the maximum time (δ seconds) of a decisive moment to submit a bid. At any equilibrium of the simultaneous-auctions game, if the global bidder wins an item at a positive price, then he wins its complement and, before buying any of them, he knows the total price for both items.

Proof of Proposition 1

Suppose a local bidder α is the first dropout at p_A , then:

1. during the decisive moment, local β 's MWTP for item B is $w_\beta := t_\beta$, and global γ 's MWTP for item B is

$$w_\gamma := \begin{cases} t_\gamma - p_A & \text{if } \alpha\text{'s action is "stop"} \\ t_\gamma - p_A/2 & \text{if } \alpha\text{'s action is "withdraw";} \end{cases} \quad (3)$$

2. at any continuation equilibrium that determines the winner of item B during the decisive moment,
 - (a) item B goes to the bidder with higher MWTP;
 - (b) the winner's expected payment given history h is

$$\mathcal{P}_i(w_i) := E[\mathbf{w}_{-i} \mid \mathbf{w}_{-i} \leq w_i; h]. \quad (4)$$

3. Suppose, when a local bidder is the first dropout, there exists a continuation equilibrium that determines the winner of the complement during the decisive moment. Then, at any equilibrium of the simultaneous auctions, both items are sold in the same decisive moment.
4. Such continuation equilibrium exists if submitting a bid takes less than half of the maximum time (δ seconds) of the decisive moment.

Proof of Step 3 (Lemma 6)

1. It suffices to show that each of bidders β and γ strictly prefers having the winner of B determined during the decisive moment to after the moment.
2. β 's preference is due to: γ 's MWTP is less in the decisive moment than after the moment.
3. γ 's preference is due to: he still has the option of not buying A during the decisive moment.

An equilibrium for Step 4 (Lemma 8)

a. Bidder β : Immediately make a jump bid for B:

$$\mathcal{P}_\beta(w_\beta) := \mathbb{E} [\mathbf{w}_\gamma \mid \mathbf{w}_\gamma \leq w_\beta; h].$$

b. Bidder γ : If β has made a jump bid x_β for B:

i. If $\mathcal{P}_\beta^{-1}(x_\beta) \neq \emptyset$, top x_β with a jump bid

$$\tilde{\mathcal{P}}_\gamma(w_\gamma \mid x_\beta) := \mathbb{E} [\mathbf{w}_\beta \mid \mathbf{w}_\beta \leq w_\gamma; \mathbf{w}_\beta \in \mathcal{P}_\beta^{-1}(x_\beta); h]$$

if $w_\gamma > \inf \mathcal{P}_\beta^{-1}(x_\beta)$, and withdraw if $w_\gamma \leq \inf \mathcal{P}_\beta^{-1}(x_\beta)$.

ii. If $\mathcal{P}_\beta^{-1}(x_\beta) = \emptyset$, update: \mathbf{w}_β is drawn from a no-atom no-gap distribution G supported by $[x_\beta, \infty)$ and follow plan (i) with $\mathcal{P}_\beta^{-1}(x_\beta)$ replaced by $[x_\beta, \infty)$, the distribution replaced by G , and h removed.

c. If γ responds to β 's jump bid with a bid x_γ , β replies by following plans (i) and (ii), with the substitutions $\mathcal{P}_\beta^{-1}(x_\beta) \rightarrow \tilde{\mathcal{P}}_\gamma^{-1}(x_\gamma \mid x_\beta)$, $\tilde{\mathcal{P}}_\gamma(w_\gamma \mid x_\beta) \rightarrow \tilde{\mathcal{P}}_\beta(w_\beta \mid x_\beta, x_\gamma)$, $\mathbf{w}_\beta \rightarrow \mathbf{w}_\gamma$, $w_\gamma \rightarrow w_\beta$, and $x_\beta \rightarrow x_\gamma$.

d. If β does not act immediately at the start of the decisive moment, γ immediately acts by following plans (a)-(c) with the roles of β and γ switched.

Explanation of the previous equilibrium

1. On the path, almost surely the auction of B ends once γ has reacted to β 's first jump bid.
 - (a) $\mathcal{P}_\beta(w_\beta) = \mathcal{P}_\beta(w'_\beta)$ iff, given h , \mathbf{w}_γ has zero weight strictly between w_β and w'_β .
 - (b) Thus, if β jump-bids to x_β , almost surely one of the following happens:
 - i. $\inf \mathcal{P}_\beta^{-1}(x_\beta) \geq w_\gamma$, then γ quits immediately;
 - ii. $\sup \mathcal{P}_\beta^{-1}(x_\beta) \leq w_\gamma$, then γ replies with a jump bid $E[\mathbf{w}_\beta \mid \mathbf{w}_\beta \in \mathcal{P}_\beta^{-1}(x_\beta)]$; seeing this, β knows $\sup \mathcal{P}_\beta^{-1}(x_\beta) \leq w_\gamma$, hence β quits immediately.
2. If the other bidder $-i$'s MWTP w_{-i} has been completely revealed, then bidder i cannot do better than crying out a bid equal to w_{-i} .
3. If w_{-i} has not been completely revealed, bidder i cannot do better than the *efficient outcome* where his winning event is $\{w_{-i} : w_{-i} < w_i\}$ and his payment conditional on winning is w_{-i} in expectation.
4. This efficient outcome is what he achieves if bidder i follows the proposed equilibrium.

Jump-Bidding Leads to Over-Concentration

Proposition 2: Any equilibrium's allocation is probably over-concentrated and is never over-diffused.

1. Due to jump-bidding, the global bidder knows whether he can profitably acquire both items before he buys any of them. Hence he faces no exposure problem and is effectively bidding for the entire package $\{A, B\}$, so he does not underbid.
2. Local bidders do not always bid up to their true values.
 - (a) A local bidder who quits may win his desired item because the other local bidder's immediate jump bid may force the global bidder to quit immediately and yield to the local bidders.
 - (b) Hence a local bidder wishes to free ride the other. This is proved by reinterpreting the impossibility of efficient provision of public goods, with the cost of public goods being the global bidder's value that local bidders' combined bid needs to top.

An Ascending Package Auctions

Local α bids only for $\{A\}$, β only for $\{B\}$, and global γ bids only for $\{A, B\}$. The price of a package starts at zero and rises continuously, with jump-bidding banned.

1. If no one has quit, the price p_A (or p_B) for package $\{A\}$ (or $\{B\}$) rises at speed \dot{p}_A (or \dot{p}_B), and the price p_{AB} for $\{A, B\}$ rises at speed $\dot{p}_A + \dot{p}_B$.
2. If global γ quits before α and β , the items are sold to α and β at the current prices p_A and p_B . (Note $p_A + p_B$ is equal to γ 's dropout price p_{AB} .)
3. If α quits at price p_A before β and γ , stop raising p_A , and raise p_B and p_{AB} at the same speed.
 - (a) If subsequently bidder β quits at price p_B before the global γ , sell both items to γ at a total price $p_A + p_B$.
 - (b) If subsequently γ quits at price p_{AB} before local β , sell B to β at its current price p_B , and sell A to α at his dropout price p_A . (Note $p_{AB} = p_A + p_B$.)
4. Symmetric treatment if β quits before α and γ .
5. If α and β quit simultaneously before global γ , γ wins.

Simultaneous Auctions Mimic Package Auctions

Corollary: The simultaneous ascending auctions which bans cross-bidding is equivalent to the package auction.

1. Due to jump-bidding, the global bidder in the simultaneous auctions is effectively bidding for the entire package $\{A, B\}$, as he does in the package auction.
2. The consequence of a local bidder's being the first dropout is the same in both auction games.
 - (a) In the package auction, once local α quits at p_A , local β finds it dominant to bid for B up to its value; if he wins, β buys B at price $t_\gamma - p_A$ (as γ bids up to t_γ) and α buys A at his dropout price p_A .
 - (b) In the simultaneous auctions, the outcome is the same in expectation.
3. If cross-bidding is allowed, the corollary becomes:

Proposition 3:

For any equilibrium of the package auction, there is an equilibrium in the simultaneous auctions that generates the same allocation.

Extension to Resale

1. The incentive for resale:

As simultaneous auctions are inefficient, resale occurs with positive probabilities (Zhèng (2002)).

2. To model auction with resale:

Assume winners in an auction selects an optimal mechanism for possible resale and assume winners in his mechanism to have the same option.

- (a) Why are the initial auctions exogenous while resale auctions are endogenous?
- (b) Recall the goal: to understand a given mechanism, simultaneous ascending auctions. Hence it is appropriate to hold this mechanism as exogenous.
- (c) But assuming exogenous resale mechanisms is not appropriate because renegotiation can take many forms, and we do not know a priori which form will be prevalent.
- (d) It is therefore natural to let resale mechanisms emerge as optimal actions chosen by some players.

The Auction-Resale Game

1. N periods; no discounting; $N \gg \#\text{bidders}$.
2. Period 1: simultaneous auctions.
3. Period 2: resale among bidders is allowed.
 - (a) If a bidder has won all items in period 1, he can commit to a mechanism for possible resale.
 - (b) If items are sold to different bidders, one of the winners is randomly selected to pick a resale mechanism; if no other winner vetoes it, the mechanism is implemented; else the mechanism is not implemented and every winner commits to a resale price for the item he currently owns. The probability with which a winner is selected to pick a resale mechanism is proportional to the number of items he currently owns.
4. If a resale mechanism results in no-sale or if period N is reached, the game ends; else in the next period the process repeats.

Over-Concentration Is Robust with Resale

1. A bidder who expects a positive probability of buying an item at resale shades his bids in the initial auctions.
2. Proposition 4:

There is a perfect Bayesian equilibrium where local bidders bid zero and the global bidder wins both items in period one; the final allocation is over-concentrated with a positive probability and never over-diffused.

- (a) Proof: skipped; in the paper.
- (b) Although the period-1 bidding strategy is weakly dominated had resale been banned, it is not weakly dominated if resale is allowed.
- (c) This equilibrium may be a focal point because the goods are over-concentrated to the global bidder before bidders learn to exploit resale opportunities.
- (d) This proposition remains true when there are n_k copies of the k -bidder, with $k = \alpha, \beta, \gamma$, such that the values of all the k -bidders are independently drawn from the same distribution F_k . (The period-1 winner is the highest-value global bidder.)

Proof of Proposition 4

1. Myerson auction and its virtual-utility allocation rule.
2. Equilibrium strategy:
 - (a) In period 1, only the global player bids.
 - (b) In period 2, the global bidder offers resale via the Myerson auction based on—
 - i. the priors if local bidders did not bid in period 1;
 - ii. the posterior that the i 's type is greater than p if a local i bid in period 1 up to price p .
3. Verification of equilibrium:
 - (a) There is no further resale after the Myerson auction.
 - i. If “no sale” in Myerson auction, the game ends.
 - ii. α never gets B and β never gets A. There is no gain to resell back to the reseller.
 - (b) Local bidders are truthful in the Myerson auction.
 - (c) The Myerson auction is optimal for the reseller, in both cases (b)-i or (b)-ii.
 - (d) Given the resale mechanism in 2(b), a local bidder does not bid in period 1.