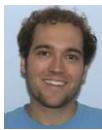


# Parallel Composition Revisited



Chris Clifton

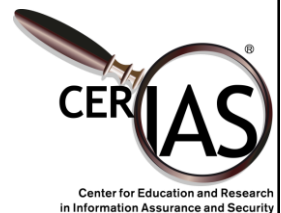
23 October 2017



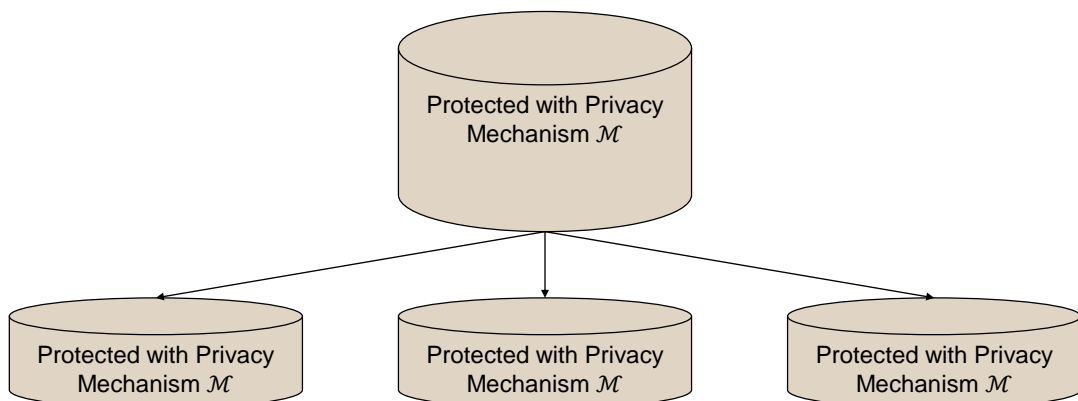
*This is joint work with Keith Merrill  
and Shawn Merrill*



*This work supported by the U.S. Census Bureau under  
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# Partitioning and Privacy



- When can we treat the databases independently?

## Definition: Parallel Composition

We say that a sanitization scheme  $A$  satisfies **parallel composition** if, given disjoint datasets  $D_1, \dots, D_n$ , with corresponding outputs  $A(D) = A(D_1), \dots, A(D_n)$  satisfies the privacy guarantee of the original scheme.

- Satisfied by:
  - Differential Privacy (*McSherry SIGMOD'09*)
    - Privacy budget treated independently for each dataset
  - Generalization-based  $k$ -anonymity,  $l$ -diversity with local recording
- Not satisfied by
  - Generalization-based anonymization with global recording
  - Differential Privacy (*Dwork, McSherry, Nissim, Smith TCC'06*):  $2\epsilon$

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## Parallel Composition: Differential Privacy

**Dwork, McSherry, Nissim, Smith  
TCC'06**

Let  $D$  be partitioned into  $d$  disjoint regions, let  $f : D^n \rightarrow \mathbb{R}^d$  be a function whose output coordinates  $f(x)_i$  depend only on those elements in the  $i$ th region. We can bound  $S(f) \leq 2 \max_i S(f_i)$ .

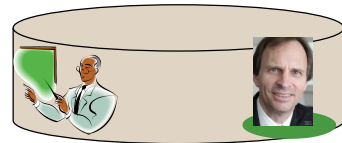
**McSherry SIGMOD'09**

Let  $M_i$  each provide  $\epsilon$ -differential privacy. Let  $D_i$  be arbitrary disjoint subsets of the input domain  $D$ . The sequence of  $M_i(X \cap D_i)$  provides  $\epsilon$ -differential privacy.

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## Why the discrepancy?

- Definition of “differ on a single entry”
  - Deletion (TAMC’08) - easy to show  $\epsilon$
  - Substitution (TCC’06) - easy to show  $2\epsilon$
  - Modifying values – Is this  $\epsilon$  or  $2\epsilon$ ?
- Disjoint datasets (’09  $\epsilon$ ) vs. Partitioned dataset (’06  $2\epsilon$ )
  - We narrow this gap



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## Definition: Partitioned Preprocessing

Choose a random partition  $\{d_i\}$  of  $|D|$  into positive integers, then partition  $D$  into pieces  $D_i$  of size  $d_i$  uniformly at random. We call  $\bigcup_{i=1}^n A(D_i)$  a **partitioned preprocessing** dataset.

- Works for parallel composition techniques
  - Including  $\epsilon$ -DP under substitution
- Potentially stronger against some types of attacks on generalization
  - Minimality
  - deFinetti
- Attack resistance arguments hold for non-parallel decomposable techniques
  - E.g., global recoding (and potential utility benefits)

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## Theorem: Parallel Composition on Random Partitions

- Let  $D$  be a dataset,  $|D| = n$ . Choose a decomposition  $\mathbf{n}$  of  $n$  and a permutation  $\pi$  on  $n$  elements uniformly at random, and partition the dataset  $D$  into  $n$  pieces  $\{D_{\pi,i}\}_{1 \leq i \leq j}$ . Let  $\mathcal{A}_1, \dots, \mathcal{A}_j$  be differentially private mechanisms with privacy budgets  $\epsilon_1, \dots, \epsilon_j$ .  
The mechanism  $\mathcal{A} = (\mathcal{A}_1(D_{\pi,1}), \dots, \mathcal{A}_j(D_{\pi,j}))$  satisfies  $\epsilon$ -differential privacy, where  $\epsilon = \max_{1 \leq i \leq j} \epsilon_i$ .

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## Proof Idea

- Partitions determined in advance, independent of data
  - Substituting a tuple affects only one partition
- For partitions without the changed tuple,  $D_{\pi,k} = D'_{\pi,k}$ , so  $P(\mathcal{A}_k(D_{\pi,k}) \in T_k) = P(\mathcal{A}_k(D'_{\pi,k}) \in T_k)$
- The changed partition  $j$  has a difference bounded by  $\epsilon_j$ 
  - This bounds the total difference between  $\mathcal{A}(D)$  and  $\mathcal{A}(D')$

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## More Differences between Deletion and Substitution

- What is the sensitivity of
  - $|D|$  ?
    - Deletion: 1
    - Substitution: 0
  - Average
- Amplification (*Li, Qardaji, Su '12*)
  - Defined under deletion

*Is there a difference in the privacy semantics?*

## Partitioned Preprocessing: Potential Utility Benefit

Age	Gender	Zip	Cancer	Age	Gender	Zip	Cancer
40-50	Male	92***	Yes	40-60	Male	925**	No
40-50	Male	92***	No	40-60	Male	925**	No
40-50	Male	92***	No	40-60	Male	925**	Yes
40-50	Male	92***	Yes	40-60	Male	925**	No

- Some benefits of local recoding
  - “Outliers” only force over-generalization in a single partition
- Each partition satisfies global recoding
  - Difficulty identifying which partition an item belongs to provides defense against attacks

## Partitioned Preprocessing: Example

Semantic Attacks: Determine likely distribution of sensitive values in an equivalence class

- Individual may belong to many equivalence classes
  - Attack gives information on one equivalence class
- Attack increases  $\Pr(x.S = S_i)$  by only a (weighted) proportion of the increase in probability for that class

k=20	Underlying Partitions	Visible Partitions	Distribution of Partitions	% of Population
Average 25,000 size	20	6 + Suppressed Class	6, 5, 6, 1, 1, 1	.244, .30, .295, .062, .048, .024 Suppress: .016

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## Partitioned Preprocessing: Example


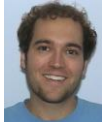
- Original Record:

ZIP	YOB	GEN	VISIT	HOSPITAL	COMP	CAT	Possible Matches
43125	1967	F	2005-08-31	Riverside Methodist	Mosquito Bite	Other	7,916

- Anonymized Versions:

ZIP	YOB	Visit Date	Hospital	Matches
43000 - 43240	1940 - 1979	2004-01-01 - 2005-12-31	Riverside Methodist Hospital	2520
43068 - 43156	1940 - 1979	2004-01-01 - 2005-12-31	Medium & Large Hospitals	3497
43068 - 43156	1900 - 1992	2004-01-01 - 2005-12-31	Riverside Methodist Hospital	1068
43119 - 43156	1940 - 1979	2004-01-01 - 2008-02-31	Large Hospitals	421
43119 - 43156	1900 - 1992	2005-07-01 - 2005-12-31	Medium & Large Hospitals	169
43068 - 43156	1900 - 1992	2004-01-01 - 2005-12-31	Large Hospitals	241

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- Implications of partitioned preprocessing on differential privacy
  - Near-optimal use of privacy budget
    - Use noise from random partitioning to satisfy differential privacy
  - Potential operational value?
  - Amplification of privacy budget through sampling
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