Secret correlation of pure automata

Olivier Gossner and Penélope Hernández

Olivier.Gossner@enpc.fr

Paris-Jourdan Sciences Économiques, and IAS Jerusalem

Universidad d'Alicante

Repeated games played by FA

Repeated games played by FA

- Non zero-sum
- Zero-sum

Repeated games played by FA

- Non zero-sum
  - n players:
  - 2 players:
- Zero-sum
  - n players:
  - 2 players:

Repeated games played by FA

- Non zero-sum
  - n players: Aumann (81), Kalai and Standford (88);
  - 2 players:
- Zero-sum
  - n players:
  - 2 players:

Repeated games played by FA

- Non zero-sum
  - n players: Aumann (81), Kalai and Standford (88);
  - 2 players: Neyman (85), Rubinstein (86), Abreu and Rubinstein (88), Papadimitriou and Yannakakis (94), Piccione and Rubinstein (93)...
- Zero-sum
  - n players:
  - 2 players:

Repeated games played by FA

- Non zero-sum
  - n players: Aumann (81), Kalai and Standford (88);
  - 2 players: Neyman (85), Rubinstein (86), Abreu and Rubinstein (88), Papadimitriou and Yannakakis (94), Piccione and Rubinstein (93)...
- Zero-sum
  - n players:
  - 2 players: Ben-Porath (93), Neyman (97), Neyman and Okada (99, 00, 00).

Repeated games played by FA

- Non zero-sum
  - n players: Aumann (81), Kalai and Standford (88);
  - 2 players: Neyman (85), Rubinstein (86), Abreu and Rubinstein (88), Papadimitriou and Yannakakis (94), Piccione and Rubinstein (93)...
- Zero-sum
  - n players: Bavly and Neyman (04), Gossner Hernández and Neyman (05).
  - 2 players: Ben-Porath (93), Neyman (97), Neyman and Okada (99, 00, 00).

n players zero-sum

n players zero-sum

- Bavly and Neyman (04):

n players zero-sum

- Bavly and Neyman (04):

  A superstrong player secretly coordinates the actions of weak members of a team against strong players.

n players zero-sum

- Bavly and Neyman (04):

  A superstrong player secretly coordinates the actions of weak members of a team against strong players.

- Gossner Hernández and Neyman (05):

n players zero-sum

- Bavly and Neyman (04):

  A superstrong player secretly coordinates the actions of weak members of a team against strong players.

- Gossner Hernández and Neyman (05):

  A superstrong player decodes the strong opponents' strategies and informs weak players of their future action plans.

n players zero-sum

- Bavly and Neyman (04):

  A superstrong player secretly coordinates the actions of weak members of a team against strong players.

- Gossner Hernández and Neyman (05):

  A superstrong player decodes the strong opponents' strategies and informs weak players of their future action plans.

What can a team achieve without superstrong players? (with players of comparable complexities)

Model: game G, $n = 3$

Model: game G, $n = 3$

Action spaces X^1, X^2, X^3. $|X^i| \geq 2$.
$X^{-i} = \Pi_{j \neq i} X^j$, $X = \Pi_i X^i$.
$g\colon X \to \mathbb{R}$ payoff to players $1, 2$.

Model: game G, $n = 3$

Action spaces X^1, X^2, X^3. $|X^i| \geq 2$.
$X^{-i} = \Pi_{j \neq i} X^j$, $X = \Pi_i X^i$.
$g \colon X \to \mathbb{R}$ payoff to players $1, 2$.

$$\begin{aligned}
v^p &= V^p(G) = \max_{x^{-3}} \min_{x^3} g \\
v^m &= V^m(G) = \max_{\delta \in \Delta(X^1) \times \Delta(X^2)} \min_{x^3} \mathrm{E}_\delta g \\
v^c &= V^c(G) = \max_{\delta \in \Delta(X^{-3})} \min_{x^3} \mathrm{E}_\delta g = \min_{s^3 \in \Delta(X^3)} \max_{x^{-3}} \mathrm{E}_{s^3} g
\end{aligned}$$

Model: game G, $n = 3$

Action spaces X^1, X^2, X^3. $|X^i| \geq 2$.
$X^{-i} = \Pi_{j \neq i} X^j$, $X = \Pi_i X^i$.
$g\colon X \to \mathbb{R}$ payoff to players $1, 2$.

$$v^p = V^p(G) = \max_{x^{-3}} \min_{x^3} g$$
$$v^m = V^m(G) = \max_{\delta \in \Delta(X^1) \times \Delta(X^2)} \min_{x^3} \mathrm{E}_\delta g$$
$$v^c = V^c(G) = \max_{\delta \in \Delta(X^{-3})} \min_{x^3} \mathrm{E}_\delta g = \min_{s^3 \in \Delta(X^3)} \max_{x^{-3}} \mathrm{E}_{s^3} g$$

$$v^c \geq v^m \geq v^p$$

Model: repeated game

Model: repeated game

An automaton of size m^i for player i, $A^i \in \Sigma_{m^i}$ consists of:

- A set of *states* Q^i of size m^i, with *initial state* $\hat{q}^i \in Q^i$
- An *action function* $f^i \colon Q^i \longrightarrow X^i$.
- A *transition function* $g^i \colon Q^i \times X^{-i} \longrightarrow Q^i$

Model: repeated game

An automaton of size m^i for player i, $A^i \in \Sigma_{m^i}$ consists of:

- A set of *states* Q^i of size m^i, with *initial state* $\hat{q}^i \in Q^i$
- An *action function* $f^i\colon Q^i \longrightarrow X^i$.
- A *transition function* $g^i\colon Q^i \times X^{-i} \longrightarrow Q^i$

It is *oblivious* if its transitions do not depend on other player's actions.

Model: repeated game

An automaton of size m^i for player i, $A^i \in \Sigma_{m^i}$ consists of:

- A set of *states* Q^i of size m^i, with *initial state* $\hat{q}^i \in Q^i$
- An *action function* $f^i \colon Q^i \longrightarrow X^i$.
- A *transition function* $g^i \colon Q^i \times X^{-i} \longrightarrow Q^i$

It is *oblivious* if its transitions do not depend on other player's actions.
A triple of automata A^1, A^2, A^3 induces an eventually periodic sequence. The average of g over a period is denoted $\gamma(A^1, A^2, A^3)$.

Model: repeated game

An automaton of size m^i for player i, $A^i \in \Sigma_{m^i}$ consists of:

- A set of *states* Q^i of size m^i, with *initial state* $\hat{q}^i \in Q^i$
- An *action function* $f^i \colon Q^i \longrightarrow X^i$.
- A *transition function* $g^i \colon Q^i \times X^{-i} \longrightarrow Q^i$

It is *oblivious* if its transitions do not depend on other player's actions.
A triple of automata A^1, A^2, A^3 induces an eventually periodic sequence. The average of g over a period is denoted $\gamma(A^1, A^2, A^3)$.
$G(m^1, m^2, m^3)$ is the game with strategy spaces Σ_{m^i} and payoff function γ to players 1 and 2.

Questions

We are concerned by the relation between the asymptotic sizes $\boldsymbol{m^1, m^2, m^3}$ and the limits of

$$\begin{aligned} \boldsymbol{V^p(m^1, m^2, m^3)} &= \boldsymbol{V^p(G(m^1, m^2, m^3))} \\ \boldsymbol{V^m(m^1, m^2, m^3)} &= \boldsymbol{V^m(G(m^1, m^2, m^3))} \\ \boldsymbol{V^c(m^1, m^2, m^3)} &= \boldsymbol{V^c(G(m^1, m^2, m^3))} \end{aligned}$$

Play against sequences

Play against sequences

A pair of automata of players 1 and 2 of sizes m^1 and m^2 that do not observe player 3's actions induce an eventually periodic sequence of actions.

Play against sequences

A pair of automata of players 1 and 2 of sizes m^1 and m^2 that do not observe player 3's actions induce an eventually periodic sequence of actions.

A periodic sequence $\tilde{x}$ of actions of $1, 2$ and A^3 induce an eventually periodic play, $\gamma(\tilde{x}, A^3)$ denotes the average of g over a period.

Play against sequences

Play against sequences

Let $\boldsymbol{\delta} \in \boldsymbol{\Delta}(\boldsymbol{X}^{-3})$, and $\tilde{\boldsymbol{x}}$ be a random $\boldsymbol{n}$-periodic sequence with $\boldsymbol{n}$ first elements i.i.d.$\sim \boldsymbol{\delta}$.

Play against sequences

Let $\delta \in \Delta(X^{-3})$, and $\tilde{x}$ be a random n-periodic sequence with n first elements i.i.d.$\sim \delta$.

Neyman (97): If $n \gg m^3 \ln m^3$ then $\forall \varepsilon > 0$

$$P(\min_{A^3} \gamma(\tilde{x}, A^3) < \min_{x^3} \mathrm{E}_\delta g - \varepsilon) \longrightarrow 0$$

Play against sequences

Let $\delta \in \Delta(X^{-3})$, and $\tilde{x}$ be a random n-periodic sequence with n first elements i.i.d.$\sim \delta$.

Neyman (97): If $n \gg m^3 \ln m^3$ then $\forall \varepsilon > 0$

$$P(\min_{A^3} \gamma(\tilde{x}, A^3) < \min_{x^3} \mathrm{E}_\delta g - \varepsilon) \longrightarrow 0$$

Probabilistic argument: Over a period, each automaton of player 3 can force a set of bounded probability of sequences to a significantly smaller payoff than $\mathrm{E}_\delta g - \varepsilon$.

Consequence on pure max min

Consequence on pure max min

If $\min(m^1, m^2) \gg m^3 \ln m^3$ then

$$V^p(m^1, m^2, m^3) \to v^c$$

Consequence on pure max min

If $\min(m^1, m^2) \gg m^3 \ln m^3$ then

$$V^p(m^1, m^2, m^3) \to v^c$$

Moreover, the same holds if players 1 and 2 are restricted to oblivious automata.

Consequence on pure max min

If $\min(m^1, m^2) \gg m^3 \ln m^3$ then

$$V^p(m^1, m^2, m^3) \to v^c$$

Moreover, the same holds if players 1 and 2 are restricted to oblivious automata.

On the other hand:

Consequence on pure max min

If $\min(m^1, m^2) \gg m^3 \ln m^3$ then

$$V^p(m^1, m^2, m^3) \to v^c$$

Moreover, the same holds if players 1 and 2 are restricted to oblivious automata.

On the other hand:

- If $m^3 \geq m^1 m^2$ then $V^p(m^1, m^2, m^3) = v^p$

Consequence on pure max min

If $\min(m^1, m^2) \gg m^3 \ln m^3$ then

$$V^p(m^1, m^2, m^3) \to v^c$$

Moreover, the same holds if players 1 and 2 are restricted to oblivious automata.

On the other hand:

- If $m^3 \geq m^1 m^2$ then $V^p(m^1, m^2, m^3) = v^p$
- If $m^3 \geq m^1$ then
  $V^p(m^1, m^2, m^3) \leq \max_{x^1, s^2} \min_{x^3} \mathrm{E}_{s^2} g$

Our main result

If $\mathbf{min}(\boldsymbol{m^1}, \boldsymbol{m^2}) \gg \boldsymbol{m^3}$ then

$$\boldsymbol{V^p(m^1, m^2, m^3) \to v^c}$$

Implementation of periodic sequences

Implementation of periodic sequences

Call a periodic sequence $\tilde{x}$ of actions of players $\mathbf{1}$ and $\mathbf{2}$ (m^1, m^2)-implementable if $\exists A^1, A^2 \in \Sigma_{m^1} \times \Sigma_{m^2}$ that do not observe player 3's actions and generate $\tilde{x}$.

Implementation of periodic sequences

Call a periodic sequence $\tilde{x}$ of actions of players 1 and 2 (m^1, m^2)-implementable if $\exists A^1, A^2 \in \Sigma_{m^1} \times \Sigma_{m^2}$ that do not observe player 3's actions and generate $\tilde{x}$.

Thus, all m-periodic sequences are (m, m)-implementable, and that an (m^1, m^2)-implementable sequence is at most $m^1 m^2$-periodic.

Periods of implementable sequences

Periods of implementable sequences

Proposition: Let $\delta \in \Delta(X^{-3})$ be rational with full support.
Let $\tilde{x}$ be random n-periodic with n first elements i.i.d.$\sim \delta$.

Periods of implementable sequences

Proposition: Let $\delta \in \Delta(X^{-3})$ be rational with full support. Let $\tilde{x}$ be random n-periodic with n first elements i.i.d.$\sim \delta$. Then $\exists C$ such that $n \leq C m \ln m$ implies

$$P(\tilde{x} \text{ is } (m,m)\text{-implementable}) \longrightarrow 1$$

Periods of implementable sequences

Proposition: Let $\delta \in \Delta(X^{-3})$ be rational with full support.
Let $\tilde{x}$ be random n-periodic with n first elements i.i.d.$\sim \delta$.
Then $\exists C$ such that $n \leq C m \ln m$ implies

$$P(\tilde{x} \text{ is } (m, m)\text{-implementable}) \longrightarrow 1$$

Hence, a pair of automata of size m can jointly implement almost every $C m \ln m$ periodic sequences.

Proof of the main result from the prop.

Proof of the main result from the prop.

Let $m = \min(m^1, m^2)$.

Proof of the main result from the prop.

Let $\boldsymbol{m} = \mathbf{min}(\boldsymbol{m}^{\mathbf{1}}, \boldsymbol{m}^{\mathbf{2}})$.

1. Choose $\boldsymbol{n}$ such that $\boldsymbol{m} \ln \boldsymbol{m} \gg \boldsymbol{n} \gg \boldsymbol{m}^{\mathbf{3}} \ln \boldsymbol{m}^{\mathbf{3}}$.

Proof of the main result from the prop.

Let $m = \min(m^1, m^2)$.

1. Choose n such that $m \ln m \gg n \gg m^3 \ln m^3$.
2. Approximate an optimal correlated strategy of players 1 and 2 in G by δ rational with full support.

Proof of the main result from the prop.

Let $m = \min(m^1, m^2)$.

1. Choose n such that $m \ln m \gg n \gg m^3 \ln m^3$.
2. Approximate an optimal correlated strategy of players 1 and 2 in G by δ rational with full support.
3. Draw $\tilde{x}$ n-periodic, with n first coordinates i.i.d.$\sim \delta$.

Proof of the main result from the prop.

Let $m = \min(m^1, m^2)$.

1. Choose n such that $m \ln m \gg n \gg m^3 \ln m^3$.
2. Approximate an optimal correlated strategy of players 1 and 2 in G by δ rational with full support.
3. Draw $\tilde{x}$ n-periodic, with n first coordinates i.i.d.$\sim \delta$.

Then for $\varepsilon > 0$

Proof of the main result from the prop.

Let $m = \min(m^1, m^2)$.

1. Choose n such that $m \ln m \gg n \gg m^3 \ln m^3$.
2. Approximate an optimal correlated strategy of players 1 and 2 in G by δ rational with full support.
3. Draw $\tilde{x}$ n-periodic, with n first coordinates i.i.d.$\sim \delta$.

Then for $\varepsilon > 0$

$$
\begin{aligned}
P(\min_{A^3} \gamma(\tilde{x}, A^3) < \min_{x^3} \mathrm{E}_\delta g - \varepsilon) &\longrightarrow 0 \\
P(\tilde{x} \text{ is } (m, m)\text{-implementable}) &\longrightarrow 1
\end{aligned}
$$

Proof of the main result from the prop.

Let $m = \min(m^1, m^2)$.

1. Choose n such that $m \ln m \gg n \gg m^3 \ln m^3$.
2. Approximate an optimal correlated strategy of players 1 and 2 in G by δ rational with full support.
3. Draw $\tilde{x}$ n-periodic, with n first coordinates i.i.d.$\sim \delta$.

Then for $\varepsilon > 0$

$$P(\min_{A^3} \gamma(\tilde{x}, A^3) < \min_{x^3} \mathrm{E}_\delta g - \varepsilon) \quad \longrightarrow \quad 0$$

$$P(\tilde{x} \text{ is } (m,m)\text{-implementable}) \quad \longrightarrow \quad 1$$

In particular, there exist (m, m)-implementable sequences that guarantee $\min_{x^3} \mathrm{E}_\delta g - \varepsilon$.

Implementation of sequences

Implementation of sequences

Let $\tilde{x}$ be n-periodic. We construct an automaton of player 1 that follows $\tilde{x}$ as long as the other player does.

Implementation of sequences

Let $\tilde{x}$ be n-periodic. We construct an automaton of player 1 that follows $\tilde{x}$ as long as the other player does. For $1 \leq l \leq n$, let ϕ be a permutation of X^2, and let $\tilde{y}$ n-periodic such that for $1 \leq t \leq n$.

$$\begin{cases} \tilde{y}_t = \tilde{x}_t, & \text{if } l \text{ does not divide } t; \\ \tilde{y}_t = (\tilde{x}^1_t, \phi(\tilde{x}^2_t)) & \text{if } l \text{ divides } t. \end{cases}$$

Implementation of sequences

Let $\tilde{x}$ be n-periodic. We construct an automaton of player 1 that follows $\tilde{x}$ as long as the other player does. For $1 \leq l \leq n$, let ϕ be a permutation of X^2, and let $\tilde{y}$ n-periodic such that for $1 \leq t \leq n$.

$$\begin{cases} \tilde{y}_t = \tilde{x}_t, & \text{if } l \text{ does not divide } t; \\ \tilde{y}_t = (\tilde{x}_t^1, \phi(\tilde{x}_t^2)) & \text{if } l \text{ divides } t. \end{cases}$$

$\tilde{y}_t^1$ is player 1's action at stage t.

Implementation of sequences

Let $\tilde{x}$ be n-periodic. We construct an automaton of player 1 that follows $\tilde{x}$ as long as the other player does. For $1 \leq l \leq n$, let ϕ be a permutation of X^2, and let $\tilde{y}$ n-periodic such that for $1 \leq t \leq n$.

$$\begin{cases} \tilde{y}_t = \tilde{x}_t, & \text{if } l \text{ does not divide } t; \\ \tilde{y}_t = (\tilde{x}^1_t, \phi(\tilde{x}^2_t)) & \text{if } l \text{ divides } t. \end{cases}$$

$\tilde{y}^1_t$ is player 1's action at stage t.
$\tilde{y}^2_t$ is player 1's anticipation at stage t, it differs from the played action $\tilde{x}^2_t$ of player 2 every l stages.

Implementation of sequences

Let $\tilde{x}$ be n-periodic. We construct an automaton of player 1 that follows $\tilde{x}$ as long as the other player does. For $1 \leq l \leq n$, let ϕ be a permutation of X^2, and let $\tilde{y}$ n-periodic such that for $1 \leq t \leq n$.

$$\begin{cases} \tilde{y}_t = \tilde{x}_t, & \text{if } l \text{ does not divide } t; \\ \tilde{y}_t = (\tilde{x}_t^1, \phi(\tilde{x}_t^2)) & \text{if } l \text{ divides } t. \end{cases}$$

$\tilde{y}_t^1$ is player 1's action at stage t.
$\tilde{y}_t^2$ is player 1's anticipation at stage t, it differs from the played action $\tilde{x}_t^2$ of player 2 every l stages.
We write the first period of $\tilde{y}$ as the concatenation of *words* $r_1 \ldots r_{\frac{n}{l}}$ in $(X^{-3})^l$.

Implementation of sequences

Let $\tilde{x}$ be n-periodic. We construct an automaton of player 1 that follows $\tilde{x}$ as long as the other player does. For $1 \leq l \leq n$, let ϕ be a permutation of X^2, and let $\tilde{y}$ n-periodic such that for $1 \leq t \leq n$.

$$\begin{cases} \tilde{y}_t = \tilde{x}_t, & \text{if } l \text{ does not divide } t; \\ \tilde{y}_t = (\tilde{x}^1_t, \phi(\tilde{x}^2_t)) & \text{if } l \text{ divides } t. \end{cases}$$

$\tilde{y}^1_t$ is player 1's action at stage t.
$\tilde{y}^2_t$ is player 1's anticipation at stage t, it differs from the played action $\tilde{x}^2_t$ of player 2 every l stages.
We write the first period of $\tilde{y}$ as the concatenation of *words* $r_1 \ldots r_{\frac{n}{l}}$ in $(X^{-3})^l$. All words are i.i.d.$\sim \rho$.

Set of states

Set of states

Let $\alpha > 1$. The set of states is a cycle $z_1, \ldots, z_m$ of elements of X^{-3} such that for every r,

$$N(r) = \#\{i, (z_i, \ldots z_{i+l}) = r\} \geq \alpha\rho(r)\frac{n}{l}$$

Set of states

Let $\alpha > 1$. The set of states is a cycle $z_1, \ldots, z_m$ of elements of X^{-3} such that for every r,

$$N(r) = \#\{i, (z_i, \ldots z_{i+l}) = r\} \geq \alpha \rho(r) \frac{n}{l}$$

Relying on DeBruijn sequences, we can construct such a cycle if $m \geq \beta \frac{n}{l}$ for some $\beta > 0$.

Programmation

Programmation

If the anticipation is correct, go to the next state in the cycle.

Programmation

If the anticipation is correct, go to the next state in the cycle.

- Start at $\hat{q}^1 = i_1$ such that $(z_{i_1}, z_{i_1+1}, \ldots, z_{i_1+l-1}) = r_1$

Programmation

If the anticipation is correct, go to the next state in the cycle.

- Start at $\hat{q}^1 = i_1$ such that $(z_{i_1}, z_{i_1+1}, \ldots, z_{i_1+l-1}) = r_1$
- At z_{i_1+l-1}, if the action of 2 does not match the anticipation, go to i_2 such that $(z_{i_2}, z_{i_2+1}, \ldots, z_{i_2+l-1}) = r_2$

Programmation

If the anticipation is correct, go to the next state in the cycle.

- Start at $\hat{q}^1 = i_1$ such that $(z_{i_1}, z_{i_1+1}, \ldots, z_{i_1+l-1}) = r_1$
- At z_{i_1+l-1}, if the action of 2 does not match the anticipation, go to i_2 such that $(z_{i_2}, z_{i_2+1}, \ldots, z_{i_2+l-1}) = r_2$
- At z_{i_2+l-1}, if the action of 2 does not match the anticipation, go to i_3 such that $(z_{i_3}, z_{i_3+1}, \ldots, z_{i_3+l-1}) = r_3$

Programmation

If the anticipation is correct, go to the next state in the cycle.

- Start at $\hat{q}^1 = i_1$ such that $(z_{i_1}, z_{i_1+1}, \ldots, z_{i_1+l-1}) = r_1$
- At z_{i_1+l-1}, if the action of 2 does not match the anticipation, go to i_2 such that $(z_{i_2}, z_{i_2+1}, \ldots, z_{i_2+l-1}) = r_2$
- At z_{i_2+l-1}, if the action of 2 does not match the anticipation, go to i_3 such that $(z_{i_3}, z_{i_3+1}, \ldots, z_{i_3+l-1}) = r_3$
- $\ldots$

Size

Size

When can we apply the construction?

Size

When can we apply the construction? Two different transitions after after two incorrect anticipations must lead to two different states.

Size

When can we apply the construction? Two different transitions after after two incorrect anticipations must lead to two different states. We thus need

$$\forall \boldsymbol{r}, \quad \#\{\boldsymbol{j}, \boldsymbol{r_j} = \boldsymbol{r}\} \leq \boldsymbol{N}(\boldsymbol{r})$$

Size

When can we apply the construction? Two different transitions after after two incorrect anticipations must lead to two different states. We thus need

$$\forall r,\ \ \#\{j, r_j = r\} \leq N(r)$$

This holds if

$$\forall r,\ \ \#\{j, r_j = r\} \leq \alpha\rho(r)\frac{n}{l}$$

Size

When can we apply the construction? Two different transitions after after two incorrect anticipations must lead to two different states. We thus need

$$\forall r, \;\; \#\{j, r_j = r\} \leq N(r)$$

This holds if

$$\forall r, \;\; \#\{j, r_j = r\} \leq \alpha\rho(r)\frac{n}{l}$$

Computation shows that this has probability close to one if $l = \gamma(\alpha)\ln n$.

Size

When can we apply the construction? Two different transitions after after two incorrect anticipations must lead to two different states. We thus need

$$\forall r, \;\; \#\{j, r_j = r\} \leq N(r)$$

This holds if

$$\forall r, \;\; \#\{j, r_j = r\} \leq \alpha \rho(r) \frac{n}{l}$$

Computation shows that this has probability close to one if $l = \gamma(\alpha) \ln n$.

Hence $m \geq \beta \frac{n}{l} = \frac{\beta}{\gamma(\alpha)} \frac{n}{\ln n}$, or for some C:

$$n \leq C m \ln m$$

Length of implementable sequences

Length of implementable sequences

What is the order of magnitude of $n(m)$ such that the set of $n(m)$ periodic (m, m)-implementable sequences has large probability?

Length of implementable sequences

What is the order of magnitude of $n(m)$ such that the set of $n(m)$ periodic (m, m)-implementable sequences has large probability?
We have proven the existence of C such that

$$n(m) \geq Cm \ln m$$

Length of implementable sequences

What is the order of magnitude of $n(m)$ such that the set of $n(m)$ periodic (m, m)-implementable sequences has large probability?
We have proven the existence of C such that

$$n(m) \geq Cm \ln m$$

We also know that if $n(m) \gg m^3 \ln m^3$ then $V^p(m, m, m^3) \to v^c$.

Length of implementable sequences

What is the order of magnitude of $n(m)$ such that the set of $n(m)$ periodic (m, m)-implementable sequences has large probability?
We have proven the existence of C such that

$$n(m) \geq Cm \ln m$$

We also know that if $n(m) \gg m^3 \ln m^3$ then $V^p(m, m, m^3) \to v^c$.
Thus we do not have

$$n(m) \gg m \ln m$$

Any number of players

Players $\{1, \ldots, I\}$ against player $I + 1$. If $\min(m^1 \ldots m^I) \gg m^{I+1}$ and at least 2 players $\{1, \ldots, I\}$ have at least two actions, then $\{1, \ldots, I\}$ possess pure strategies that guarantee the correlated $\max\min$ against $I + 1$.

On the power of a team

On the power of a team

One player of size m can implement all m-periodic sequences.

On the power of a team

One player of size m can implement all m-periodic sequences.

Two players of size m can implement almost all $Cm \ln m$-periodic sequences.

On the power of a team

One player of size m can implement all m-periodic sequences.

Two players of size m can implement almost all $Cm \ln m$-periodic sequences.

More than two players cannot implement a large set of sequences of significantly larger period (or they could obtain v^c against a player of the same size as theirs).

Correlated strategies 1

Correlated strategies 1

We derive results from two player games.

Correlated strategies 1

We derive results from two player games.

From Ben Porath (93): If $\ln m^3 \ll m$ then

$$V^c(m, m, m^3) \to v^c$$

Correlated strategies 1

We derive results from two player games.

From Ben Porath (93): If $\ln m^3 \ll m$ then

$$V^c(m, m, m^3) \to v^c$$

Furthermore, the same limit obtains when players $1, 2$ use oblivious strategies only.

Correlated strategies 1

We derive results from two player games.

From Ben Porath (93): If $\ln m^3 \ll m$ then

$$V^c(m, m, m^3) \to v^c$$

Furthermore, the same limit obtains when players $1, 2$ use oblivious strategies only.
Over a period, each initial state of an automaton of player 3 can force a set of bounded probability of sequences to a significantly smaller payoff than $\mathrm{E}_\delta g - \varepsilon$.

Correlated strategies 1

We derive results from two player games.

From Ben Porath (93): If $\ln m^3 \ll m$ then

$$V^c(m, m, m^3) \to v^c$$

Furthermore, the same limit obtains when players $1, 2$ use oblivious strategies only.
Over a period, each initial state of an automaton of player 3 can force a set of bounded probability of sequences to a significantly smaller payoff than $\mathrm{E}_\delta g - \varepsilon$. The asymptotic condition on m^3 and n is that this probability times the number m^3 of states for 3 goes to 0.

Correlated strategies improved

Correlated strategies improved

Since two players of size m can implement a large set of sequences of size $m \ln m$, applying the same method shows.

Correlated strategies improved

Since two players of size m can implement a large set of sequences of size $m \ln m$, applying the same method shows.

If $\ln m^3 \ll m \ln m$ then

$$V^c(m, m, m^3) \to v^c$$

Correlated strategies 2

Correlated strategies 2

From Neyman (97): With $\boldsymbol{K = \ln|X^1 \times X^2|}$, if $\boldsymbol{\ln m^3 \geq K m^1 m^2}$ then

$$\boldsymbol{V^c(m^1, m^2, m^3) \to v^p}$$

Correlated strategies 2

From Neyman (97): With $K = \ln|X^1 \times X^2|$, if $\ln m^3 \geq K m^1 m^2$ then

$$V^c(m^1, m^2, m^3) \to v^p$$

There is a (mixed) strategy of player 3 that eventually plays a best response to almost all sequences of actions of players 1 and 2.

Correlated strategies 2

From Neyman (97): With $K = \ln|X^1 \times X^2|$, if $\ln m^3 \geq K m^1 m^2$ then

$$V^c(m^1, m^2, m^3) \to v^p$$

There is a (mixed) strategy of player 3 that eventually plays a best response to almost all sequences of actions of players 1 and 2. This automaton is capable of finding which sequence of actions is implemented by players 1 and 2 with high probability.

Conjecture

Conjecture

There exists K such that, if $\ln m^3 \geq K m \ln m$ then

$$V^c(m, m, m^3) \to v^p$$

Conjecture

There exists K such that, if $\ln m^3 \geq K m \ln m$ then

$$V^c(m, m, m^3) \to v^p$$

Indeed, this size of m^3 is sufficient for beating all sequences of period $m \ln m$.