Introduction to BCP – MCF Example

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BCP: Branch-Cut-Price

- Software for branch-and-cut-and-price
- Parallel code
- LP solver : Clp, Cplex, Xpress, . . .
- Most flexible in COIN-OR
- Research code (no stand-alone executable)

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BCP code split into four directories: (see coin-Bcp/Bcp/src)

- include: all header files
- Tree Manager (TM): Maintain the LP associated with each node, manage cuts and variables
- Node level operations (LP): cutting, branching, heuristics, fixing, column generation
- Utilities (Member): code for interface between TM and LP, initialization



Tree Manager

Solver

• read data

Tree Manager

Solver

- read data
- pack module data

• unpack module data

• read data

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Tree Manager Folver Fread data Fread data

Tree Manager

Solver

• select node

Tree Manager

Solver

- select node
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• select node • pack node LP data • unpack node LP data • solve • generate cuts/vars • branch • create LP data for sons

Solver Tree Manager select node • pack node LP data • unpack node LP data solve • generate cuts/vars branch create LP data for sons pack node LP data for

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BCP Constraints/Variables

Types of Constraints/Variables:

- Core : present at all nodes
- Algorithmic : separation/generation algorithm
- Indexed: e.g. stored in a vector

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Representation: Constraints are stored as ranged constraints:

$$lb \le ax \le ub$$

with
$$lb = -DBL_MAX$$
 or $ub = DBL_MAX$ possible

Implementing a Column Generation Application

Member:

- Read input
- Implement variables

TM:

- Set up the LP at the root node
- display of a solution

LP:

- Test feasibility of a solution
- Column generation method
- Computation of a lower bound
- Branching decision

Col. Gen. Example: Multicommodity Flow (MCF-1)

- Directed graph G = (V, E)
- N commodities
- (s^i, t^i) : source-sink pair, i = 0, ..., N-1
- d^i : supply/demand vector for $s^i t^i \mathit{flow}, \ i = 0, \dots, N-1$

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For each arc $e \in E$:

- 0 : lower bound for total flow on arc
- u_e : finite upper bound for total flow on arc $(0 \le u_e)$
- w_e : unit cost $(0 \le w_e)$

MCF: ILP Formulation

Solution:

- f^i : $s^i t^i$ -flow with supply/demand vector d^i
- $\sum_{i} f_e^i \le u_e$ for all $e \in E$

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II P Formulation:

$$\min \sum_{i} w^{T} f^{i}$$

$$\sum_{i} f^{i} \le u \tag{1}$$

$$\sum_{e=(v,w)\in E} f_e^i - \sum_{e=(w,v)\in E} f_e^i = d_v^i \quad \forall v \in V, \forall i$$
 (2)

$$0 \le f^i \le u \qquad \forall i \tag{3}$$

$$f^i$$
 integral $\forall i$ (4)

MCF: Input data

Class MCF_data (see Member/MCF_data.hpp):

- arcs: vector of struct (tail, head, lb, ub, weight)
- commodities : vector of struct (source, sink, demand)
- numarcs
- numnodes
- numcommodities
- Setup by MCF_data::readDimacsFormat()

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Parameter MCF_AddDummySourceSinkArcs: Add numcommodities dummy arcs with large weight to ensure feasibility

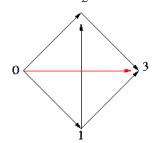
Master Problem:

- Column : $s^i t^i$ -flow satisfying d^i for some i
- F^i : matrix of all generated $s^i t^i$ -flows (+ dummy flow)
- λ^i : multiplier for generated $s^i t^i$ -flows

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Example: all arcs upper capacity 2, source = 0, sink = 3, d = 2.

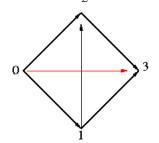


01	C
02	C
12	C
13	C
23	C
03	2

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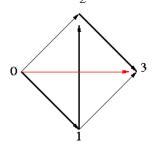


01	0	1
-	_	
02	0	1
12	0	1
13	0	0
23	0	1
03	2	0

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01	0	1	2
02	0	1	0
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13	0	0	0
23	0	1	2
03	2	0	0

$$\min \sum_{i} w^{T} F^{i} \lambda^{i}$$

$$\sum_{i} F^{i} \lambda^{i} \leq u$$
(5)

$$e^T \lambda^i = 1 \quad \forall i \tag{6}$$

$$\lambda^i \ge 0 \quad \forall i$$
 (7)

$$F^i \lambda^i$$
 integer $\forall i$ (8)

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$$\begin{bmatrix} F^0 & F^1 & F^2 \end{bmatrix} \begin{bmatrix} \lambda^0 \\ \lambda^1 \\ \lambda^2 \end{bmatrix} \le u$$

$$\begin{bmatrix} 1^T & . & . \\ . & 1^T & . \\ . & . & 1^T \end{bmatrix} \begin{bmatrix} \lambda^0 \\ \lambda^1 \\ \lambda^2 \end{bmatrix} = 1$$

$$\min \sum_{i} w^{T} F^{i} \lambda^{i}$$

$$\sum_{i} F^{i} \lambda^{i} \leq u \qquad (\pi)$$

$$e^{T} \lambda^{i} = 1 \quad \forall i \quad (\nu^{i})$$
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$$\lambda^i \ge 0 \quad \forall i$$
 (7)

$$F^i \lambda^i \text{ integer } \forall i$$
 (8)

Pricing of feasible $s^i t^i$ -flow f:

 $\begin{array}{ll} \text{weight of flow}: & \quad w^T f \\ \text{dual activity:} & \quad \pi^T f + \nu^i \end{array}$

Reduced cost of flow $f = w^T f - \pi^T f - \nu^i = (w^T - \pi^T) f - \nu^i$



Class MCF_vars

MCF_var:

- int commodity: index of commodity
- CoinPackedVector flow: positive flow on arcs
- weight: objective coefficient

See include/MCF_var.hpp, Member/MCF_var.cpp

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MCF_lp::vars_to_cols(): generate columns of the master
problem for vars

MCF: Setting the Master at the Root

Variables:

- Dummy flow variables are algorithmic variables $(\lambda_0^i \ \forall \ i)$
- All generated variables are algorithmic

See in TM/MCF_tm.cpp:

MCF_tm::initialize_core

MCF_tm::create_root

MCF: Setting the Master at the Root

Variables:

- Dummy flow variables are algorithmic variables $(\lambda_0^i \ \forall \ i)$
- All generated variables are algorithmic

Constraints:

- All constraints are core constraints
- Upper bound constraints: $0 \le u_e \ \forall \ e \in E$
- Dummy upper bound constraints: $dem(i)\lambda_0^i \leq dem(i) \ \forall i$
- Convexity constraints: $\lambda_0^i = 1 \ \forall \ i$

See in TM/MCF_tm.cpp:

MCF_tm::initialize_core

MCF_tm::create_root

Class MCF_tm: Derived from BCP_tm_user

Data:

MCF_data data

Methods:

- pack_module_data(): pack data needed at the node level.
 Called once for each processor used as a solver.
- initialize_core() : Transmit core constraints/variables to BCP.
- create_root : set up the problem at the root node
- pack_var_algo() : pack algorithmic vars
- unpack_var_algo(): unpack algorithmic vars
- display_feasible_solution(): display solution

Node operations

- 1. Initialize new node
- 2. Solve node LP
- 3. Test feasibility of node LP solution
- 4. Compute lower bound for node LP
- 5. Fathom node (if possible)
- 6. Perform fixing on vars
- 7. Update row effectiveness records
- 8. Generate cuts, Generate vars
- 9. Generate heuristic solution
- 10. Fathom node (if possible)
- 11. Decide to branch, fathom, or repeat loop
- 12. Add to node LP the cuts/vars generated, if loop is repeated
- 13. Purge cut pool, var pool



Class MCF_LP: Derived from BCP_lp_user

Data:

- OsiSolverInterface* cg_lp: pointer on Osi LP solver used for column generation
- MCF_data data: problem data
- vector<MCF_branch_decision>* branch_history:
 branch_history[i]: vector of branching decision involving commodity i (arc, 1b, ub)
- map<int,double>* flows: flows[i]: map between index
 of arc and positive flow for commodity i in LP solution
- BCP_vec<BCP_var*> gen_vars: vector holding generated vars
- bool generated_vars: indicator for success in column generation

See LP/MCF_lp.cpp, include/MCF_lp.hpp



Class MCF_LP: Derived from BCP_lp_user (cont)

Methods:

- unpack_module_data()
- pack_var_algo(), unpack_var_algo()
- initialize_new_search_tree_node(): Natural place for initializing user defined variables of MCF_lp.
- test_feasibility(): Test feasibility of current LP solution.
- compute_lower_bound(): Lower bound on optimal value of subproblem
- generate_vars_in_lp(): Pass new variables to BCP
- vars_to_cols(): Function generating a column from the var representation
- select_branching_candidates(): Generate rules for creating potential sons

MCF: Computing a Lower Bound

Initially, lower bound of a node is set to the lower bound of its father

- Try to generate a variable with negative reduced cost
- If successful, lower bound is currently known lower bound
- If unsuccessful, lower bound is the current LP value

See MCF_lp::compute_lower_bound() in LP/MCF_lp.cpp

• π, ν^i : optimal dual solution of the Master

(see MCF_lp::compute_lower_bound in LP/MCF_lp.cpp)

• π, ν^i : optimal dual solution of the Master

Column generation:

$$\sum_{e=(w,v)\in E} f_e^i - \sum_{e=(w,v)\in E} f_e^i = d_v^i \quad \forall v \in V$$
 (9)

$$\ell^i \le f^i \le u^i \tag{10}$$

$$f^i$$
 integral (11)

If solution is negative, then f^i is the new column

(see MCF_lp::compute_lower_bound in LP/MCF_lp.cpp)



• π, ν^i : optimal dual solution of the Master

Column generation:

$$\sum_{e=(w,v)\in E} f_e^i - \sum_{e=(w,v)\in E} f_e^i = d_v^i \quad \forall v \in V$$

$$(9)$$

$$\ell^i \le f^i \le u^i \tag{10}$$

$$f^i$$
 integral (11)

If solution is $\langle v^i \rangle$, then f^i is the new column

(see MCF_lp::compute_lower_bound in LP/MCF_lp.cpp)



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Column generation:

$$\sum_{e=(w,v)\in E} f_e^i - \sum_{e=(w,v)\in E} f_e^i = d_v^i \quad \forall v \in V$$
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If solution is $\langle v^i$, then f^i is the new column

Minimum cost flow problem ⇒ Solve as an LP (see MCF_lp::compute_lower_bound in LP/MCF_lp.cpp)



Branching

MCF_lp::select_branching_candidates(): Called at the end of each iteration. Possible return values are:

- BCP_DoNotBranch_Fathomed: fathomed without branching
- BCP_DoNotBranch : continue to work on this node
- BCP_DoBranch: Branching must be done. Must create the candidates

- Solution of the Master is fractional
- No new column is generated

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- ⇒ Must branch

Branching rule:

- Select an arc e (not dummy) and i with $F^i \lambda^i = z$ fractional
- First child: Use only columns where flow of i on e is > z
- Second child: Use only columns where flow of i on e is < z

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- Need to know ℓ_e^i and u_e^i for all i and e for col. gen.
 - ⇒ use branch_history[i]

Class MCF_branching_var

MCF_branching_var:

- artificial variable used to keep branching history around
- weight 0
- coefficients 0
- upper: 1, lower 0: identify child

See include/MCF_var.hpp, Member/MCF_var.cpp

Class MCF_branching_var

MCF_branching_var:

- artificial variable used to keep branching history around
- weight 0
- coefficients 0
- upper: 1, lower 0: identify child

Data:

- commodity: commodity i used in branching
- arc_index: arc e used in branching
- lb_child0, ub_child0, lb_child1, ub_child1: bounds for commodity i on e in the children

See include/MCF_var.hpp, Member/MCF_var.cpp

Branching object

Create the candidates using:
BCP_lp_branching_object::BCP_lp_branching_object()
Its relevant parameters are:

- int children: # children
- BCP_vec<BCP_var*> *new_vars : vector for new vars
- BCP_vec<BCP_cut*> *new_cuts : vector for new cuts
- BCP_vec<int> *fvp: vector for indices of variables whose bounds are changed
 Negative indices: vars from new_vars, index -i 1 corresponding to entry i
- BCP_vec<int> *fcp: vector for indices of cuts whose bounds are changed.



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 Negative indices: vars from new_vars, index -i 1 corresponding to entry i
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 Negative indices: cuts from new cuts



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Branching object (cont)

- BCP_vec<double> *fvb : vector for lower/upper bounds for each vars in fvp, for each child
- BCP_vec<double> *fcb : vector for lower/upper bounds for each constraint in fcp, for each child
- 4 additional parameters (implied parts)

Branching object (cont)MCF

- BCP_vec<double> *fvb: vector for lower/upper bounds for each vars in fvp, for each child
 [0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1]
- BCP_vec<double> *fcb : vector for lower/upper bounds for each constraint in fcp, for each child
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Branching object: Forced vs. Implied

Forced changes:

- Used during strong branching
- Sent to the tree manager if branching object is selected
- Used in the children if branching object is selected

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Many implied changes ⇒ storing them is costly. If implied changes are used, implement them also in MCF_lp::_initialize_new_search_tree_node()

MCF: Parameter File

Predefined parameters:
Class MCF_par (see include/MCF_par.hpp)
Class BCP_lp_par Class BCP_tm_par

MCF: Parameter File

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Class MCF_par (see include/MCF_par.hpp)
Class BCP_lp_par Class BCP_tm_par
```

Some parameters with their default values:

- MCF_AddDummySourceSinkArcs: 1
- MCF_InputFilename: small
- BCP_VerbosityShutUp: 0
- BCP_MaxRunTime: 3600
- BCP_Granularity: 1e-8
- BCP_IntegerTolerance: 1e-5
- BCP_TreeSearchStrategy: 1